4.1) Make a table listing the general solutions of all steady, uni-dimensional, constant properties, heat conduction problems in Cartesian, cylindrical, and spherical coordinates, with and without uniform heat generation. (This table should prove a very useful tool in future problem work. It should include 18 solutions, all told.)

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Solution w/o heat generation</th>
<th>Solution with heat generation</th>
</tr>
</thead>
<tbody>
<tr>
<td>cartesian</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x-dir.</td>
<td>( T = C_1x + C_2 )</td>
<td>( T = C_1x + C_2 - \frac{q}{2k} ) ( x^2 )</td>
</tr>
<tr>
<td>y-dir.</td>
<td>( T = C_1y + C_2 )</td>
<td>( T = C_1y + C_2 - \frac{q}{2k} ) ( y^2 )</td>
</tr>
<tr>
<td>z-dir.</td>
<td>( T = C_1z + C_2 )</td>
<td>( T = C_1z + C_2 - \frac{q}{2k} ) ( z^2 )</td>
</tr>
<tr>
<td>cylindrical</td>
<td></td>
<td></td>
</tr>
<tr>
<td>r-dir.</td>
<td>( T = C_1lnr + C_2 )</td>
<td>( T = C_1lnr + C_2 - \frac{q}{4k} ) ( r^2 )</td>
</tr>
<tr>
<td>( \theta )-dir.</td>
<td>( T = C_1\theta + C_2 )</td>
<td>( T = C_1\theta + C_2 - \frac{\hat{q}r^2}{2k} ) ( \theta^2 ) (where ( r ) is some constant value)</td>
</tr>
<tr>
<td>z-dir.</td>
<td>( T = C_1z + C_2 )</td>
<td>( T = C_1z + C_2 - \frac{q}{2k} ) ( z^2 )</td>
</tr>
<tr>
<td>spherical</td>
<td></td>
<td></td>
</tr>
<tr>
<td>r-dir.</td>
<td>( T = C_1 + \frac{C_2}{r} )</td>
<td>( T = C_1 + \frac{C_2}{r} - \frac{q}{6k} ) ( r^2 )</td>
</tr>
<tr>
<td>( \theta )-dir.</td>
<td>( T = C_1ln \tan \frac{\theta}{2} + C_2 )</td>
<td>( T = C_1ln \tan \frac{\theta}{2} + C_2 + \frac{\hat{q}r}{k} \ln \sin \theta )</td>
</tr>
<tr>
<td>( \phi )-dir.</td>
<td>( T = C_1\phi + C_2 )</td>
<td>( T = C_1\phi + C_1 - \left( \frac{q}{2k} \right) r^2 \sin^2 \theta ) ( \phi^2 )</td>
</tr>
</tbody>
</table>

Some of these solutions will have limited practical value. For example, the \( \theta \)-dir. solutions will be applicable only to thin cylindrical and spherical shells whose radius is virtually constant. This must also be the case for the \( \phi \)-dir. solution in spherical configurations, but it is also restricted to a narrow longitudinal swath.

4.2 Develop a dimensionless equation for the temperature in the wall shown:

General solution:

\[
\frac{d^2T}{dx^2} = -\frac{A}{k} (T - T_\infty)
\]

so

\[
T - T_\infty = C_1 \cos \frac{\sqrt{A/k} x}{x} + C_2 \sin \frac{\sqrt{A/k} x}{x}
\]

b.c.'s: LHS: \(-T_\infty = C_1\)

RHS: \(-\frac{2k}{\partial x} = \frac{h}{k} (T - T_\infty) \) or \( T_\infty \sin \frac{\sqrt{A/k} L - C_2 \cos \frac{\sqrt{A/k} L}{x}}{x} = \frac{h_{RHS}}{k} \)\( T_\infty \cos \frac{\sqrt{A/k} L}{x} \)

\( Bic \) + \( C_{es} \)\( A_k \)\( L \)
4.2 (continued)

\[ C_2 \left( B_e \sin \sqrt{\frac{A}{k}} L + \cos \sqrt{\frac{A}{k}} L \right) = B_e T_\infty \cos \sqrt{\frac{A}{k}} L - T_\infty \sin \sqrt{\frac{A}{k}} L \]

\[ C_2 = \frac{B_e T_\infty}{B_e \sin \sqrt{\frac{A}{k}} L + \cos \sqrt{\frac{A}{k}} L} \]

Thus:

\[ \frac{T_\infty - T}{T_\infty} = \cos \sqrt{\frac{A}{k}} x - \frac{B_e \cos \sqrt{\frac{A}{k}} L - \sin \sqrt{\frac{A}{k}} L}{B_e \sin \sqrt{\frac{A}{k}} L + \cos \sqrt{\frac{A}{k}} L} \sin \sqrt{\frac{A}{k}} x \]

Check the limit as \( A \to 0 \):

\[ \frac{T_\infty - T}{T_\infty} = 1 - \frac{B_e}{B_e \sqrt{\frac{A}{k}} L + 1} \sqrt{\frac{A}{k}} x = 1 - \left( 1 + \frac{1}{B_e \sqrt{\frac{A}{k}} L + 1} \right) \frac{h_{h_s L}}{1} \]

4.3) A long wide plate of known size, material, and thickness, \( L \), is connected across the terminals of a power supply and serves as a resistance heater. The voltage, current, and \( T_\infty \) are known. The plate is insulated on the bottom and transfers heat out the top by convection. The temperature, \( T_\infty \), of the bottom is measured with a thermocouple. Obtain expressions for a) temperature distribution in the plate, b) \( h \), at the top, c) temperature at the top. (Note that your answers must depend on known information, only.)

\[ \dot{q} = \frac{EI}{L B k} = B k \]

So:

\[ \frac{d^2 T}{dx^2} + B = 0 \]

General solution:

\[ T = C_1 \left( \frac{x}{L} \right) + C_2 - \frac{B L^2}{2} \left( \frac{x}{L} \right)^2 \]

b.c.'s: \( T(x = 0) = T_{tc} \), the thermocouple reading

\[ \left. \frac{dT}{dx} \right|_{x=0} = 0 \), since insulated.

(The b.c.'s are interesting in that both are at \( x = 0 \). We might have replaced the second one with:

\[ -k \frac{dT}{dx} \bigg|_{x=0} = \frac{EI}{B k} \]

Apply the b.c.'s:

\[ \begin{cases} T_{tc} = 0 + C_2 - 0; & C_2 = T_{tc} \\ 0 = C_1 - 0; & C_1 = 0 \end{cases} \]
4.3 (continued)

Therefore:

a.) \[ \frac{T - T_{tc}}{BL^2 / 2} = -\left(\frac{x}{L}\right)^2 \]

b.) \[ h = -\frac{k}{T_x = L - T_{\infty}} = \frac{EI/\text{bl}}{T_{\text{top}} - T_w}, \]

\[ c.) \quad T_{\text{top}} = T(x = L) = T_{tc} - \frac{BL^2}{2} \]

4.4, 4.5, 4.6 Write the dimensionless functional equation for each of the following situations.

4.4 Heat transfer to a fluid flowing over a plate of length, L.

\[ \bar{h} = \bar{h}(u_{\infty}, \mu, \rho, c_p, k, L) \]
\[ \frac{W}{m^2 \cdot ^\circ C} \quad \frac{m}{s} \quad \frac{kg}{m^3} \quad \frac{kg}{m^3 \cdot ^\circ C} \quad \frac{W}{m^2 \cdot ^\circ C} \quad \frac{m}{s} \]

7 var. in 4 dimensions \( \rightarrow \) 7-4 or 3 pi-groups. We choose:

\[ \frac{\bar{h}L}{k} = fn \left( \frac{\rho u_{\infty} L}{\mu}, \frac{\mu c_p}{k} \right) \text{ see eqn. (6.58) and others that follow it.} \]

4.5 Vapor condensing from a pipe. (Call the wavelength, \( \lambda \).)

\[ \lambda = \lambda([\rho_f - \rho g], \sigma, g) \]
\[ m \quad \frac{kg}{m^3} \quad \frac{N}{m} \quad \frac{m}{s^2} ; \text{ 4 var in 3 dim } \rightarrow 1 \text{ pi-group.} \]

\[ \frac{\sqrt{g(\rho_f - \rho g)}}{\sigma} = fn (\text{nothing else}) = \text{constant} \quad \text{see equation (9.6b)} \]

4.6 Velocity in a condensate film

\[ u \text{ m/s} = u(\gamma \text{ m, } g \text{ m/s}^2, \nu \text{ m}^2 / \text{s, } \xi \text{ m}) \]

so

\[ \frac{u}{\sqrt{g} \xi} = fn(\frac{\gamma}{\xi}, \frac{\nu}{\xi \sqrt{g} \xi}) \]

We find this situation described by eqn. (8.51) which takes this form when the vapor density is negligible.
4.7 Find the dimensionless temperature distribution in the cylindrical shell shown and plot it for \( r_i/r_o = 2/3 \). Establish criteria for neglecting convection and internal resistance.

General solution: \[ T = C_1 \ln \frac{r}{r_o} + C_2 - \frac{q}{4k} r^2 \]

if \( \Theta = \frac{T-T_{\infty}}{q/r_o^2/4k} \) \( \frac{q}{r_o} \) this becomes \[ \Theta = C_1 \ln \rho + C_2 - \rho \]

with b.c.'s: \[ \frac{dT}{dr} \bigg|_{r=r_o} = 0 \quad \text{or} \quad \frac{d\Theta}{d\rho} \bigg|_{\rho=1} = 0 \]

and: \[ \frac{3(T-T_{\infty})}{r} \bigg|_{r=r_i} = \frac{h_i}{k} (T-T_{\infty}) \quad \text{or} \quad \frac{d\Theta}{d\rho} \bigg|_{\rho=\rho_i} = B_i \Theta \quad \rho = \rho_i \]

impose the first b.c. on the gen'l. soln.: \[ C_1 = C_3 - 2 \quad C_2 = 2 \]

impose 2nd b.c.: \[ \frac{C_i^3}{\rho_i} - 2\rho_i = B_i (C_1 \ln \rho_i + C_2 - \rho_i) \]

so \[ C_4 = \frac{2}{B_i} \left( \frac{1}{\rho_i} - \frac{1}{\rho_i^2} \right) - 2 \ln \rho_i + \rho_i^2 - \rho_i \]

Return to the gen'l solution with these constants:

\[ \Theta = 2 \ln \rho + \frac{2}{B_i} \left( \frac{1}{\rho_i^2} - \frac{1}{\rho_i} \right) - 2 \ln \rho_i + \rho_i^2 - \rho_i \]

or

\[ \Theta = -\left( \rho_i^2 - \rho_i \right) + 2 \ln \rho_i + \frac{2}{B_i} \left( \frac{1}{\rho_i^2} - \frac{1}{\rho_i} \right) \]

Notice that for \( B_i \) large this approaches \( \Theta = \frac{\rho_i^2 \rho_i^2}{B_i} + 2 \ln \rho_i \)

and \( \rho_i \) small \( \Theta = \frac{2}{B_i} \left( \frac{1}{\rho_i} - \frac{1}{\rho_i^2} \right) \)
4.1 (continued)

For $\beta = 2/3$ the equation reduces to:

$$\Theta = -\left(\beta^2 - 0.9999\right) + 2\beta n_{15} + \frac{1.6667}{\beta}$$

When $\beta = 0.25$, the temperature distribution within the tube wall is within 4 percent of uniform (0.8 percent at $\beta = 0.1$, etc.)

When $\beta = 100$, the temperature drop through $h_i$ is 6 percent of that inside the tube wall (3 percent at $\beta = 200$, etc.)
4.8 Steam condenses in a small pipe keeping the inside at a temperature, $T_i$. The pipe releases $q$ W/m$^2$ within its walls as a result of electric current flowing through it. The outside temperature is $T_\infty$ and there is a heat transfer coefficient $h$ on the outside. a) Evaluate the dimensionless temperature distribution in the pipe. b) Plot the result for an inside radius that is 2/3 of the outside radius. c) Discuss interesting aspects of the result.

\[ T = T(r) \]

1. \[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) = -\frac{q}{k} \]
2. \[ T = -\frac{q}{4k} r^2 + C_1 \ln r + C_2 \]
3. \[ T(r = r_i) = T_i \]
4. \[ \ln \left( \frac{T - T_\infty}{T - T_\infty} \right)_{r = r_o} = -k \frac{\partial T}{\partial r} \bigg|_{r = r_o} \]

5. \[ C_2 = \frac{q}{4k} r_i^2 - C_1 \ln r_i \]

\[ -\frac{q}{4k} r_o^2 + C_1 \ln r_o + C_2 - T_\infty = \frac{k}{h} \left( \frac{q r_o}{2k} - \frac{C_i}{r_o} \right) \]

Combine:

\[ \frac{q}{4k} (r_i^2 - r_o^2) + C_1 \ln r_o/r_i + (T_i - T_\infty) = \frac{q r_o}{2k} - \frac{C_i}{r_o} \]

\[ \therefore C_1 = \frac{\frac{q}{4k}}{\Delta T} \left[ \frac{2}{\ln \frac{r_i}{r_o}} - \left( \frac{r_i}{r_o} \right)^2 \right] - \frac{1}{\Delta T} \ln \frac{r_i}{r_o} + \frac{1}{B_i} \]

6. \[ \frac{T - T_i}{T_i - T_\infty} = -\frac{q}{4k} \ln \left[ \frac{2}{\ln \frac{r_i}{r_o}} - \frac{1}{\ln \frac{r_i}{r_o}} \right] + \frac{1}{B_i} \]

\[ \frac{T - T_\infty + T_i}{T_i - T_\infty} = \frac{1}{4} \left( \frac{\rho^2 - \rho_i^2}{-\ln (\rho/\rho_i)} + \frac{\rho_i^2 - \rho^2}{-\ln (\rho_i/\rho)} \right) - \frac{\ln \rho / \rho_i}{\frac{1}{B_i} - \ln \rho_i} \]
4.8 (continued)

This can be rewritten as

\[
\Theta = 1 - \frac{\Gamma}{4} \left[ \rho^2 \varphi^2 + \frac{2}{\varphi^2} + 1 - \frac{\rho^2 \ln \rho}{\rho^2 - \varphi^2} \right] \ln \frac{\rho}{\rho_c} - \frac{\rho^2 - \varphi^2}{\rho^2 - \varphi^2} \frac{\rho^2}{\rho^2 - \varphi^2} \ln \frac{\rho}{\rho_c} \]

b) for \( \rho_c = \frac{2}{3} \) \( \equiv 13 : = 1 \), \( \Theta = 1 - \frac{\ln 3 \rho/2}{1.405} - \frac{\Gamma}{4} \left[ \rho^2 - \frac{4}{3} - 1.918 \ln \frac{3 \rho_c^2}{2} \right] \)

![Graphs showing \( \Theta \) as a function of \( \rho \) for different \( \Gamma \) values.](image)

We see that internal heat generation becomes important for \( \Gamma > O(1) \). For \( \Gamma \gg 2 \), that the temperature maximizes within the shell.
4.9 Solve Problem 2.5, putting it in dimensionless form first. With reference to the Problem 2.5 solution, we repeat the steps as follows.

Step 3) \( \Gamma = C_1 \ln r + C_2 \) becomes \( \frac{T - T_{in}}{T_{in} - T_{out}} \equiv \Theta = C_3 \ln \rho + C_4, \quad \rho = \frac{r}{r_c} \)

Step 1) \( \frac{\partial \Theta}{\partial \rho} \bigg|_{\rho=1} = B_i \frac{\partial \Theta}{\partial \rho} \bigg|_{\rho=1} \)
\( \frac{\partial \Theta}{\partial \rho} \bigg|_{\rho=0} = B_o \left( 1 - \Theta \right) / \rho \)

Step 5) \( C_3 = B_i C_4 \) or \( \frac{C_3}{\rho_o} = B_o \left( 1 - C_3 \ln \rho_o - C_4 \right) / \rho_o \)
\( C_3 = C_3 / B_i \) or \( C_3 = \frac{1}{\left( \frac{1}{B_i} + \frac{1}{B_i} + \ln \rho_o \right)} \)

Step 6) \( \Theta = \frac{\ln \rho + 1/B_i}{\frac{1}{B_o} + \frac{1}{B_i} + \ln \rho_o} \) Some result as in Prob. 2.5 with a lot less algebra

When we allow \( B_i \) and \( B_o \) \( \Rightarrow \infty \), \( \Theta \Rightarrow \frac{\ln r/r_o}{\ln r/r_i} \) E-24

4.10 Complete the algebra leading to equation (4.41).

we have: \( \frac{\partial (T-T_o)}{\partial x} = \frac{\partial (T-T_o)}{\partial x} \quad \text{or} \quad \frac{\partial \Theta}{\partial \xi} = (mL)^2 \Theta, \quad \text{so} \quad \Theta = C_1 e^{mL} + C_2 e^{-mL} \)

subject to: \( (T-T_o) \bigg|_{\xi=0} = (T-T_o) \quad \text{or} \quad \Theta(\xi=0) = 1 \), \( \text{so} \quad 1 = C_1 + C_2 \)

and to: \( \frac{\partial (T-T_o)}{\partial \xi} \bigg|_{\xi=L} = 0 \quad \text{or} \quad \frac{\partial \Theta}{\partial \xi} \bigg|_{\xi=L} = 0 \), \( \text{so} \quad 0 = C_1 e^{mL} - C_2 e^{-mL} \)

Now put \( C_1 = 1 - C_2 \) from 1st b.c., in 2nd b.c. i get: \( C_2 = \frac{e^{mL}}{e^{mL} - e^{-mL}} = \frac{1}{2} \cos mhL \)

Then: \( \Theta = \frac{2e^{mL} \cosh mL - e^{mL(1+\xi)} + e^{mL(1-\xi)}}{2 \cosh mL} = \frac{e^{mL(1+\xi)} + e^{mL(1-\xi)} e^{mL(1-\xi)} e^{mL(1+\xi)}}{2 \cosh mL} \)

So: \( \Theta = \cosh mL (1-\xi) / \cosh mL \)
Problem 4.11 Derive eqn. (4.48)

Solution

We already have the dimensionless form of the general solution of eqn. (4.30) in eqn. (4.35)

\[ \Theta = C_1 e^{mL\xi} + C_2 e^{-mL\xi} \]

and the dimensionless form of the b.c.s. (eqn. 4.31a) given in eqn. (4.46).

We put the solution in the two b.c.'s and get:

\[ \Theta(\xi=0) = 1 \Rightarrow 1 = C_1 + C_2 \quad \text{or} \quad C_1 = 1 - C_2 \]

\[ \frac{d\Theta}{d\xi}\bigg|_{\xi=1} = -Bi_{ax} \Theta(\xi=1) \Rightarrow mL e^{mL} C_1 - mL e^{-mL} C_2 = -Bi_{ax} (C_1 e^{mL} + C_2 e^{-mL}) \]

We put \( C_1 = 1 - C_2 \) in this, rearrange it, and get:

\[ C_2 = \frac{e^{mL} + \frac{Bi_{ax}}{mL} e^{mL}}{2 \left( \cosh mL + \frac{Bi_{ax}}{mL} \sinh mL \right)} \]

Put this \( C_2 \) in \( \Theta = (1-C_2) e^{mL\xi} + C_2 e^{-mL\xi} \) and get:

\[ \Theta = \frac{2 e^{mL\xi} (\cosh mL + \frac{Bi_{ax}}{mL} \sinh mL) - (e^{mL} + \frac{Bi_{ax}}{mL} e^{mL}) e^{mL\xi} + (e^{mL} + \frac{Bi_{ax}}{mL}) e^{-mL\xi}}{2 (\cosh mL + \frac{Bi_{ax}}{mL} \sinh mL)} \]

\[ = \frac{e^{mL(1+\xi)} + e^{-mL(1-\xi)} + \frac{Bi_{ax}}{mL} e^{mL(1+\xi)} - e^{mL(1+\xi)} - \frac{Bi_{ax}}{mL} e^{-mL(1+\xi)} + \frac{Bi_{ax}}{mL} e^{-mL(1+\xi)}}{2 (\cosh mL + \frac{Bi_{ax}}{mL} \sinh mL)} \]

\[ = \frac{e^{mL(1-\xi)} + \frac{Bi_{ax}}{mL} e^{mL(1-\xi)}}{2 (\cosh mL + \frac{Bi_{ax}}{mL} \sinh mL)} \]

\[ \Theta = \frac{1}{2} \left[ e^{mL(1-\xi)} + e^{mL(1-\xi)} \right] + \frac{1}{2} \frac{Bi_{ax}}{mL} \left[ e^{mL(1-\xi)} - e^{-mL(1-\xi)} \right] \]

or, finally:

\[ \Theta = \frac{\cosh mL(1-\xi) + \frac{Bi_{ax}}{mL} \sinh mL(1-\xi)}{\cosh mL + \frac{Bi_{ax}}{mL} \sinh mL(1-\xi)} \]
4.12 Obtain the infinite fin result: \( \Theta = e^{-mL \xi} \) by starting with the general fin solution: \( \Theta = C_1 e^{mL \xi} + C_2 e^{-mL \xi} \) and using the b.c.'s \( T(x=0) = T_0 \) or \( \Theta(\xi=0) = 1 \) and \( T(x=L) = T_{\infty} \) or \( \Theta(\xi=L) = 0 \). Discuss fully.

From the first b.c. we get \( C_1 = 1 - C_2 \) as before. From the second b.c. we obtain: \( 0 = C_1 e^{mL} + C_2 e^{-mL} \)

or \( 0 = e^{mL} - C_2 (e^{mL} - e^{-mL}) \)

\[ \Rightarrow 2 \sinh mL \]

or \( C_2 = \frac{e^{mL}}{2 \sinh mL} \)

Then the general solution becomes:

\[ \Theta = e^{-mL \xi} - C_2 (e^{mL \xi} - e^{-mL \xi}) = \frac{e^{mL \xi} (e^{mL} - e^{-mL}) - e^{mL (1+\xi)} + e^{mL (1-\xi)}}{2 \sinh mL} \]

\[ \Theta = \frac{e^{mL (1+\xi)} - e^{mL (1-\xi)} - e^{mL (1+\xi)} + e^{mL (1-\xi)}}{2 \sinh mL} \]

\[ \Theta = \frac{\sinh mL (1-\xi)}{\sinh mL} \]

When \( x \) is large, \( \sinh x = \frac{e^x - e^{-x}}{2} \rightarrow \frac{e^x}{2} \). Therefore, when \( mL \) is large

\[ \Theta \rightarrow \frac{e^{mL (1-\xi)}}{e^{mL}} = e^{-mL \xi} \quad \text{which is eqn. (4.50)} \]

It follows that these b.c.'s are not enough, by themselves, to get the infinite fin result. We must also require that the fin be very long.
4.13 How long must \( l \) be to guarantee an error less than than 0.5 percent in the thermometer well shown.

Find \( ml \)

\[
ml = \sqrt{\frac{\pi P l^2}{kA}} = \sqrt{\frac{k \pi d_i l^2}{4 k \pi (d_0 - d_i^2)}}
\]

\[
= \sqrt{\frac{300(0.02)}{17(0.0009 - 0.0000663)}} \quad l = 174 l
\]

As long as \( l > 0.0172 \text{ m} \), \( ml \) will be greater than 3 and we can use the "finite fin" approximation.

\[
\Theta = \frac{1}{\cosh ml} < 0.005 \quad \therefore e^{ml} + e^{-ml} = 400
\]

This is true for \( ml = 5.992 \).

Therefore \( 5.992 = 174 l \) so \( l = 0.03444 \text{ m} \)

This means that the well must only be 3.44 cm in length to guarantee the required accuracy.

4.14 What is the maximum possible heat flux from the fin shown.

\[
Q = \sqrt{kA h P (T_o - T_0)} \text{tanh} \frac{ml}{\text{max value for long } l \text{ is } l}
\]

\[
= \sqrt{50(0.0009)(600)2(0.02)(0.02)} (170-20) = 34.5 \text{ W}
\]
A thin rod is anchored at a wall at \( T = T_0 \) on one end, and is insulated at the other end. Plot the dimensionless temperature distribution in the rod as a function of dimensionless length:

a) if the rod is exposed to an environment at \( T_\infty \) through a heat transfer coefficient, and b) if the rod is insulated but heat is consumed in it at the uniform rate \( -q = \overline{h}P(T_0 - T_\infty) / A \). Comment on the implications of the comparison.

**Case a)**

We already know the solution. It is

\[
\Theta_a = \frac{\cosh mL(1-\xi)}{\cosh mL}
\]

**Case b)**

\[
\frac{d^2 \Theta}{d\xi^2} = (mL)^2 \text{ constant}
\]

so we integrate twice and get:

the general solution:

\[
\Theta = \frac{(mL)^2}{2} \xi^2 + C_1 \xi + C_2
\]

Apply 1st b.c.:

\( \Theta(\xi=0) = C_2 = 1 \)

Apply 2nd b.c.:

\[
\left. \frac{d^2 \Theta}{d\xi^2} \right|_{\xi=1} = (mL)^2 + C_1 = 0 \quad \text{and} \quad C_1 = -(mL)^2
\]

Therefore

\[
\Theta_b = (mL)^2 (\frac{\xi^2}{2} - \xi) + 1
\]

When \( \Theta \) is close to unity, \( \overline{h}P(T_0 - T_\infty) / A \approx \overline{h}P(T - T_\infty) / A \), or \((mL)^2 \Theta \approx (mL)^2\), and the problems (and their solutions) become identical. As \( \Theta \) becomes \( < 1 \) the solutions diverge. The energy consumption in case b is unabated and \( \Theta_b \) is generally \( < \Theta_a \).
4.16 Consider the tube shown below. Fluid enters the tube on the left at \( T = T_0 \). Assume its temperature to remain constant. Evaluate and plot the temperature distribution in the tube.

\[
\frac{d^2(T-T_0)}{dx^2} = \frac{q}{k} = \frac{\pi d_i h_i (T_0 - T)}{\pi \left( \frac{d_0^2 - d_i^2}{d_0^2} \right) k}
\]

or \( \frac{d^2 \Theta}{d\xi^2} = (mL)^2 \Theta \)

where \( \Theta = \frac{T-T_0}{T_r-T_0} \), \( m = \sqrt{\frac{4h_i}{kd_i \left( \frac{d_0^2}{d_0^2} - 1 \right)}} \)

General solution: \( \Theta = C_1 e^{mL\xi} + C_2 e^{-mL\xi} \)

Left boundary condition: \( \Theta(\xi=0) = 0 = C_1 + C_2 \) so \( C_1 = -C_2 \)

Right boundary condition: \( \Theta(\xi=1) = 1 = C_2 (e^{mL} - e^{-mL}) \) so \( C_2 = \frac{1}{2 \sinh mL} \)

Thus: \( \Theta = \frac{\sinh mL \xi}{\sinh mL} \) or we might better define \( \Phi = \frac{T - T_r}{T_0 - T_r} \)

So: \( \Phi = 1 - \Theta = 1 - \frac{\sinh mL \xi}{\sinh mL} \)

Notice that when \( m \) is small, the influence of convection is also small and the temperature distribution is almost linear as it would be in pure conduction. In the other extreme -- the convection dominated or large \( m \) case -- the temperature distribution remains near \( T_0 \), except as it approaches the right-hand wall.
4.17 Plot the temperature distribution in the fin shown and evaluate \( \eta_f \).

Eqn. (4.57) becomes:

\[
\frac{d}{dx} \left[ 2B(x^2) b \frac{d^2 T - T_\infty}{d x^2} \right] = \frac{h P L^2}{k} \frac{T - T_\infty}{T_\infty - T_\infty} \Rightarrow 0
\]

or:

\[
\frac{d^2 \Theta}{dx^2} + \frac{2 \Theta}{x} = \frac{h P L^2}{k (2 \Theta b)} \Rightarrow 0
\]

To solve this (Euler's d.e.) we look for a solution of the form: \( \Theta = C \xi^p \)

so:

\[
P (p-1) \xi^p + 2 P \xi^p - m (m + 1) \xi^p = 0 \quad \text{or} \quad P^2 + P - (m + 1) = 0
\]

This has two solutions:

\[ P_1 \quad \text{and} \quad P_2 = \pm \sqrt{1 + (m + 1)} - \frac{1}{2} \]

so the general solution is:

\[ \Theta = C_1 \xi^{P_1} + C_2 \xi^{P_2} \]

and the usual b.c.'s give:

\[ \Theta(x=1) = 1 = C_1 + C_2 \]

\[ \Theta(x=0) = 0 = [C_1 P_1 + C_2 P_2] \xi = 0 \]

Notice that \( P_2 \) must be negative so \( C_2 \) must be zero to satisfy this. Therefore \( C_2 = 0 \) and \( C_1 = 1 \), and:

\[ \Theta = \xi^{P_1} \]

The efficiency is:

\[ \eta_f = \int_0^1 \frac{\Theta(x)}{h(T_x - T_\infty)} b L \, dx = \int_0^1 \Theta(x) \, d\xi \]

\[ = \int_0^1 \xi^{P_1 - \frac{1}{2}} \, d\xi = \frac{1}{\frac{3}{4} + (m + 1)^2 - \frac{1}{2}} \left[ 1 \right] \]

so:

\[ \eta_f = \frac{2}{\sqrt{1 + 4 (m + 1)^2} + 1} \]

(cont'd.)
Before we plot these results, we note that mL in this case is the same as mL(L/P)^{1/2} in Fig. 4.13.

4.18 Problem 4.18 was solved under a full nondimensionalization in the solution given for 2.21. We do not repeat it here.

4.19 A fin connects two walls as shown. How much heat is removed from its surface?

\[ \Theta = C_1 e^{\frac{mL}{x}} + C_2 e^{-\frac{mL}{x}} \]

bc’s: \[ \Theta |_{x=0} = 1 = C_1 + C_2 \quad ; \quad C_1 = 1 - C_2 \]

\[ \Theta |_{x=\infty} = 0 = C_1 e^{\frac{mL}{x}} + C_2 e^{-\frac{mL}{x}} \]

Then: \[ \Theta |_{x=\infty} = (1-C_2) e^{\frac{mL}{x}} + C_2 e^{-\frac{mL}{x}} \quad ; \quad C_2 = \frac{e^{mL} - \Theta |_{x=\infty}}{2 \sinh mL} \quad ; \quad C_1 = 1 - \frac{e^{mL} - \Theta |_{x=\infty}}{2 \sinh mL} \]

So: \[ \Theta = \left(1 - \frac{e^{-\Theta |_{x=\infty}}}{2 \sinh mL}\right) e^{\frac{mL}{x}} + \frac{e^{mL} - \Theta |_{x=\infty}}{2 \sinh mL} e^{-\frac{mL}{x}} \]
4.19 (continued)

\[ Q = -kA \Delta T \left[ \frac{\partial \Theta}{\partial x} \bigg|_{x=0} - \frac{\partial \Theta}{\partial x} \bigg|_{x=L} \right] = -kwA\Delta T \left[ \left( -\frac{e^{-\Theta}}{2\sinh(mL)} \right) \left( 1 - e^{-mL} \right) + \frac{e^{-\Theta}}{2\sinh(mL)} \left( 1 - e^{-mL} \right) \right] \]

\[ = 390 \pi (0.00625)^2 (200 - 30) \left[ \frac{17(9)}{300(0.0125)} \right] \left[ \left( -\frac{305.5}{226.25} \right) + 0.5952 \right] \left( 1 - e^{-1.125} \right) \frac{0.9552 - 2.065}{0.6785} \]

\[ = 19.13 \text{ W} \]

4.20 How much error does the insulated tip assumption give rise to in example 4.8?

\[ \text{Calculate } \% \text{ error} = \frac{Q_{\text{ins}} - Q_{\text{ unins}}}{Q_{\text{ unins}}} \text{ using eqns. (4.44) and (4.48)} \]

\[ \% \text{ error} = \frac{\text{tanh} mL (1 + \frac{B_{\text{ins}}}{mL}) - \frac{B_{\text{ins}}}{mL}}{\frac{B_{\text{ins}}}{mL} + \text{tanh} mL} \times 100 \]

\[ = \frac{\frac{B_{\text{ins}}}{mL} (1 - \text{tanh}^2 mL - 1)}{\frac{B_{\text{ins}}}{mL} + \text{tanh} mL} \times 100 \]

From Example 4.8, \( mL = 0.8656 \), \( \frac{B_{\text{ins}}}{mL} = 0.0460 = 0.0541 \), so

\[ \text{the error} = 3.67 \% \]

4.21 Compute the heat removed from the fin shown, considering root depression. Assume the tip to be insulated.

\[ T_0 = 35 \degree C \]

\[ \text{MAGNESIUM, } k = 153 \]

\[ \text{d} = 0.006 \text{ m} \]

\[ h = 1.3 \]

\[ x = 0 \]

\[ x = 0.06 \text{ m} \]

\[ mL = \sqrt{130(\pi)(0.006)} \sqrt{153(\pi)(0.003)}(0.06) = 1.428 \]

\[ Q = \sqrt{kA \Delta T \text{tan} mL} = \sqrt{153 \pi^2(0.003)(0.006)(1.3)}(265)(0.8913) = 24.38 \text{ W} \]

The fin efficiency, \( \eta_f = \tanh(mL)/mL = 0.8913/1.428 = 0.624 = 62.4\% \)

The fin effectiveness, \( \epsilon = \eta_f (\text{fin surface area})/\text{fin cross-sectional area} \)

\[ \epsilon = 0.624(2\pi r L/\pi r^2) = 1.248 L/\pi = 25 \]

18
4.22 A 2 cm dia. horizontal 1.0% steel rod connects a block of ice with a block of dry ice (CO₂) in a 30°C room. The frozen blocks are insulated from the room. The rod is embedded in each block with a 20 cm span between the blocks. The heat transfer coefficient between the rod and the room is 10 W/m²K. Will the ice begin to melt when the rod is at steady state?

Solution We need to determine whether the temperature gradient in rod is positive or negative where it enters the ice on the left. If it is positive, heat will flow into the ice and it will begin to melt. Fortunately, we have already solved for the temperature distribution in a “fin” with specified temperatures at two ends, in Problem 4.19. We need only differentiate that expression for temperature, and determine whether the slope is positive or negative.

\[ mL = \sqrt{\frac{hPL^2}{kA}} \]

\[ \text{but } \frac{P}{A} = \frac{2\pi r}{r^2} = \frac{2}{r} \]

\[ mL = \sqrt{\frac{2hL^2}{k} r} = \sqrt{\frac{2(10)(0.2)}{43 (0.01)}} = 1.364 \]

We obtain from the solution of Problem 4.19:

\[ \Theta = \left(1 - \frac{e^{\frac{ML}{25\sinh (mL)}}}{2 \sinh (mL)}\right) e^{\frac{ML}{25\sinh (mL)}} + \left(\frac{e^{-\Theta_2}}{2 \sinh (mL)}\right) e^{-\Theta_2} \]

Where

\[ \Theta = \frac{T - T_{\infty}}{T_0 - T_{\infty}} = \frac{T - 303}{273 - 303} \]

so \( \Theta_1 = 1, \Theta_2 = 3.612 \)

Then

\[ \frac{d\Theta}{dx} \bigg|_{x=0} = m \left[1 - \frac{e^{\frac{ML}{25\sinh (1.364)}}}{\sinh (1.364)}\right] = 0.836m \]

The slope is thus positive & the ice will melt.
4.23 Compute the heat removed by the fin shown.

\[ m = \sqrt{\frac{hP}{kA}} \quad \text{where} \quad A = \frac{1}{2}(0.1)(0.02) = 0.001 \text{m}^2 \]

\[ mL^2/kA = \sqrt{\frac{hL^2}{kA}} = \frac{230(0.1)}{52(0.001)} = 2.1 \]

This gives an efficiency, from Fig. 4.13b, of:

\[ \eta_f = 0.415 \]

\[ Q = \eta_f \left( A \bar{h} [200 - 40] \right) = 0.415 \left( 2 - \sqrt{0.1^2 + 0.01^2} \right)(230)(160) = 3070 \text{ W} \]

4.24 The initial temperature distribution in a slab of width, \( L \), is:

\[ \frac{T - T_w}{qL^2/k} = \frac{1}{2} \left( \frac{\chi}{L} - \chi^2 \right) \quad \text{where} \quad qL^2/k \text{ can be viewed as a constant} A, \]

with the units of temperature. The sides are kept at \( T_w \) and the slab is permitted to cool. Predict \( (T - T_w)/A \) as a function of \( x \sqrt{t} \).

The maximum amplitude of the parabolic distribution is \( \frac{T - T_w}{A} = \frac{1}{8} \), so we approximate the initial distribution with \( \frac{T - T_w}{A} = \frac{\sin \pi x}{\pi} \).

The heat diffusion equation can be written as:

\[ \frac{d^2(T - T_w)}{dx^2} = \frac{1}{\alpha} \frac{d(T - T_w)}{dt} \]

so the general solution (eqn. 4.11) becomes:

\[ \frac{T - T_w}{A} = \left( 1 + \sin \left( \frac{\pi x}{L} \right) \right) e^{-\alpha L^2 t} \]

Then:

b.c. at \( x = 0 \): \[ \frac{T - T_w}{A} = 0 = (0 + E) e^{-\alpha L^2 t} \quad \text{so} \quad E = 0 \]

b.c. at \( x = L \): \[ \frac{T - T_w}{A} = 0 = (0 + E) e^{-\alpha L^2 t} \quad \text{so} \quad E = 0 \]

i.c.:

\[ \frac{T - T_w}{A} = \frac{\sin \pi (x/L)}{\pi} = D \sin \pi (x/L) \quad \text{so} \quad D = \frac{1}{8} \]

Thus:

\[ \frac{T - T_w}{A} = \frac{\sin \pi (x/L)}{8} e^{-\frac{\alpha L^2}{\pi^2} t} \]
4.25 A 1.5 m length of pipe is finned as shown. Find the rate at which steam at 10 atm, within the tube, will be condensed.

First evaluate the heat removal
\[ r_z/r_1 = 3.5/2 = 1.75 \]
\[ m_k \sqrt{\frac{k_l}{h}} = \sqrt{\frac{h_l}{k_h}} = \frac{G(0.0075)}{\sqrt{190(0.0008 \times 0.0075)}} = 0.04712 \]

So from Fig. 4.13a we read, \( \eta = 0.98 \)

Then:
\[ Q_{fin} = 0.98 \left[ \frac{2\pi}{4}(0.035^2 - 0.02^2)(180.5 - 18) \right] = 1.24 W \]

and:
\[ Q_{pipe} = Q_{fin} \left( \frac{1.5}{0.005} \right) + h_n A_{pipe} \left( \frac{0.50 - 0.09}{0.50} \right) \Delta T \]
\[ = 1.24 \times 300 + 6(\pi)(0.02)(1.5)(0.99)(180.5 - 18) \]
\[ = 448 W \]

The mass rate of condensate is \( m_{cond} = \frac{Q_{pipe}}{h_f g} \)

At 10 atm, \( h_f g = 2.013 \times 10^6 J/kg \) so

\[ m_{cond} = \frac{448 \times 1.5}{2.013 \times 10^6} = 0.000223 \frac{kg}{s} = 0.802 \frac{kg}{hr} \]

4.26 How long must the fin shown be, if:
\[ \frac{T - T_\infty}{T_0 - T_\infty} \bigg|_{x=L} = 0.2 \]

Using eqn. (4.95):
\[ 0.2 = \frac{1}{cosh} \sqrt{\frac{h_l P}{k_h}} L = \frac{1}{cosh} \sqrt{\frac{2h_l}{k_h}} L \]

or:
\[ S = \cosh \sqrt{\frac{2(28)}{3098(0.002)}} L \]

solving by trial and error we obtain \( L = 0.2734 m \)
4.27 A 2 cm ice cube sits on a shelf of aluminum rods, 3 mm in diam., in a refrigerator at 10°C. How rapidly, in mm/min, does the ice cube melt through the wires if \( h \) between the wires and the air is 10 W/m²°C. (Be careful that you understand the physical mechanism before you make the calculation.) Check your result experimentally. (\( h_{fs} = 333,300 \text{ J}/\text{kg.} \))

Solution. The rods act as infinite fins. Each carries heat off in both directions at a rate given by eqn. (4.42). This is balanced by the rate of melt:

\[ 2\sqrt{(kA)(h_P)(T_x - T_{sat})} = (2R)(2cm) \hat{i} \quad \rho_{ice} \ h_{fs} \]

where \( \hat{i} \) is the rate the rod advances. Thus

\[ \hat{i} = \frac{2\sqrt{200(0.0015)^2 10(2) m^2 (10-0)}}{2(0.0015)(0.02)(917)333,300} = 12.9 \times 10^{-6} \frac{m}{s} \]

\[ = 0.772 \text{ mm/min} \]

I did this in my refrigerator and found about 1 cm advance after 15 min. This gave 0.667 mm/min which is a reasonable comparison.

4.28 The highest heat flux that can be achieved in nucleate boiling (called \( q_{max} \) -- see the qualitative discussion in Section 9.1) depends upon: \( \rho_g \), the saturated vapor density; \( h_{fg} \), the latent heat of vaporization; \( \sigma \), the surface tension; a characteristic length, \( L \); and the gravity force per unit volume, \( g(\rho_f - \rho_g) \), where \( \rho_f \) is the saturated liquid density. Develop the dimensionless functional equation for \( q_{max} \) in terms of a dimensionless length.

\[ q_{max} = f_n(\frac{\rho_g}{\rho_f}, \ h_{fg}, \ \sigma, \ \frac{g(\rho_f - \rho_g)}{L}) \]

\[ \frac{J}{m^2 \cdot s} \quad \frac{k_g}{m^2} \quad \frac{J}{k_g} \quad \frac{k_g}{s^2} \quad \frac{k_g}{m^2 \cdot s^2} \quad m \]

There are 6 variables in 4 dimensions \( (J, m, k_g, s) \). This gives 2 \( \Pi \) groups. To find them we first eliminate \( J \) from the dimensional functional equation:

\[ \frac{q_{max}}{h_{fg}} = f_n \left( \frac{\rho_g}{\rho_f}, \ h_{fg}, \ \sigma, \ \frac{g(\rho_f - \rho_g)}{L} \right) \]

\[ \frac{k_g}{m^2 \cdot s} \quad \frac{k_g}{m^3} \quad \frac{J}{k_g} \quad \frac{k_g}{s^2} \quad \frac{k_g}{m^2 \cdot s^2} \quad m \]
4.28 (continued)

Next, get rid of \( m \):

\[
\frac{q_{\text{max}}}{h_\text{fg} g(p_f - p_g)} = f_n \left( \frac{\rho_\text{c}}{[g(p_f - p_g)]^{1/2}} \cdot \sigma \cdot \frac{1}{\sqrt{\text{g}(p_f - p_g)}} \right) \cdot \frac{\text{s}^2/\text{kg} \cdot \text{kg/s}^2 \cdot \text{kg/m}^2 \cdot \text{s}^{1/2}}{\text{s}}
\]

Get rid of \( \text{kg} \):

\[
\frac{q_{\text{max}}}{h_\text{fg} g(p_f - p_g)} = f_n \left( \frac{\rho_\text{c} \text{g}}{[g(p_f - p_g)]^{1/2}} \cdot \frac{1}{\text{s}^2} \right) \cdot \frac{\text{kg/s}^2}{\text{kg/s}}
\]

Finally we get rid of \( s \) and the term \( f \), and obtain the result:

\[
\frac{q_{\text{max}}}{(\rho_\text{c} g)^{1/2} h_\text{fg} \sqrt{\text{g}(p_f - p_g)}} = f_n \left( \frac{L}{\sqrt{\text{g}(p_f - p_g)}} \right)
\]

This is called the Kutateladze No. after S. S. Kutateladze, head of the heat transfer lab. in Novosibirsk, Siberia.

This is the square root of what is called a Bond No.

4.29

You want to rig a handle for a door in the wall of a furnace. The door is at 160°C. You consider bending a 16 in. length of 1/4 in. mild steel rod into a U-shape, and welding the ends to the door. Surrounding air at 24°C will cool the handle (\( \hat{h} = 12 \text{ W/m}^2 \cdot \text{°C} \)). What is the coolest temperature of the handle? How close to the door can you grasp it without being burned? How might you improve the handle?

This handle is like \( Z \), 8 in. (0.2032 m) long fins with insulated tips.

\[
(mL)^2 = \frac{\hat{h} \pi D L^2}{k A} = \frac{\hat{h} \pi D L^2}{k \pi D^2/4} = \frac{4(12)(0.2032)^2}{5 \pi (0.00635)} = 6.00
\]

\[
\Theta_{\text{tip}} = \frac{1}{\cosh mL} = \frac{1}{\cosh 2.45} = 0.1713, \quad T_{\text{tip}} = 24 + 0.1713(160 - 24) = T_{\text{coolest}} = 47.3^\circ \text{C}
\]

\[
\Theta_{\text{burn}} = \frac{65 - 24}{160 - 24} = 0.3015 = \frac{\cosh 2.45(1 - \epsilon)}{\cosh 2.45}
\]

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4.29 (continued)

\[ \cosh 2.45(1-\xi) = 1.76, \ 2.45(1-\xi) = 1.165, \ \xi = 0.5245 \]

Therefore if you touch this handle within 0.5245 inches of the door, you'll be burned.

To improve the design you need a far smaller rod diameter -- maybe a mere wire loop. Better still, weld on two short steel studs and connect them with a (low conductivity) piece of wood. The proposed design is not a good one.

4.30 A 14 cm long, 1 cm by 1 cm square brass rod is supplied with 25 W at its base. The other end is insulated. It is cooled by air at 20°C with \( h = 68 \text{ W/m}^2\cdot\text{°C} \). Develop a dimensionless expression for \( \Theta \) as a function of \( \xi \) and other known information. Calculate the base temperature.

We know that:

\[ \frac{d(T-T_\infty)}{d\xi} \bigg|_{\xi=0} = \frac{Q_{\text{base}}}{AK}, \quad \frac{d(T-T_\infty)}{d\xi} \bigg|_{\xi=1} = 0 \]

so:

\[ C_3 - C_4 = -\frac{Q_{bl}}{AKmL} \]

\[ C_3 e^{mL\xi} = C_4 e^{mL(1-\xi)} \]

so

\[ C_3 = C_4 e^{-2mL} = C_3 e^{-2mL} + \frac{Q_{bl}L}{Ak\xiL} \]

or

\[ C_3 = \frac{Q_{bl}L}{Ak\xiL} \frac{e^{-2mL}}{1-e^{-2mL}} = C_4 \frac{Q_{bl}L}{kAk\xiL} \]

\[ \frac{T-T_\infty}{Q_{bl}/kAkL} = \frac{e^{-mL\xi} + e^{-mL(1-\xi)}}{2\sinh mL} \]

\[ = \frac{e^{-mL(1-\xi)} - e^{-mL(1+\xi)} + e^{-mL(1-\xi)} + e^{-mL(1+\xi)}}{2\sinh mL} \]

\[ \Theta \equiv \frac{T-T_\infty}{Q_{bl}/kAkL} = \cosh mL(1-\xi) \]

\[ \frac{mL}{2 \sinh mL} \]

then:

\[ \frac{T_{\text{base}}-T_\infty}{Q_{bl}/kAkL} = \frac{1}{mL \tanh mL} \]

\[ T_{\text{base}} = T_\infty + \frac{Q_{bl}L}{kAkL \tanh mL} \]

so

\[ T_{\text{base}} = 20 + \frac{25(0.14)}{109(0.012) \sqrt{68.4 + 0.14 \tanh 2.2116}} \]

\[ = 20 + 148.7 = 168.7 °C \]

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4.31 A cylindrical fin has a constant imposed heat flux of \( q_1 \) at one end and \( q_2 \) at the other end, and it is cooled convectively along its length. Develop the dimensionless temperature distribution in the fin. Specialize this result for \( q_2 = 0 \) and \( L \to \infty \), and compare it with equation (4.50).

The general solution is
\[
T - T_\infty = C_1 e^{mL\xi} + C_2 e^{-mL\xi},
\]
with
\[
q_1 = -k \frac{d(T - T_\infty)}{dx} \bigg|_{\xi = 0} \quad \text{and} \quad q_2 = -k \frac{d(T - T_\infty)}{dx} \bigg|_{\xi = 1}
\]
or:
\[
\frac{q_1}{k} = -C_1 + C_2 \quad \text{and} \quad \frac{q_2}{k} = C_1 e^{mL} - C_2 e^{-mL}
\]
so
\[
C_2 = \frac{q_1}{k} + C_1 \quad \text{and} \quad \frac{q_2}{k} = -\frac{q_1}{k} e^{-mL} + C_1 z \sinh mL
\]
thus:
\[
C_1 = \frac{q_1}{k} \left( e^{-mL} + \frac{q_2}{q_1} \right) / 2 \sinh mL
\]

And we have:
\[
\frac{T - T_\infty}{q_1/k} = \frac{(e^{-mL} + \frac{q_2}{q_1}) e^{mL\xi} + 2 \sinh mL e^{-mL\xi} + (e^{-mL} + \frac{q_2}{q_1}) e^{-mL\xi}}{2 \sinh mL}
\]
so
\[
\frac{T - T_\infty}{q_1/k} = \frac{q_2}{q_1} \frac{\cosh mL\xi + e^{-mL(1-\xi)}}{\sinh mL} + \frac{\cosh mL(1-\xi)}{\sinh mL}
\]

for \( q_2 = 0 \), the insulated tip, \( \frac{T - T_\infty}{q_1/k} \frac{\cosh mL(1-\xi)}{\sinh mL} \), (which is the solution of Problem 4.30). As \( mL \to \infty \) this becomes:
\[
\frac{T - T_\infty}{q_1/k} = e^{-mL\xi}
\]

which is equation (4.50) with \( q_1/k \) serving in lieu of the characteristic temperature: \( T_0 - T_\infty \).
4.32 A thin metal cylinder of radius, \( r_0 \), serves as an electrical resistance heater. One axial line in one side is kept at \( T_1 \). Another line, \( \Theta_2 \) radians away, is kept at \( T_2 \). Develop a dimensionless expressions for the temperature distributions in the two sections.

\[
\frac{1}{r_0^2} \frac{d^2(T-T_1)}{d\Theta^2} = -\frac{q}{k} \quad \text{so} \quad T-T_1 = -\frac{q r_0^2}{2 k} \Theta^2 + C_1 \Theta + C_2
\]

b.c.'s: \( T-T_1 = 0 \) at \( \Theta = 0 \) so \( C_2 = 0 \)

\[
T-T_1 = T_2-T_1 \quad \text{at} \quad \Theta = \Theta_2 \quad \text{so} \quad C_1 = \frac{T_2-T_1}{\Theta_2} + \frac{q r_0^2}{2 k} \Theta_2
\]

Then:

\[
\frac{T-T_1}{T_2-T_1} = -\frac{q r_0^2}{2 k \Delta T} \Theta^2 + \frac{\Theta}{\Theta_2} + \frac{q r_0^2}{2 k \Delta T} \Theta_2 \Theta
\]

Call: \( \Theta = \frac{T-T_1}{T_2-T_1} \); \( \frac{q r_0^2}{2 k \Delta T} \Theta_2^2 = \Gamma \); \( \frac{\Theta}{\Theta_2} = \gamma \). Then

\[
\Theta = \sqrt{(\gamma-y)^2 + y^2} \quad \left( 0 < \gamma < \Theta_2 \right)
\]

For the other segment - \( \Theta = \Theta_2 - \Theta_\lambda \) at \( T-T_1 = T_2-T_1 \), so the solution becomes:

\[
\Theta = -\frac{q r_0^2 (\Theta_2 - \Theta_\lambda)^2}{2 k \Delta T} \beta^2 - \beta - \frac{q r_0^2 (\Theta_2 - \Theta_\lambda)^2}{2 k \Delta T} \beta
\]

where \( \beta = \Theta/([\Theta_2 - \Theta_\lambda]) \). so if we define \( \mathcal{B} = q r_0^2 (\Theta_2 - \Theta_\lambda)^2 / 2 k \Delta T \),

\[
\Theta = \mathcal{B} (\beta - \beta^2) - \beta \quad (\Theta_2 < \Theta < \Theta_\lambda)
\]
Heat transfer is augmented, in a particular heat exchanger, with a field of 0.007 m diameter fins protruding 0.02 m into a flow. The fins are arranged in a hexagonal array with a minimum spacing of 1.8 cm. The fins are bronze and $h_f$ around the fins is 168 W/m²°C. On the wall itself, $h_w$ is only 54 W/m²°C. Calculate $h_{eff}$ for the wall with its fins ($h_{eff} = Q_{wall}$ divided by $A_{wall}$ and $(T_{wall} - T_{∞})$.

In this case:

\[ mL = \sqrt{\frac{k \pi D}{4 \cdot 0^2}} \times L = \sqrt{\frac{168 \times 4}{(0.007)}} \times 0.02 = 1.215 \]

Next define $h_A = \frac{Q_{fin}}{A} = \frac{4}{T_{wall} - T_{∞}} = k(mL) \tanh(mL)/L = \frac{2\pi}{0.02} \times 1.215 \tanh 1.215 \times \frac{0.8382}{0.8382}$

\[ h_A = 1324 \text{ W/m}^2\text{-°C} \]

This characterizes heat removal where the fin replaces the wall.

$A_{triangle} = \frac{1}{2}(0.018)^2 \cos 60^\circ = 0.0001403 \text{ m}^2$

$A_{fin within Δ} = 3 \left[ \frac{1}{2} \left( \frac{1}{4} (0.007)^2 \right) \right] = 1.924 \times 10^{-5} \text{ m}^2$

\[ h_{eff} = \frac{1}{A_{Δ}} \left[ h_A \times A_{fin} + h_w \times (A_{Δ} - A_{fin}) \right] \]

\[ = \left[ 1324 \times 1.924 \times 10^{-5} + 54 (0.0001403 - 1.924 \times 10^{-5}) \right] / 0.0001403 \]

\[ h_{eff} = 228 \text{ W/m}^2\text{-°C} \]

The fins therefore yield a considerably improved heat removal.
4.34 An engineer seeks to study the effect of temperature on the curing of concrete by controlling the temperature of curing in the following way. A sample slab of thickness, $L$, is subjected to a heat flux, $q_w$, on one side, and it is cooled to temperature, $T_1$, on the other. Derive a dimensionless expression for the steady temperature in the slab. Plot the expression and offer a criterion for neglecting the internal heat generation in the slab.

General solution: $T - T_1 = - \frac{q_w L^2}{2k} \xi^2 + C_1 \xi + C_2$ where $\xi = \frac{x}{L}$

Boundary Conditions:

\[ -k \frac{dT}{d\xi} \bigg|_{\xi=0} = q_w = \left[ \frac{q_w L}{k} \xi - \frac{k C_1}{L} \right] \bigg|_{\xi=0} \]

So $C_1 = -q_w L/k$

\[ \left[ T - T_1 \right] \bigg|_{\xi=1} = 0 = -\frac{q_w L^2}{2k} - \frac{q_w L}{k} + C_2 \]

So $C_2 = \frac{q_w L^2}{2k} + \frac{q_w L}{k}$

So: $T - T_1 = -\frac{q_w L^2}{2k} \xi^2 + \frac{q_w L^2}{2k} - \frac{q_w L}{k} \xi + \frac{q_w L}{k}$

or: $\frac{T - T_1}{q_w L/k} = \Phi = \frac{q_w L}{2q_w L} \left( 1 - \xi^2 \right) + \left( -\xi \right)$

Call $\frac{q_w L}{2q_w L} = \Gamma$

When $\Gamma < 0.1$ we can neglect internal heat generation with only 10 percent error.
4.35 Develop the dimensionless temperature distribution in a spherical shell with the inside wall kept at one temperature, and the outside wall at a second temperature. Reduce your solution to the limiting cases in which \( r_{\text{outside}} \gg r_{\text{inside}} \) and in which \( r_{\text{outside}} \) is very close to \( r_{\text{inside}} \). Discuss these limits.

The general solution is:

\[
T = \frac{C_1}{r} + C_2 \quad \text{with b.c.'s} \quad T(r_i) = T_i, \quad T(r_o) = T_o
\]

Then:

\[
\begin{align*}
T_i &= \frac{C_1}{r_i} + C_2 \\
T_o &= \frac{C_1}{r_o} + C_2
\end{align*}
\]

\[
T_i - T_o = C_1 \left[ \frac{r_o - r_i}{r_o r_i} \right] \quad ; \quad C_1 = \frac{\Delta T r_o r_i}{\Delta r}
\]

\[
T_i = \frac{C_1}{r_i} + C_2 \quad ; \quad C_2 = T_i - \Delta T \frac{r_o}{\Delta r}
\]

So:

\[
T - T_i = \Delta T \frac{r_o}{\Delta r} \left[ \frac{r_i}{r} - 1 \right] \quad ; \quad \Theta = \frac{r_o - r_i}{r_o r_i} \left[ \frac{r - r_i}{r} \right]
\]

where \( \Theta = (T - T_o)/(T_i - T_o) \) and we have switched the signs to make everything positive. Then:

\[
\lim_{r_o \to r_i} \Theta = \frac{r - r_i}{r} \quad \text{which is the result for a semi-infinite region. (See e.g., the solution to 2.15)}
\]

\[
\begin{bmatrix}
\frac{T - T_o}{T_i - T_o} - \frac{T_i - T_o}{T_i - T_o} = \frac{r - r_i}{r} \quad \text{or as} \quad \frac{T - T_o}{T_i - T_o} = \frac{r_i}{r}
\end{bmatrix}
\]

And:

\[
\lim_{r_o \to r_i} \Theta = \frac{r_o}{r_o r_i} \left[ \frac{r - r_i}{r} \right] = \frac{r - r_i}{r_o - r_i} \quad \text{which is the result for a plane slab of thickness} \quad r_o - r_i
\]

4.36 Does the temperature distribution during steady heat transfer in an object, with b.c.'s of only the first kind, depend on \( k \)? Explain.

For such a problem we have: \( \nabla^2 T = 0 \) or \(-q/k\) and \( T(x = h) = T_1, \ T(x = h) = T_2, \) etc. Thus \( T_i = \Theta_i(T_i - T_1, \ x_1, \ h_1, \ h_2, \ q/k \). \) This gives 6 var. in \( \Theta, m, \) only so with \( q \) there are 6-2 or 4 \( \Theta \) groups:

\[
\frac{T_i - T_2}{T_2 - T_1} = \Theta_i \left( \frac{x_1}{h_1}, \ \frac{h_2}{h_1}, \ \frac{q}{k}, \ T_i - T_1 \right) \quad \text{\( k \) can only enter if \( q \) is}\]

in the problem also.
4.37 A long, 0.005 m diameter, duralumin rod is wrapped with an electrical resistor over 3 cm of its length. The resistor imparts a surface flux of 40 kW/m². Evaluate the temperature of the rod on either side of the heated section, if \( h = 150 \text{ W/m}^2\cdot\text{°C} \), and \( T_{\text{ambient}} = 27^\circ\text{C} \).

\[ Q \text{ to either side} = \frac{1}{2} (0.03 [\pi (0.005)]) 40,000 = 9.42 \text{ W} \]

\( q_{\text{to}} \) at the base of the rod is

\[ Q = \frac{q_{\text{to}}}{A} = \frac{9.42}{(0.03) (0.005)^2} = 480 \text{ kW/m}^2 \]

From equation (4.51) we have:

\[ q_{\text{to}} = k \frac{m \Delta T}{d} \]

\( m = \sqrt{\frac{4k}{k \Delta T}} = \sqrt{\frac{A (150)}{164 (0.005)}} = 27.05 \text{ m}^{-1} \)

so \( \Delta T = \frac{4.8 (10)^5}{164 (23.86)} = 108.2^\circ\text{C} \)

Therefore the base temperature is \( T_{\text{to}} = 108.2 + 27 = 135.2^\circ\text{C} \)

4.38 The heat transfer coefficient between a cool surface and a saturated vapor, when the vapor condenses in a film on the surface, depends on: the liquid density and specific heat, the temperature difference, the buoyant force per unit volume \( (g [\rho_f - \rho_g]) \), the latent heat, the liquid conductivity and kinematic viscosity, and the position \( (x) \) on the cooler. Develop the dimensionless functional equation for \( h \).

\[ h = h (\rho_f, c_p_f, \frac{(T_{\text{sat}} - T_w)}{\Delta T}, h_c g, g (\rho_f - \rho_g), k, 2f, x) \]

\[ \frac{J}{\text{m}^2\cdot\text{s} \cdot \text{°C}}, \frac{\text{kg}}{\text{m}^3}, \frac{J}{\text{kg} \cdot \text{°C}}, \frac{J}{\text{kg}}, \frac{\text{kg}}{\text{s} \cdot \text{m}^2}, \frac{J}{\text{m}^2 \cdot \text{s}^2 \cdot \text{g}}, \frac{\text{m}^2}{\text{s}} \]

We have 9 variables in 5 dimensions \( (J, m, s, °C, kg) \)

This gives 4 TT-groups. The method in the text will yield in the correct sequence -- give:

\[ Nu_x = f_n (TT, Pr, Ja) \text{ where: } Pr = \frac{\mu c_p}{k}, Ja = \frac{c_p \Delta T}{h c_g} \]

\[ Nu_x = \frac{h x}{k}, \text{ or } \frac{\mu h}{k} \frac{x^3}{\rho_f c_p} \]

(Of course other combinations are also acceptable. See details in Section 8.5.)
4.39 A duralumin pipe through a cold room has a 4 cm ID and a 5 cm OD. It carries water which sometimes sits stationary. It is proposed to put electric heating rings around the pipe to protect against freezing during cold periods of -7°C. The heat transfer coefficient outside the pipe is 9W/m²·°C. Neglect the presence of the water in the conduction calculation, and determine how far apart the heaters would have to be if they brought the pipe temperature to 40°C, locally. How much heat do they require?

\[
\text{Find } m: \quad m = \sqrt{\frac{hP}{kA}} = \left(\frac{9 \pi (0.05)}{164 \pi^2 (0.05^2 - 0.04^2)}\right)^{1/2} = 3.49
\]

\[
\Theta = \frac{O - (-7)}{4C - (-7)} = 0.149 = \frac{1}{\cosh ml} \text{ at the midpoint}
\]

so, by trial and error, \( ml = 2.592 \) or \( l = 0.743 \text{ m} \)

Thus the heaters must be spaced every \( \underline{1.486 \text{ m}} \)

and:

\[
\frac{Q}{2} = \sqrt{164 \pi^2 (0.05^2 - 0.04^2)} \cdot 9 \pi (0.05) (-7) \cdot \tanh 2.592 = 18.82 \text{ W}
\]

For heat flow both left & right, \( Q = 2(18.82) = \underline{37.64} \)
4.40 Evaluate \( \frac{d\tanh x}{dx} \).

\[
\frac{d\tanh x}{dx} = \frac{d}{dx} \left( \frac{e^x - e^{-x}}{e^x + e^{-x}} \right) = \frac{e^x (e^x + e^{-x}) - e^{-x} (e^x - e^{-x})}{(e^x + e^{-x})^2} = \frac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{(e^x + e^{-x})^2} = \frac{4}{(e^x + e^{-x})^2} = \frac{1}{(\cosh x)^2}
\]

4.41 The specific entropy of an ideal gas depends on its specific heat at constant pressure, its temperature and pressure, the ideal gas constant and reference values of the temperature and pressure. Obtain the dimensionless functional equation for the specific entropy and compare it with the known equation.

\[
S = S \left( \frac{C_p}{k_B}, T, \text{ref}, \gamma, \frac{p}{p_{\text{ref}}} \right)
\]

7 var in \( \frac{J}{kg}, ^oC, \frac{N}{m^2} \) ⇒ 7-3 or 4 \( \pi \)-groups

Thus:

\[
\frac{S}{R} = \sum_n \left( \frac{C_p}{k_B}, \frac{T}{T_{\text{ref}}}, \frac{p}{p_{\text{ref}}} \right)
\]

This is in the form of the known result:

\[
\frac{S}{R} = \frac{C_p}{R} \ln \frac{T}{T_{\text{ref}}} - \ln \frac{p}{p_{\text{ref}}}
\]

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5.1 A body at $T_b = T_i$ with $Bi << 1$ is immersed in a bath at $t = 0$. If $T_{bath} = T_i + bt$, plot $T_{body} = f(t)$ for the case in which $T_{bi} < T_i - bT$.

As in Example 5.1, the general solution is given by eqn. (5.13)

$$T_b - T_i = C_1e^{-t/T} + b(t - T)$$

and $C_1 = T_{bi} - T_i + bT$ so the particular solution is

$$T_b = T_i + (t - T) + (T_{bi} - T_i + bT)e^{-t/T}$$

5.2 A body of known volume and temperature, initially at $T_i$, is suddenly immersed in a bath for which

$$T_{bath} = T_i + (T_o - T_i)e^{-t/\tau}$$

where $\tau = 10T$. Plot $T_{body}$ from $t = 0$ to $t = 2\tau = 20T$.

$$\frac{dT}{dt} = \frac{T - T_o}{T_i} \quad \text{or} \quad \frac{d(T - T_i)}{dt} = -\frac{T - T_i}{T} + (T_o - T_i)e^{t/\tau}$$

The gen'l. soln. of the homo. eqn. is $T - T_i = C_1e^{-t/T}$ and the particular soln. of the complete eqn. might be found by substituting $T - T_i = Ae^{t/\tau} + Be^{-t/\tau}$ in the d.e. and adjusting $A \neq B$ to satisfy it. We get $B > 0$ and $A = (T_o - T_i)(e^{t/\tau})$ so

$$\frac{T - T_i}{T_o - T_i} = e^{t/\tau} + \frac{C_1e^{-t/T}}{e^{1/\tau}}$$

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5.2 (continued)

The i.e., \( (T-T_c)_{t=0} = 0 \), gives \( C_1 = -\frac{T_o - T_c}{T} + 1 \)

\[
\frac{T-T_c}{T_0-T_c} = \frac{t}{\tau} \left( e^{\frac{t}{\tau}} - e^{-\frac{t}{\tau}} \right)
\]

\[
\frac{T-T_c}{T_0-T_c}_{\text{long time}} = \frac{t}{\tau} e^{\frac{t}{\tau}}
\]

5.3 A body of known volume and area is immersed in a bath whose temperature varies \( T_\infty = T_{\text{mean}} + A \sin \omega t \). Find the steady periodic response of the body if its Biot number is small.

Define: \( \Theta = \frac{T-T_m}{A} \), \( T = \rho c V/\text{ha} \), \( \tau = t/T \), \( \Delta T = \omega T \). Then the d.e.

\[
\frac{dT}{dt} = -\frac{T-T_m}{\tau} + A \sin \tau t \]

becomes

\[
\frac{d\Theta}{dT} + \Theta = \sin \omega t
\]

The general sol'n of the homogeneous eq'n is: \( \Theta = C_1 e^{-\tau} \). The particular sol'n of the complete eq'n can be found by trying \( \Theta = C_2 \cos \Delta t + C_3 \sin \Delta t \) in the complete eq'n. This gives

\[-\Delta C_2 \sin \Delta t + \Delta C_3 \cos \Delta t + C_2 \cos \Delta t + \Delta C_3 \sin \Delta t = \sin \omega t\]

or

\[-\Delta C_2 + C_3 + 1 \sin \Delta t + (\Delta C_3 + C_2) \cos \Delta t = 0\]

This will be true if \( C_2 = -\Delta C_3 \) and \( C_3 = \frac{1}{\Delta^2 + 1} \). Then the
5.3 (continued)

Particular solution of the complete equation is

\[ \Theta = C_1 e^{-t} \frac{\partial}{\partial t^2} \cos \omega t + \frac{i}{\partial t^2} \sin \omega t \]

or

\[ \Theta = C_1 e^{-t} - \frac{1}{\partial t^2+1} \left[ \partial \cos \omega t - \sin \omega t \right] \]

At time \( t=0 \), \( \Theta = \Theta_0 = C_1 \frac{\partial}{\partial t^2} \) so \( C_1 = \Theta_0 \frac{\partial}{\partial t^2} \), where \( \Theta_0 \) might be anything, between 0 and 1, we might wish to

specify. Thus

\[ \Theta = \Theta_0 e^{-t} \frac{1}{\partial t^2+1} \left[ \partial \cos \omega t - \sin \omega t - \partial e^{-t} \right] \]

After a long time (\( t > 3T \) or \( t > 3 \)) this reduces to the steady periodic solution:

\[ \Theta \Rightarrow -\frac{1}{\partial t^2+1} \left( \partial \cos \omega t - \sin \omega t \right) \]

Now use the trigonometric identity

\[ A \cos x + B \sin x = \sqrt{A^2 + B^2} \sin \left(x + \cos^{-1} \frac{1}{\sqrt{A^2 + B^2}} \right) \]

In this case:

\[ \frac{\partial \cos \omega t - \sin \omega t}{\partial t^2+1} = \sqrt{\partial t^2+1} \sin \left(\omega t - \cos^{-1} \frac{1}{\sqrt{\partial t^2+1}} \right) \]

we identify \( \beta \) the phase lag angle. Then

\[ \Theta_{\text{periodic}} = -\frac{1}{\partial t^2+1} \sin \left(\omega t - \beta \right) \]

0 < amplitude < 1 \quad 0 < \beta < 90^\circ

Suppose, for example, that \( \omega T = \theta = \frac{\pi}{2} \). Then \( \beta = \cos^{-1} 0.707 \approx 45^\circ \)
or \( \pi/4 \) radians and the amplitude is 0.707.
5.3 (continued)

\[
\begin{align*}
\text{body temperature} & \quad \text{bath temperature} \\
\text{Dimensionless time, } t/T & = \tau \\
\text{Notice that } \Theta_{\text{periodic}} = f(\tau \text{ and } \Omega). \quad \text{When } \Omega \text{ (or } \omega T) \text{ is large, the process can be regarded as slow and } \Theta = \Theta_{\text{bath}} = \sin(\Omega t). \quad \text{When } \Omega \text{ is small the process is rapid, } \Phi \approx 90^\circ \text{ and the amplitude of the response goes to zero. In a rapid oscillation } \Theta \Rightarrow 0.
\end{align*}
\]

5.4 A copper block of volume, \( V \), floats in mercury contacting it over an area, \( A_c \), and exchanging heat with it by convection. The mercury container also exchanges heat with the mercury itself by convection. The entire system is initially in equilibrium at temperature, \( T_i \).

Predict the response of the copper if the container temp. is suddenly raised to \( T_s \) and if \( B_{\text{Cu}} \) and \( B_{\text{Hg}} \ll 1 \).

For the copper:
\[
\rho c V_c \frac{dT_c}{dt} = (h A_c)(T_m - T_c)
\]
\[\text{or } \frac{dT_c}{dt} = \frac{T_m - T_c}{T_c} \tag{1}\]

For the mercury:
\[
\frac{dT_m}{dt} = \frac{T_s - T_m}{T_m} \tag{2}
\]

This is exactly the second-order lumped capacitance problem solved in the text. The solution is eqn. (5.23) which we paraphrase as follows:

\[
\frac{T_c - T_s}{T_c - T_i} = \frac{b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 - c} e^{\left(\frac{b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 - c}\right)t} + \frac{-b + \sqrt{\left(\frac{b}{2}\right)^2 - c}}{2} e^{\left(\frac{-b}{2} - \sqrt{\left(\frac{b}{2}\right)^2 - c}\right)t}
\]

where \( b = \frac{1}{T_c} + \frac{1}{T_s} + \frac{h_c}{h_a T_m} \) and \( c = \frac{1}{T_c T_m} \).

This can be rewritten as
\[
\frac{T_c - T_s}{T_c - T_i} = A_1 e^{a_1 t} + A_2 e^{a_2 t}
\]

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5.4 (continued)

at \( t = 0 \), \( \frac{T_c - T_i}{T_c - T_i} = \frac{A_1 + A_2}{2\sqrt{(h_1) - c}} = 1 \) \( \therefore T_c = T_i \)

at \( t = 0 \), \( T_m = T_c \) so from eqn. (1) \( \frac{dT_c}{dt} = 0 = (T_c - T_m)(A_1a_1 + A_2a_2) \)

Thus: \( \frac{A_1}{A_2} = -\frac{a_2}{a_1} \). We see that both sides equal \( \frac{\sqrt{b + \sqrt{c}}}{\sqrt{b - \sqrt{c}}} \) so the

second bc. is also satisfied. Finally, we expect \( T_c \rightarrow T_m \)

or \( \frac{T_c - T_m}{T_c - T_i} \rightarrow 0 \) as \( t \rightarrow \infty \). This means that \( a_1 \) & \( a_2 \) must both

be negative. \( a_2 \) obviously is. \( a_1 \) is also negative because \( b \) is a positive number greater than \( c \).

5.5 Sketch the electrical circuit that is analogous to the second order lumped capacity system shown in Fig. 5.5.

![Electrical Circuit Diagram]

To see that this is valid we write the nodal equations

for nodes A and B as an E.E. might.

**node A**

\[
(pcv)_2 \frac{dT_c}{dt} + \frac{T_2 - T_1}{1/hcA} + \frac{T_c - T_\infty}{1/hA} = 0
\]

**node B**

\[
(pcv)_1 \frac{dT_i}{dt} + \frac{T_i - T_2}{1/hcA} = 0
\]

These equations are identical to equations (5.16) and (5.15), respectively, so the circuit is correct.
5.6 Plot $\overline{h}$ vs. $(T_{\text{sph}} - T_{\text{sat}})$ for the sphere quench in the figure with the problem in the text.

\[ \frac{dU}{dt} = \rho c V \frac{d(T_{\text{sph}})}{dt} = \overline{h} A (T_{\text{sph}} - T_{\text{sat}}); \quad \overline{h} = \frac{\rho c R}{3} \frac{dT_{\text{sph}}}{dt} \]

\[ \rho c \frac{R}{3} = 8934(389) \cdot \frac{0.0254}{2 \times 3} J \frac{0.0009418 \text{ Btu}}{\text{ft}^2} \frac{0.39414 \text{ in}^2}{\text{ft}^2} \frac{\text{OK}}{f + 2} \frac{1.8 \text{ F}}{1.8 \text{ F}} = 0.712 \text{ Btu/ft}^2 \text{hr} \text{F} \]

Then, scaling points from the figure:

<table>
<thead>
<tr>
<th>$(T_{\text{sph}} - T_{\text{sat}})$</th>
<th>$\frac{dT_{\text{sat}}}{dt}$</th>
<th>$\overline{h}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>855 - 211</td>
<td>320/40</td>
<td>51.8</td>
</tr>
<tr>
<td>750 - 211</td>
<td>370/40</td>
<td>35.9</td>
</tr>
<tr>
<td>711 - 211</td>
<td>375/40</td>
<td>35.2</td>
</tr>
<tr>
<td>611 - 211</td>
<td>405/40</td>
<td>64.0</td>
</tr>
<tr>
<td>511 - 211</td>
<td>620/25</td>
<td>222.1</td>
</tr>
<tr>
<td>411 - 211</td>
<td>893/18.5</td>
<td>584</td>
</tr>
<tr>
<td>311 - 211</td>
<td>843/11.2</td>
<td>1920</td>
</tr>
<tr>
<td>30</td>
<td>843/8.2</td>
<td>6157</td>
</tr>
<tr>
<td>10</td>
<td>943/11.2</td>
<td>1897</td>
</tr>
<tr>
<td>5</td>
<td>550/40</td>
<td>3525</td>
</tr>
<tr>
<td></td>
<td>40/40</td>
<td>512</td>
</tr>
</tbody>
</table>

$\overline{h}$ at the peak value of $\overline{h}$ is:

\[ Bi = \frac{9000 \text{ Btu/ft}^2 \text{hr} \text{F}}{3 \cdot 225 \text{ Btu/ft}^2 \text{hr} \text{F}} \]

\[ = 0.43 \] This is somewhat high but only at the peak.

5.7 The temperature of a butt-welded 36 gage (0.127 mm diam.) thermocouple in a gas flow rises at 20 degC/s, and stays 2.4 degC below the gas flow temperature. Find $h$ between the wire and the gas if $\rho c = 3800 \text{ kJ/m}^3 \text{degC}$.

\[ T = \frac{\rho c V}{h A} = \frac{\rho c R}{2h} = \frac{3.8(10)^6(0.000127)}{4 h} = \frac{120.7}{h} \]

but

\[ \frac{dT_W}{dt} = 20 = \frac{T_g - T_W}{T} = 2.4 - \frac{120.7}{h}; \quad h = \frac{1006}{W \text{ m}^2 \text{degC}} \]
Predict the temperature at the point $x_0 = 0.2$, $Bi = 10$ or $Bi^{-1} = 0.1$, and $x/L = 0$, and compare it with the graphical value in Fig. 5.7.

To do this we use eqn. (5.34) with $\langle \lambda L \rangle$ values generated by eqn. (5.35): $\text{ctn} \langle \lambda L \rangle = \lambda L/Bi = 0.1 \langle \lambda L \rangle$. By trial and error we get: $\langle \lambda L \rangle_1 = 1.42887$, $\langle \lambda L \rangle_2 = 4.30580$, $\langle \lambda L \rangle_3 = 7.22811$, etc. Using these numbers in eqn. (5.34) we get:

$$\Theta = e^{-1.42887^2(0.2)} \frac{2 \sin(1.42887) \cos(1.42887)}{1.42887 + \sin(1.42887) \cos(1.42887)} + e^{-4.3058^2(0.2)} \frac{2 \sin(4.3058) \cos 0}{4.3058 + \sin(4.3058) \cos(4.3058)} + \cdots$$

$\Theta = 0.8339 - 0.00065 + O(e^{-7.23^2(0.2)}) \approx 0.8293$

From Fig. 7 we read $\Theta \approx 0.82$ or 0.83 so the results agree within the accuracy with which we can read the graphs.

Prove that when $Bi$ is large, and the b.c. of the 3rd kind therefore reduces to a b.c. of the 1st kind, eqn. (5.34) reduces to (5.33)

The eigenvalue eqn. (5.35) becomes $\text{ctn} \lambda L = 0$ or $\tan \lambda L = \infty$, so $\lambda L = \frac{n \pi}{2}$, $3 \frac{n \pi}{2}$, ..., $n \frac{n \pi}{2}$ where $n$ is odd. Therefore eqn. (5.34) becomes:

$$\Theta = \sum_{n=\text{odd}}^{\infty} e^{-\left(\frac{n \pi}{2}\right)^2 \Phi_0} \frac{2 \sin \frac{n \pi}{2} \cos \frac{n \pi}{2} \left(\zeta - 1\right)}{\frac{n \pi}{2} + \sin \frac{n \pi}{2} \cos \frac{n \pi}{2} \left(\zeta - 1\right)}$$

$$\Theta = \frac{4}{\pi} \sum_{n=\text{odd}}^{\infty} e^{-\left(\frac{n \pi}{2}\right)^2 \Phi_0} \frac{\sin \frac{n \pi}{2} \xi}{n}$$

(5.33)
5.10 Check the point \( B_1 = 0.1, F_O = 2.5 \) on the graph for slabs in Fig. 5.10.

First we go to eqn. (5.35) ctn \( \lambda L = \frac{\lambda L}{0.1} \) and get \( \lambda L = 0.31105, 3.1731, \text{ etc.} \), by trial and error. Then we put eqn. (5.34) in eqn. (5.36) and get:

\[
\phi = + \int_{0}^{F_O} \frac{(\lambda L)^2 F_O}{\lambda L + \sin \lambda L \cos \lambda L} \sin \lambda L (\xi - 1) dF_O = -e^{-(\lambda L)^2 F_O - 1} \frac{2(\sin^2 \lambda L)}{\lambda L + \sin \lambda L \cos \lambda L}
\]

\[
\phi = \frac{1 - e^{-0.31105^2(2.5)}}{0.31105} \frac{2 \sin^2(0.31105)}{.31105 + \sin(.31105)\cos(.31105)} + \frac{1 - e^{-3.173^2(2.5)}}{3.173} \frac{2 \sin^2(3.173)}{3.173 + \sin(.31105)\cos(.31105)} + \ldots
\]

or

\[
\phi = 0.2148 - 0.0002 + \ldots = 0.2146
\]

From Fig. 5.10 we read \( \phi = 0.22 \), which agrees within graphical accuracy.
5.11 Show, in Fig. 5.7, 5.8, or 5.9, where b.c.s of the third kind may be replaced with b.c.s of the first kind, where we can assume lumped capacity, and where the solid may be seen as semi-infinite.

**Solution** We choose the chart for a point midway between the center and surface of a sphere.

This region will effectively be semi-infinite as long as the change of surface temperature does not penetrate all the way to the center. That will be true for very low values of \( F_0 \).

![Graph showing temperature distribution](image)

- We may treat situation as lumped capacity in this region of low Biot numbers (less than 0.1).
- We may treat a situation as a b.c. of the first kind in this region of high Biot numbers (greater than 50 or so.).
5.12 A ribbon is heated by a.c. as shown:

How much does its temperature fluctuate?

\[
\begin{align*}
\text{Bi} & = \frac{\bar{h}S}{k} = \frac{2000(0.00005)}{13} = 0.00769 \\
\psi & = \frac{\partial^2 \xi}{\partial \alpha^2} = \frac{(20.6)(0.00005)^2}{0.34(10)^5} = 0.2772
\end{align*}
\]

From Fig. 5.11 we then read \( \frac{T_{\max} - T_{avg}}{T_{avg} - T_0} \approx 0.014 \)

so the temperature fluctuation is just a little over one percent of the average temperature difference between the wall and the stream.

5.13 Resolve eqn. (5.58) into appropriate dimensionless groups.

In this case: \( R = R(k, \Delta T, \rho_f h_f, \rho C_p, t) \). Thus there are 4 basic variables in \( T, m, k, g, \sigma_c \) so we look for two \( \Pi \)-groups:

\[
\frac{R}{\sqrt{\alpha t}} = f(m) \left( \frac{\rho C_p \Delta T}{\rho g h_f} \right), \text{ a modified } \text{Ja}
\]

The eqn. (5.52) can be rearranged as:

\[
\frac{R}{\sqrt{\alpha t}} = \frac{2}{\text{Re}} \text{Ja}_{\text{mod}}
\]

which confirms the dim. analysis.

5.14 The water column shown is initially at 102°C. Then it is suddenly depressurized to 1 atm.

a) When will the temperature reach 101.95 at the bottom?

b) Plot the height of the column vs. time, up to this time.

a) \( \Theta = \frac{101.95-100}{102-100} = 0.975 \) and \( \frac{k}{h_f} = \frac{k/L}{\alpha} = 0 \)

from Fig 5.7, \( F_0 = 0.06 \frac{\sigma f}{k} \); \( t = \frac{0.06(0.07)^2}{1.69 \times 10^{-3}} \)

b) \[ \int_0^t q \, dt = \frac{k\Delta T}{\rho C_p} \int_0^t \frac{dt}{\sqrt{t}} = \frac{2k\Delta T}{\rho C_p} \sqrt{t} \]

where we have used eqn. (5.48) for \( q \)

reduction in height \( \Delta h = \left( \frac{\int_0^t q \, dt}{\beta h_f \frac{d}{m^2}} \right) \sqrt{\frac{t}{\rho C_p \alpha \sqrt{\alpha}}} = \frac{2k\Delta T}{\rho C_p \sigma f \sqrt{\alpha \sqrt{\alpha}}} = \frac{2(0.68)2}{961(2.26)10^6 \sqrt{17(1.48)10^{-3}}} \)

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5.14 (continued)
we therefore obtain:

$$\text{height} = (0.07 - 1.72(10)^{-6} \sqrt{t}) \, \text{mm}$$

5.15 A slab with $Bi = 2$ is cooled. Compare the exact and semi-infinite region solutions for $\Theta$, on the surface.

<table>
<thead>
<tr>
<th>$Fo$</th>
<th>$Bi$</th>
<th>$\Theta$ from Fig. 5.17</th>
<th>$\Theta$ from Fig. 5.16</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>$0$</td>
<td>$1$</td>
<td>$1$</td>
</tr>
<tr>
<td>$0.1$</td>
<td>$0.4$</td>
<td>$0.54$</td>
<td>$0.56$</td>
</tr>
<tr>
<td>$0.2$</td>
<td>$0.8$</td>
<td>$0.46$</td>
<td>$0.465$</td>
</tr>
<tr>
<td>$0.3$</td>
<td>$1.2$</td>
<td>$0.395$</td>
<td>$0.38$</td>
</tr>
<tr>
<td>$0.4$</td>
<td>$1.6$</td>
<td>$0.346$</td>
<td>$0.33$</td>
</tr>
<tr>
<td>$0.5$</td>
<td>$2.0$</td>
<td>$0.305$</td>
<td>$0.24$</td>
</tr>
<tr>
<td>$0.75$</td>
<td>$3.0$</td>
<td>$0.23$</td>
<td>$0.20$</td>
</tr>
<tr>
<td>$1.0$</td>
<td>$4.0$</td>
<td>$0.175$</td>
<td>$0.26$</td>
</tr>
<tr>
<td>$1.5$</td>
<td>$6.0$</td>
<td>$0.098$</td>
<td>$0.218$</td>
</tr>
</tbody>
</table>

Since the semi-infinite approximation does not reflect the influence of the insulated wall at $x/L = 0$, it eventually shows a slower cooling than the correct solution.

5.16 Derive eqn. (5.62) from:

$$\frac{d^2 \Theta}{dx^2} = \frac{d\Theta}{dx}$$

$\Theta(\xi = 0) = \cos \Omega$

$\Theta(\xi = \infty) = \text{finite}$

Assume:

$$\Theta = \alpha(\xi) e^{im}$$

so:

$$\frac{d^2 \Theta}{d\xi^2} = 2i(\xi)$$

hence:

$$f = C_1 e^{i\Omega \xi} + C_2 e^{-i\Omega \xi}$$

but:

$$\sqrt{2 i} = 1 + i$$

$$e^{im} = \cos m + i \sin m$$

so:

$$\Theta = (\cos \xi \cos \Omega + \sin \xi \sin \Omega)(C_3 e^{i\xi} + C_4 e^{-i\xi})$$

new forms.

To accommodate the second b.c. we must get $C_3 = 0$.

To accommodate the first b.c.:

$$\Theta(\xi = 0) = \cos(-\Omega)C_4 e^{-\Omega} = \cos \Omega$$

It follows that $C_4 = 1$ so we get:

$$\Theta = e^{-\xi} \cos(\xi - \Omega)$$

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5.17 A "steel" cylinder wall is 1 cm thick. (Take $\alpha = \frac{k}{\rho c} = \frac{32}{(7800)(473)} = 0.000008.67^*$/s.) The inside wall temp. is 
\(650 + 100\cos(\omega t)^\circ C\) and $\omega = 2\pi B = 50.26$ rad/sec.
Plot the envelope of the temperature disturbance in the wall.

$$\Theta = e^{-x} \cos(\pi - \xi)$$

so the envelope is given by $\Theta = \pm e^{-x}$ where $\xi = \sqrt{\frac{2\pi B}{k}} = 170.25\times$

and where: $\Theta = \frac{T - T_0}{\Delta T} = \frac{T - 650}{300}$, so $T_{\text{envelope}} = 650 \pm 300e^{-170.25(x,m)}$

It is clear that the outer wall is effectively undisturbed.

5.18 A 750°C, 0.4 m dia. pipe is buried in Portland cement
\(k = 17, \) (It is parallel to a 15°C surface and 1 m away from it. Plot T along a vertical line through the center of the pipe and compute the heat loss per meter of pipe.

\[ Q = k \Delta T S = 1.7(75-15)S \]

\[ = 102S \]

where: $S_{\text{graphical}} = \frac{N}{3} = \frac{8}{3} \]

\[ = \frac{2.67}{2.67} \]

\[ S_{\text{analytic}} = \frac{2\pi}{\cosh(y)}(S) = \frac{2\pi}{2.292} \]

\[ = 2.74 \]

\[ [\text{error} = \frac{2.74-2.67}{2.74} = 2.7\%] \]

\[ Q_{\text{graphical}} = 272 \text{ W/m} \]
\[ Q_{\text{analytic}} = 279.5 \text{ W/m} \]
5.18 (continued) The variation of temp. with depth obtained from the flux plot.

5.19 Obtain $S$ for a sphere buried in an infinite medium.

General solution for this case (cf. Example 5.10) is

\[ T = C_1 / r + C_2 \]

w/b.c.'s: \[ T(r=\infty) = T_{\infty}, \quad \text{so} \quad C_2 = T_{\infty} \] and \[ T(r=R) = T_w, \quad \text{so} \quad C_1 = (T_w - T_{\infty})R \]

Thus: \[ T - T_{\infty} = (T_w - T_{\infty}) \frac{R}{r} \] and \[ Q = 4\pi R^2 \left( -k \frac{\partial T}{\partial r} \right)_{r=R} = 4\pi R^2 k (T_w - T_{\infty}) \frac{1}{R} \]

It follows that \[ S = \frac{Q}{k\Delta T} = 4\pi R \]

5.20 Find $S$ for parallel cylinders in an infinite medium. One has twice the diameter of the other. The centers are one diameter of the larger cylinder, apart.

According to Fig 5.2, \[ m_1/R_1 = 1 - \frac{1}{3} = \frac{2}{3} \quad \text{and} \quad m_2/R_2 = 2 \left(1 - \frac{1}{2} + \frac{1}{2^2} \right) = \frac{13}{8} \]

Then \[ S_{\text{analytical}} = \frac{2\pi}{\cosh^{-1} \frac{13}{8}} = \frac{2\pi}{0.60 + 1.01} = 3.76 \]

And \[ S_{\text{flux plot}} = \frac{2(7.3)}{4} = 3.65 \]

Error \[ \frac{3.76 - 3.65}{3.76} = \frac{2.92}{3.76} \]

\[ = 0.25 \]
5.21 The 3 in. by 3 in. copper slab (1 in. thick) shown below conducts heat from the 1000°F surface on the left to the 1 in. portion of the right side which is kept at 900°F.

\[ Q = SL\Delta T = \frac{6}{3} \times 226 \frac{\text{Btu}}{\text{ft}^2 \cdot \text{hr} \cdot \text{°F}} \times (100 - 90)\text{°F} \]

\[ = 1507 \frac{\text{Btu}}{\text{ft}^2 \cdot \text{hr}} \times \frac{\text{ft}^2}{12} = 125.6 \frac{\text{Btu}}{\text{hr}} = 38.8 \text{ W} \]

5.22a Obtain the shape factor for the following shape.

\[ S = \frac{N}{4} = \frac{6}{4} = 1.67 \]
5.22b Obtain the shape factor for the configuration shown. Evaluate $T$ at point A.

$$S = \frac{N}{l} = \frac{3.35}{3.25} = 1.03$$

$$T_A = 200 + \frac{1.25}{3.25}(100)$$

$$T_A = 238.5^\circ F$$

5.22c Find $S$ for the inside of the form shown.

$$S = \frac{2(2)}{6.8} = 0.59$$

5.22d Find the shape factor for the region outside the same form.

$$S = \frac{N}{l} = \frac{2(2.4)}{8} = 0.6$$
5.22e Find S for the shape shown and the center temperature.

\[ S = \frac{N}{I} = \frac{0.4 + 3 + 0.6}{4.25} = 0.94 \]

\[ T_C = T_1 + \frac{2.75}{4.25} (T_2 - T_1) \]

\[ = T_1 + 0.65(T_2 - T_1) \]

5.22f Find S for the shape below, and evaluate T at point A.

\[ S = \frac{N}{I} = \frac{2(4)}{\infty} = 0 \]

because the thermal resistance is infinite in length.

\[ T_A = 0 + \frac{2}{5} (50 - 0) = 20^\circ C \]

5.22g Find S

Notice the symmetry:

N must equal I so \( S = 1 \)

(There are some interesting ramifications to this problem. See J. Heat Transfer, Vol. 103, No. 3, 1981, pp. 600-1.)
5.22h Find S

\[ S = \frac{N}{I} = \frac{4.6}{2} = 2.3 \]

5.22i Find S and \( T_A \)

\[ S = \frac{N}{I} = \frac{2}{2} = 1 \]

Thus the singular point of infinite resistance in the center blocks all heat flow from \( T_1 \) to \( T_2 \).

\( T_A = T_2 \) since all temperature drop occurs across the point of infinite resistance in the center.

5.22j Find S for the form shown

This form has 4 axes of symmetry. We therefore isolate the stippled area and do a flux plot for it. We get (see Prob. 5.22g)

\[ S_{\Delta} = 1.00 \]

Then for the total shape

\[ S_{\text{total}} = \frac{\text{twice as many channels}}{\text{twice as many isotherms}} = \frac{2}{2} \cdot S_{\Delta} = 1.00 \]
5.23 The two copper slabs shown are suddenly laid together as shown. Find the temperature of the left-hand adiabatic side after 2.3 sec elapses.

By symmetry, we see that the interface must immediately assume—and retain—the average temperature of 50°C.

\[ P_o = \frac{a t}{L^2} = \frac{11.57(10)^{-5}(2.3)}{0.03^2} = 0.296 \quad \text{and} \quad B_i^{-1} = \frac{k}{hL} = \frac{k}{L} = 0 \]

Then from 5.7 we read

\[ \frac{T_{\text{ad.wall}} - T_{\text{int.}}}{T_i - T_{\text{left}}} = 0.61 \]

so

\[ T_{\text{ad.wall}} = 0.61(100 - 50) + 50 = 80.5^\circ C \]
5.25 Prove that temperature cannot oscillate in a 2nd order lumped capacity system.

If the system is to oscillate \( \sqrt[2]{(\frac{\delta}{2})^2} \) in eqn. (5.23) must be imaginary or

\[
b^2 = \left( \frac{1}{T_1} + \frac{1}{T_2} + \frac{hc}{nT_2} \right)^2 < \frac{4}{T_1T_2}
\]

or:

\[
\sqrt{\frac{T_2}{T_1}} + \sqrt{\frac{T_1}{T_2}} (1 + \frac{hc}{n}) - 2 < 0
\]

If we call \( \frac{T_2}{T_1} = x \), then this says:

\[
x^2 - 2x + (1 + \frac{hc}{n}) < 0
\]

or:

\[
(x - 1)^2 + \frac{hc}{n} < 0
\]

But everything on the left must be positive so this cannot be! Therefore the system cannot oscillate.

5.26 When the isothermal and adiabatic lines in a flux plot are interchanged, \( N \) turns into \( I \) and \( I \) into \( N \). It follows that:

\[
S_{\text{interchanged}} = \frac{I}{N} = \frac{1}{S_{\text{original}}}
\]
5.28 Plot $T(x=0)$ as a function of time when the strip heater shown is turned on. What is $q$ at $x = 0.01$ m when $T(x=0) = 200^\circ C$?

From eqn. (5.50),

$$T_w(t) = T_0 + 2 \frac{Q_w}{k} \sqrt{\frac{\pi t}{x}} = 15 + \frac{8000}{0.1} \sqrt{\frac{5.4 \times 10^{-8}}{\pi}} \sqrt{t}$$

$$= 15 + 10.49 \sqrt{t}$$

$T_w = 200^\circ C$ when $t = 310.4$ sec.

Then from eqn. (5.49):

$$\frac{q_w - q(x,t)}{q_w} = \frac{1000 - q}{1000} = \text{erf} \frac{x}{2 \sqrt{kt}} = \text{erf} 1.22 = 0.9155$$

$$\Rightarrow q(x=0.01, t=310.4) = 338 \frac{W}{m^2}$$

5.29 There as many answers to this problem as there are students. (The most common error students will make is that of touching items in a room that are not at room temperature -- rings on fingers, window panes, ice cubes, etc.)
5.30 What is the maximum \( Q \) from the container shown if \( T_s \) cannot exceed 30°C

\[
Q = k \Delta T S = 2(30 - 0)S
\]

And from Table 5.2, No. 7,

\[
S = \frac{4\pi R}{1 - \frac{R}{2h}} = \frac{4\pi}{1 - \frac{1}{2(1)}} = 16.76
\]

Therefore the maximum \( Q \) is \( 2(30)16.76 = 1005 \text{ W} \)

5.31 A semi-infinite slab of ice at -10°C is exposed to air at 15°C through a heat transfer coefficient of 10 W/m²°C. What is the initial rate of melting in kg of water/m²-s? What is the asymptotic rate of melting? Describe the melting process in physical terms. (The latent heat of fusion of ice \( h_f s = 333,300 \text{ J/kg} \).)

**Solution.** The surface must first be brought up to the melting temperature. During this period \( m_{\text{melt}} = 0 \text{ kg/m}^2\text{-s} \).

Once the saturation temperature, 0°C, has been reached at the surface, heat will flow into the interior of the slab in accordance with equation (5.48) which shows that \( q \sim 1/\sqrt{t} \). Thus, after a long time, that portion of the heat reaching the interface, which flows to the interior, becomes negligible. Then a simple energy balance yields:

\[
\bar{h}(T_{\text{air}} - 0°C) = h_{fs} m_{\text{melt}}
\]

or:

\[
m_{\text{melt}} = \frac{10(15-0)/333,300}{0.00045} \text{ kg/m}^2\text{-s}
\]

5.32 One side of a firebrick wall, 10 cm thick, initially at 20°C is exposed to a 1000°C flame through a heat transfer coefficient of 230 W/m²°C. How long will it be before the other side is too hot to touch? (Estimate properties at 500°C).

**Solution.** \( k/\bar{h}L = 0.15/230(0.1) = 0.00652 \)

\[
\frac{T_{\text{burn}} - T_{\infty}}{T_i - T_{\phi}} = \frac{65 - 1000}{20 - 1000} = 0.954
\]

Then from Fig. 5.7, upper left, we get

\[
\frac{at}{L^2} = 0.075, \quad t = \frac{0.075(0.1)^2}{5.4 \times 10^{-8}} = 13,690 \text{ sec}
\]

\[= 3 \text{ hr } 51 \text{ min} \]
**Problem 5.33**  A lead bullet travels for 0.5 seconds within a shock wave that heats the air near the bullet to 300°C. Approximate the bullet as a cylinder 0.8 mm in diameter. What is its surface temperature at impact if \( h = 600 \text{ W/m}^2\text{K} \) and if the bullet was initially at 20°C? What is its center temperature?

**Solution**  The Biot number \( 600(0.004)/35 = 0.0685 \), so we can first try the lumped capacity approximation. See eqn. (1.22):

\[
\frac{T_{sfc} - 300}{20 - 300} = \exp\left(-\frac{t}{T}\right), \text{ where } T = \frac{mc}{hA}
\]

So \( T = \rho c(aREA)/h(circumf.) = 11,373(130)\pi(0.004)^2/h\pi(0.008) = 4.928 \text{ seconds} \)

And \( (T_{sfc} - 300)/(20 - 300) = \exp(-0.5/4.928) \).

So \( T_{sfc} = 300 - 0.903(280) = 47.0^\circ\text{C} \)

In accordance with the lumped capacity assumption, 

\( 47.0^\circ\text{C} \) is also the center temperature.

Now let us see what happens when we use the exact graphical solution, Fig. 5.8:

for \( Fo = \alpha t/r_o^2 = 2.34(10^{-5})(0.5)/0.004^2 = 0.731 \) and \( r/r_o = 1 \), we get:

\[
\frac{(T_{sfc} - 300)}{(20 - 300)} = 0.90, \quad \text{So } T_{sfc} = 48.0^\circ\text{C}
\]

And at \( r/r_o = 0, \quad \frac{(T_{ctr} - 300)}{(20 - 300)} = 0.92, \quad & \text{So } T_{ctr} = 42.4^\circ\text{C} \)

We thus have good agreement within the limitations of graph-reading accuracy. It also appears that the lumped capacity assumption is accurate within around 6 degrees in this situation.
5.34 A loaf of bread as shown, is at 125°C when it is removed from an oven and put to cool on an insulating counter. \( k = 0.05 \text{ W/m-°C} \) and \( 5\times10^{-7} \text{ m}^2/\text{s} \). \( h = 10 \text{ W/m-°C} \). When will the bottom center reach 60°C.

\[
\Theta = \frac{60 - 25}{125 - 25} = 0.35
\]

\[
\Theta_x \Theta_y \Theta_z
\]

Let us guess times, calculate \( \Theta_x, \Theta_y, \Theta_z \), and see how close to 0.35 we come.

\[
F_0 = \frac{\alpha t}{L^2}, \quad k/\delta L = 0.005/L
\]

<table>
<thead>
<tr>
<th>Guess</th>
<th>( t )</th>
<th>3600 sec</th>
<th>4000 sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_0 )</td>
<td>( k/\delta L )</td>
<td>( \Theta_x )</td>
<td>( \Theta_y )</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0333</td>
<td>0.92</td>
<td>0.90</td>
</tr>
<tr>
<td>0.556</td>
<td>0.38</td>
<td>0.089</td>
<td>0.95</td>
</tr>
</tbody>
</table>

It looks like about 3900 seconds or 1 hr and 5 min.
5.35 A lead cube, 50 cm on each side, is initially at 20°C. The surroundings are suddenly raised to 200°C and $h$ around the cube is 272 W/m²·°C. Plot the cube temperature along a line from the center to the middle of one face, after 20 minutes have elapsed.

\[ B_i = \left( \frac{h}{k} \right)^{-1} = \frac{34}{272(0.25)} = 0.5 \quad ; \quad F_o = \frac{\alpha t}{L^2} = \frac{2.35 \times 10^{-5} (20 \times 60)}{0.25^2} = 0.451 \]

Then:

\[ \Theta = \frac{T - T_0}{T_i - T_0} = 6 \left( B_i = 0.5, F_o = 0.451, \frac{x}{L} = 0 \right) \times 9 \left( B_i = 0.5, F_o = 0.451, \frac{x}{L} \right) \]

and:

\[ T = 200 - 88 \times 2 \times 0.70 = 138.3°C \quad \text{at} \quad \frac{x}{L} = 0 \]
\[ T = 200 - 88 \times 2 \times 0.67 = 140.9°C \quad \text{at} \quad \frac{x}{L} = 0.2 \]
\[ T = 200 - 88 \times 2 \times 0.63 = 144.4°C \quad \text{at} \quad \frac{x}{L} = 0.4 \]
\[ T = 200 - 88 \times 2 \times 0.55 = 151.5°C \quad \text{at} \quad \frac{x}{L} = 0.6 \]
\[ T = 200 - 88 \times 2 \times 0.46 = 159.4°C \quad \text{at} \quad \frac{x}{L} = 0.8 \]
\[ T = 200 - 88 \times 2 \times 0.32 = 171.8°C \quad \text{at} \quad \frac{x}{L} = 1.0 \]

Notice that the calculated points vary a bit owing to the error inherent in graph-reading.
A jet of clean water superheated to 150°C issues from a (1/16)in. diameter sharp-edged orifice into air at 1 atm., moving at 27 m/s.

The coefficient of contraction of the jet is 0.611. Evaporation at \( T = T_{\text{sat}} \) begins immediately the outside of the jet. Plot the center-line temperature of the jet, and \( T(r/r_0=0.6) \), as functions of distance from the orifice, up to about 5 m. Neglect any axial conduction and any dynamic interactions between the jet and the air.

Any element of the jet cools approximately as an infinite cylinder would, while it moves. Therefore we can use Fig. 5.8 with \( r_0/\theta_0 \) and \( r/r_0 = 0.6 \), and (since we have a b.c. of the first kind, \( T(r=r_0) = T_{\text{sat}} \)) \( \text{Bi} = 0 \). Then:

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
 t \text{ sec} & x = U_\text{jet} t & F_0 = \frac{\rho_\text{f} U_\text{jet}}{\rho_\text{a}} & \Theta_\infty \Theta_{r_0} & \Theta(\theta_0) & \Theta(\theta_0=0.6) \\
\hline
 0.01 & 0.27 & 0.0120 & 1.00 & 0.99 & 150°C & 148°C \\
 0.1 & 2.7 & 0.1201 & 0.80 & 0.44 & 140 & 122 \\
 0.15 & 4.05 & 0.1801 & 0.158 & 0.29 & 129 & 119.5 \\
 0.2 & 5.4 & 0.2402 & 0.42 & 0.20 & 121 & 110 \\
\hline
\end{array}
\]

where \( r_0 = \frac{1}{2}(0.0254/16)^{0.611} = 0.000 \)

\( U_\text{jet} = 27 \text{ m/s} \); \( T_\infty = T_{\text{sat}} = 100°C \); \( T_i = T_\theta = 150-100°C \); and \( \rho_\text{f} = 1.201 \text{ g} \text{c}^{-1} \text{ with } \rho_\text{a} = 150°C \);

\( \Theta_\infty = 2.018 \text{ g} \text{c}^{-1} \text{ evaluated at } T_\infty = 127°C = 400^\circ \text{F} \).

centerline{temp.
A 3 cm thick slab of aluminum (initially at 50°C) is slapped tightly against a 5 cm slab of copper (initially at 20°C). The outsides are both insulated and the contact resistance is negligible. What is the initial interfacial temperature? Estimate how long the interface will keep its initial temperature.

In accordance with equation (5.60), we get:

\[
\frac{T_i - T_c}{T_u - T_c} = \frac{k_a/\sqrt{\alpha_a}}{k_c/\sqrt{\alpha_c} + k_a/\sqrt{\alpha_a}}
\]

or

\[
\frac{T_i - 20}{50 - 20} = \frac{237/\sqrt{9.61(10)^{5}}}{392/\sqrt{11.57(10)^{5}} + 237/\sqrt{9.61(10)^{5}}} = 0.39
\]

The solution is \( T_i = 31.86°C \)

and this will be valid as long as the slabs behave as though they were semi-infinite regions. This will be as long as \( \mathcal{G} > 3.65 \) (See eqn. (5.45))

\[
\mathcal{G} = \frac{x}{\alpha t} \quad \text{so} \quad 3.65 = \mathcal{G}_a = \frac{0.02}{\sqrt{9.61(10)^{5}} t} \quad ; \quad t < 0.703 \text{ sec.}
\]

\[
3.65 = \mathcal{G}_{Cu} = \frac{0.03}{\sqrt{11.57(10)^{5}} t} \quad ; \quad t < 1.622 \text{ sec.}
\]

The shorter of the two times dictates how long it will take the first insulated wall to be felt. Consequently, the interface temperature of \( T_i = 31.86°C \) will remain constant for:

\[
\frac{0.703 \text{ sec}}{\text{(at least)}}
\]
5.38 A cylindrical underground gasoline tank, 2m in diameter and 4m long, is embedded in 10°C soil with \( k=0.8 \) W/m·°C and \( \alpha = 1.3(10)^{-6} \) m²/s. Water at 27°C is injected into the tank to test it for leaks. It is well-stirred with a submerged, (1/2)kW pump. We observe the water level in a 10cm ID transparent standpipe and measure its rate of rise or fall. What rate of change of height will occur after one hour if there is no leakage? Will the level rise or fall? Neglect thermal expansion and deformation of the tank which should be complete by the time the tank is filled. (Hint: see eqn. (8.7))

\[
\text{Area} = \frac{4}{3} \pi (2)^2 + 2 \pi (4) = 58.64 \text{ m}^2, \quad V = \frac{4}{3} \pi (2)^3 = 67.02 \text{ m}^3
\]

There are two energy transfers: 500 J/s of work to the water, and:

\[
Q = \frac{kA \Delta T}{\pi \alpha t} = \frac{0.8(58.64)(27-10)}{1 \pi (1.3)(10)^{-6}(3600)} = 6577 \frac{\text{J}}{\text{s}} \text{ of heat from the water.}
\]

Now, using eqn. (8.7),

\[
\beta = \frac{1}{V} \frac{\partial V}{\partial T} \bigg|_p = \frac{1}{V} \frac{\partial V}{\partial T} \bigg|_p, \quad \frac{\partial V}{\partial T} = 0.000275 \frac{67.02}{100} = 0.01843 \frac{\text{m}^3}{\text{s}}
\]

And:

\[
\frac{\partial t}{\partial T} = \frac{C_v V}{Q_{\text{net}}} = \frac{526.4(4177)(67.02)}{(500 - 6577)} = -45.909 \frac{\text{sec}}{\text{°C}}
\]

So

\[
\frac{\partial V}{\partial t} \bigg|_{T} = -\frac{0.01843}{45.909} = -0.401 \times 10^{-6} \frac{\text{m}^3}{\text{s}} \approx -0.001445 \frac{\text{m}^3}{\text{hr}}
\]

Then:

\[
\frac{\partial h}{\partial t} = \frac{d(V/A_{\text{pipe}})}{\partial t} = -\frac{0.001445}{\pi \left(0.1\right)^2} = -0.184 \frac{\text{m}}{\text{hr}} = -18.4 \frac{\text{cm}}{\text{hr}}
\]

Thus the tank is leak-free when the standpipe drops at the rather substantial rate of 18.4 cm/hr. The drop is the result of thermal contraction.
A 47°C copper cylinder, 3 cm in diameter, is suddenly immersed horizontally in water at 27°C. Plot \( T_{\text{cyl}} \) as a function of time if \( g = 0.76 \text{ m/s}^2 \), 
\[ H = [2.753 + 10.448(\Delta T°C)^{1/6}]^2 \text{ W/m}^2°C. \] (Do it numerically if you cannot integrate the resulting equation analytically.)

In this case: 
\[ \frac{d(T_{\text{cyl}} - T_w)}{dt} = \frac{-hA}{\rho c V} (T_{\text{cyl}} - T_w) \]
\[ \frac{\rho c D}{4} \frac{1}{\Delta T} \frac{d \Delta T}{dt} = -1 \]

so:
\[ \Delta T = \left( \frac{236.2 \frac{d \Delta T}{dt}}{\Delta T(0.2616 + \Delta T^{1/6})^2} \right) \]

\[ \int \Delta T \]

\[ \begin{array}{c|c|c}
\Delta T°C & I & \int \Delta T \text{sec} \\
\hline
20 & 3.29 & 0 \\
18 & 3.73 & 7.32 \\
16 & 4.32 & 15.44 \\
14 & 5.13 & 24.94 \\
12 & 6.25 & 36.44 \\
10 & 7.50 & 54.94 \\
8 & 10.51 & 72.64 \\
6 & 15.20 & 97.36 \\
4 & 25.50 & 136.4 \\
3 & 61.66 & 167.4 \\
\end{array} \]

Area between 20 & 14°C (at 2 sec/fin square) 
15 24.84 sec
5.40 The mechanical engineers at the University of Utah end Spring semester by roasting a pig and having a picnic. The pig is roughly cylindrical and about 26 cm in diameter. It is roasted over a propane flame, whose products have properties similar to those of air, at 280°C. The hot gas flows across the pig at about 2 m/s. If the meat is cooked when it reaches 95°C, and if it is to be served at 2:00 P.M., what time should cooking commence? Assume Bi to be large, but note Problem 7.40. The pig is initially at 25°C.

In this case, \( T_0 = \frac{95 - 280}{25 - 280} = 0.725 \), \( Bi = 0 \), so from Fig 5.8 we read: \( Fo = 0.13 \)

Using \( \alpha \) for beef we get:

\[
T = Fo \frac{\alpha^2}{\alpha} = 0.13 \frac{0.13^2}{1.35(10)^{-7}} = 16274 \text{ sec}
\]

So the pig must be cooked for 4 1/2 hours. Cooking should begin at about 9:30 A.M. in the morning. \( \text{If Bi is not large, this will be low.} \)

5.41 People from cold Northern climates know not to grasp metal with their bare hands in subzero weather. A very slightly frosted piece of, say, cast iron will stick to your hand like glue in, say, -20°C weather, and you can tear off patches of skin. Explain this quantitatively.

Equation (5.60) tells us what to expect for the interfacial temperature when we touch ice. \( (T_{\text{body}} = k_{\text{body}}, \alpha_{\text{body}} = \alpha_{\text{beef}}) \)

\[
\frac{T_{\text{sc}} - (-20)}{37 - (-20)} = \frac{0.6/1.35(10)^{-7}}{2.215/1.35(10)^{-6} + 1850} = 0.973 \quad \text{so} \quad T_{\text{sc}} = 7°C
\]

Thus, in immediate contact the ice (or frost) will melt to water (very quickly because it is thin.) Then:

\[
\frac{T_{\text{sc}} - (-20)}{0°C - (-20)} = \frac{2.215/1.35(10)^{-6}}{52/1.35(10)^{-5} + 2065} = 0.128 \quad \text{so} \quad T_{\text{sc}} = -17.4°C
\]

Thus the iron will immediately refreeze the water, causing it to "glue" your hand to the iron.
5.42 A 4 cm dia. No. 304 stainless steel rod has a very small hole down its center. The hole is clogged with wax that has a melting point of 60°C. The rod is at 20°C. In an attempt to free the hole, a workman swirls the end of the rod — and about a meter of its length — in a tank of water at 80°C. If $h$ is 688 W/m²°C on both the end and the sides of the rod, plot the depth of the melt front as a function of time, up to, say, 4 cm.

\[ \Theta(r=0, x) = \Theta(x) \cdot \Theta(r=0) \]

At melt front \( \Theta = \frac{40-80}{20-80} = 0.667 \)

The rod: \( \text{Bi}^{-1} = \frac{k}{h\tau_o} = \frac{13.8}{688(0.02)} = 1.00 \)

\[ F_0 = \frac{\alpha t}{\tau_o^2} = \frac{0.000004}{0.02^2} t = 0.01t \]

The semi-inf. region: \( \beta^2 = \frac{h^2 \alpha t}{k^2} = \frac{688^2(0.000004)}{13.8^2} \)

\[ \beta g = \frac{h x}{k} = \frac{688}{13.8} x = 49.86 x \]

Then,

<table>
<thead>
<tr>
<th>$t$ (s)</th>
<th>$F_{0\text{cy}l}$</th>
<th>$\Theta_{\text{cy}l}(F_{0\text{cy}l})$</th>
<th>$\Theta_{\text{semi-inf}}0.667$</th>
<th>$\beta^2 = 0.000934 t$</th>
<th>$\beta g$</th>
<th>$x$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.15</td>
<td>0.54</td>
<td>0.71</td>
<td>off-scale, Melt has not begun</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.20</td>
<td>0.88</td>
<td>0.76</td>
<td>0.20</td>
<td>0.20</td>
<td>0.004</td>
</tr>
<tr>
<td>30</td>
<td>0.30</td>
<td>0.72</td>
<td>0.844</td>
<td>0.32</td>
<td>0.32</td>
<td>0.012</td>
</tr>
<tr>
<td>32</td>
<td>0.32</td>
<td>0.73</td>
<td>0.844</td>
<td>0.32</td>
<td>0.32</td>
<td>0.014</td>
</tr>
<tr>
<td>35</td>
<td>0.35</td>
<td>0.75</td>
<td>0.844</td>
<td>0.35</td>
<td>0.35</td>
<td>0.014</td>
</tr>
<tr>
<td>40</td>
<td>0.40</td>
<td>0.640</td>
<td>1.04</td>
<td>off-scale, Melt is complete for all $x$'s</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Melting begins between 15 & 20 sec after quench. Between 35 & 40 sec after quench, the center melts everywhere by conduction from the sides, and the melt front disappears.
5.43 A cylindrical insulator contains a single, very thin, electrical resistor wire that runs along a line halfway between the center and the outside. The wire liberates 480 W/m. The thermal conductivity of the insulation is 3 W/m·°C, and the outside perimeter is held at 20°C. Develop a flux plot for the cross section, considering carefully how the field should look in the neighborhood of the point through which the wire passes. Evaluate the temperature at the center of the insulation.

The wire emits heat equally in all directions. Therefore we set up 16 adiabatic lines -- 8 on one symmetrical side -- all converging at equal angles (22.5°) on the wire. There are an infinite number of isotherms -- T goes to infinity at the wire. (A real wire with finite diameter would alleviate this problem.)

Now if the wire liberates Q W then through each square:

\[
\frac{Q}{16} = k \Delta T, \quad \Delta T = \frac{480}{16k} = 10°C
\]

The center is 1.85 squares in from the perimeter. Therefore it is at

\[
T_{\text{center}} = 20 + 1.85(10) = 38.5°C
\]
5.45 Lord Kelvin made an interesting estimate of the age of the earth in 1864. He assumed the earth originated as a mass of molten rock at 4144°K (7000°F) and that it has been cooled by outer space at 0°K, ever since. To do this, he assumed that Bi for the earth is very large, and that cooling has thus far penetrated only through a relatively thin (one-dimensional) layer. Using \( \alpha_{\text{Rock}} = 1.18 \times 10^{-6} \, \text{m/s}^2 \) and the measured surface temperature gradient of the earth, \( (1/27) \, ^\circ \text{C/m}, \) find Kelvin’s value of Earth’s age.

(Kelvin’s result turns out to be much less than the accepted value of 4 billion years. His calculation fails because internal heat generation by radioactive decay of the material in the surface layer causes the surface temperature gradient to be higher than it would otherwise be.)

Solution. Since we take the problem to be unidimensional and since, with a large Bi, we may approximate the earth’s surface as 0°K (with respect to 4144°K core), we may therefore use eqn. (5.48) for the heat flux

\[
q = k \frac{\partial T}{\partial x}\bigg|_{x=0} = k(T_i - T_\infty) \text{erf} \left( \pi \alpha t \right)^{-1/2}
\]

where the derivative is given as \( (1/27) \, ^\circ \text{C/m}. \) Then

\[
t = \frac{(T_i - T_\infty)^2}{(1/27)^2} \frac{1}{\pi \alpha} = 27^2(4144 - 0)^2/\pi(1.18 \times 10^{-6})
\]

\[
= 3.38 \times 10^{15} \text{ seconds} = 107 \text{ million years}
\]

It is interesting that, though Kelvin used 4144°K as the temperature of molten rock, he revised this estimate downward to 1473°K in the late 1890’s, giving an even smaller age of the earth. Further discussion can be found in Carslaw and Jaeger [1.14] or various geophysics textbooks.
**Problem 5.52** Suppose that \( T_\infty(t) \) is the time-dependent temperature of the environment surrounding a convectively-cooled, lumped object.

a) When \( T_\infty \) is not constant, show that eqn. (1.19) leads to

\[
\frac{d}{dt} (T - T_\infty) + \frac{(T - T_\infty)}{T} = -\frac{dT_\infty}{dt}
\]

where the time constant \( T \) is defined as usual.

b) If the object’s initial temperature is \( T_i \), use either an integrating factor or Laplace transforms to show that \( T(t) \) is

\[
T(t) = T_\infty(t) + \left[ T_i - T_\infty(0) \right] e^{-t/T} - e^{-t/T} \int_0^t e^{s/T} \frac{dT_\infty(s)}{ds} ds
\]

**Solution**

a) From eqn. (1.19) for constant \( c \), with \( T_\infty(t) \) not constant:

\[
-hA(T - T_\infty) = \frac{d}{dt} \left[ \rho c V(T - T_{\text{ref}}) \right] = mc \frac{dT}{dt} = mc \frac{d(T - T_\infty)}{dt} + mc \frac{dT_\infty}{dt}
\]

Setting \( T = mc / hA \) and rearranging, we obtain the desired result:

\[
\frac{d}{dt} (T - T_\infty) + \frac{(T - T_\infty)}{T} = -\frac{dT_\infty}{dt}
\]

b) The integrating factor for this first-order o.d.e. is \( e^{t/T} \). Multiplying through and using the product rule, we have

\[
\frac{d}{dt} \left[ e^{t/T} (T - T_\infty) \right] = -e^{t/T} \frac{dT_\infty}{dt}
\]

Next integrate from \( t = 0 \) to \( t \):

\[
e^{t/T} (T - T_\infty) - \left[ T_i - T_\infty(0) \right] = - \int_0^t e^{s/T} \frac{dT_\infty}{ds} ds
\]

Multiplying through by \( e^{-t/T} \) and rearranging gives the stated result:

\[
T(t) = T_\infty(t) + \left[ T_i - T_\infty(0) \right] e^{-t/T} - e^{-t/T} \int_0^t e^{s/T} \frac{dT_\infty}{ds} ds
\]

**Alternate approach:** To use Laplace transforms, we first simplify eqn. (1) by defining \( y(t) \equiv T - T_\infty \) and \( f(t) \equiv -dT_\infty/dt \):

\[
\frac{dy}{dt} + \frac{y}{T} = f(t)
\]

Next, we apply the Laplace transform \( \mathcal{L} \{ . \} \), with \( \mathcal{L} \{ y(t) \} = Y(p) \) and \( \mathcal{L} \{ f(t) \} = F(p) \):

\[
\mathcal{L} \left\{ \frac{dy}{dt} \right\} + \mathcal{L} \left\{ \frac{y}{T} \right\} = \mathcal{L} \{ f(t) \}
\]

\[
pY(p) - y(0) + \frac{1}{T} Y(p) = F(p)
\]

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Solving for $Y(p)$:
\[ Y(p) = \frac{1}{p + 1/T} y(0) + \frac{1}{p + 1/T} F(p) \]

Now take the inverse transform, $\mathcal{L}^{-1}\{\ldots\}$:
\[ \mathcal{L}^{-1}\{Y(p)\} = \mathcal{L}^{-1}\left\{ \frac{1}{p + 1/T} \right\} y(0) + \mathcal{L}^{-1}\left\{ \frac{1}{p + 1/T} F(p) \right\} \]

With a table of Laplace transforms, we find
\[ \mathcal{L}^{-1}\left\{ \frac{1}{p + 1/T} \right\} = e^{-t/T} \quad \equiv G(p) \quad \equiv g(t) \]

and with $G(p)$ and $g(t)$ defined as shown, the last term is just a convolution integral
\[ \mathcal{L}^{-1}\left\{ \frac{1}{p + 1/T} F(p) \right\} = \mathcal{L}^{-1}\{G(p)F(p)\} = \int_0^t g(t-s)f(s) \, ds \]

Putting all this back into eqn. (2), we find
\[ y(t) = e^{-t/T} y(0) + \int_0^t e^{-(t-s)/T} f(s) \, ds \]

and putting back the original variables in place of $y$ and $f$, we have at length obtained:
\[ T(t) = T_\infty(t) + [T_i - T_\infty(0)] e^{-t/T} - e^{-t/T} \int_0^t e^{s/T} \frac{dT_\infty}{ds} \, ds \]

**Extra credit.** State which approach is more straightforward!
Problem 5.61

Fourier number, $Fo$

Theta

Bi = 0.5
Bi = 1
Bi = 2
Bi = 5
Bi = 10

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Problem 6.1 Verify that eqn. (6.13) follows from eqns. (6.11a) and (6.12).

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{dp}{dx} + \frac{\partial^2 u}{\partial y^2} \]

or

\[
\left( \frac{\partial u}{\partial x} + \frac{u \frac{\partial v}{\partial x}}{\partial y} \right) + \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{dp}{dx} + \frac{\partial^2 u}{\partial y^2} \]

then multiply the continuity equation by \( u \) get:

\[
u \frac{\partial u}{\partial x} + u \frac{\partial v}{\partial y} = 0
\]

and subtract it to get

\[
\frac{u \frac{\partial u}{\partial x}}{\partial y} + \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{dp}{dx} + \frac{\partial^2 u}{\partial y^2}
\]

Problem 6.2 Complete the algebra between eqns. (6.16) and (6.20).

Start with

\[
\frac{\partial^4}{\partial y^2} \frac{\partial^2}{\partial x^2} - \frac{\partial^4}{\partial x^2} \frac{\partial y^2}{\partial y^2} = \frac{\partial^2}{\partial y^2} \frac{\partial^2}{\partial x^2} \]

Substitute \( \psi(x,y) = \sqrt{u_0/x} f(y) \) and get eqn. (6.18).

\[
\frac{\partial^4}{\partial y^2} \frac{\partial^2}{\partial x^2} = \left( \sqrt{u_0/x} \right) \frac{\partial^2}{\partial y^2} \frac{\partial^2}{\partial x^2} = u_0 f' = u
\]

\[
\frac{\partial^4}{\partial x^4} + \frac{\partial^4}{\partial x^2 y^2} = \frac{\partial^2}{\partial x^2} \frac{\partial^2}{\partial y^2} + \frac{1}{2} \frac{\partial^2}{\partial x^2} f
\]

\[
= -\frac{1}{2} \frac{\partial}{\partial x} \left( f' \frac{\partial f}{\partial y} - f \right) = u
\]

\[
\frac{\partial^4}{\partial x^4} + \frac{\partial^4}{\partial x^2 y^2} = u \left( \frac{\partial^2}{\partial x^2} \frac{\partial^2}{\partial y^2} \right)
\]

\[
\frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial x^2} \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial x^2} \frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial x^2} \frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial x^2} \frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial x^2} \frac{\partial^2}{\partial y^2}
\]

Combine these in the 3rd order mom. eqn. in \( f \), and get

\[
-u_0 f' \left( \frac{u_0}{2} f'' \right) + \frac{1}{2} \sqrt{\frac{u_0}{x}} (f' f - f) \frac{u_0}{\sqrt{x}} = \frac{1}{2} \frac{u_0^2}{x} f''
\]

or

\[
-\frac{u_0^2}{2x} f f'' + \frac{u_0^2}{2x} \frac{u_0}{f} f' f' f' - \frac{u_0^2}{2x} f f'' = \frac{u_0^2}{x} f''
\]

So

\[
ff'' + 2f'' = 0
\]
6.3 Solve \( f'' + 2f''' = 0 \) subject to the b.c.'s:
\( f(0) = f'(0) = 0 \) and \( f'(+\infty) = 1 \).

We begin by mapping the b.c., \( f'(+\infty) = 1 \) into the origin, thus:

\[
\begin{align*}
\text{set } f &= a F(x) \quad \text{and} \quad g = a y \\
\quad \text{then } f' &= a^2 F' \\
\quad f'' &= a^3 F'' \\
\quad f''' &= a^4 F'''
\end{align*}
\]

so \( f'' + 2f''' = 0 \) becomes \( a^4 F'' + 2a^3 F''' = 0 \) or \( FF'' + 2FF''' = 0 \)

with the b.c.s
\[
\begin{align*}
F(0) &= a F'(0) = 0 \\
F'(0) &= a^2 F''(0) = 0 \\
F'(+\infty) &= a^3 F'''(+\infty) = 1
\end{align*}
\]

Now we would normally have to guess \( F''(0) = a^3 F'(0) \). What we shall do is to set \( F'(0) = 1 \) so that \( F'(0) = a^3 \), and solve \( FF'' + 2FF''' = 0 \) subject to \( F(0) = F'(0) = 0 \). This solution will give a certain value of \( F(+\infty) \) from which we can calculate \( a = [F(+\infty)]^{-1/2} \). Once we know \( a \), we return to our calculated values of \( F \) for given values of \( f \) and \( g \) and correct these back to \( f \) and \( g \) using \( f = aF \) and \( g = F/a \).

There are many ways to solve the system \( FF'' + 2FF''' = 0 \), \( F(0) = F'(0) = 0 \), \( F'(+\infty) = 1 \).

The simplest is probably to reduce it to three first order d.e.'s thus: let \( y_1 = F, \ y_2 = F', \) and \( y_3 = F'' \) so

\[
\begin{align*}
\frac{dy_1}{dx} &= y_2 \\
\frac{dy_2}{dx} &= y_3 \\
\frac{dy_3}{dx} &= y_2
\end{align*}
\]

\( y_1(0) = 0 \quad y_2(0) = 0 \quad y_3(0) = 1 \)

Runga-Kutta integration schemes are available in computer libraries and can easily be called in to solve such systems of first-order equations.
6.4 Verify that the Blasius solution (given in Table 6.1) satisfies eqn. (7.25). Do this by showing graphically that

\[
\frac{d}{dx} \left[ 5 \int_0^1 \frac{u}{u_\infty (1-\frac{u}{u_\infty})} d\left( \frac{y}{\delta} \right) \right] = - \frac{d}{u_\infty \delta} \frac{d(u/u_\infty)}{d(y/\delta)} \bigg|_{y=0}
\]

is satisfied by the numbers in Table 6.1. We begin by converting the equation with the help of \( \xi = \frac{4.92x}{\sqrt{\text{Re}_x}} \):

\[
\frac{u_\infty}{2^{\xi}} \frac{4.92x}{\sqrt{\text{Re}_x}} \frac{d}{dx} \left[ \frac{4.92x}{\sqrt{\text{Re}_x}} \int_0^1 \frac{u}{u_\infty (1-\frac{u}{u_\infty})} d\left( \frac{y}{4.92^{\xi/2}} \right) \right] = \frac{d(u/u_\infty)}{d(y/\delta)} \bigg|_{y=0}
\]

or

\[
\frac{4.92}{2^{\xi}} \frac{u_\infty}{\delta} \frac{d}{dx} \left[ \frac{2^{\xi/2}}{u_\infty} \int_0^1 \frac{u}{u_\infty (1-\frac{u}{u_\infty})} d\eta \right] = 0.3320 \xi = 4.92
\]

or

\[
= \text{Constant, } K
\]

so \( K \text{ should } = 0.66412 \)

There are about 330, \( \frac{1}{2} \times \frac{1}{2} \) squares

with an numerical value of

\( 0.01 \times 0.2 \) or \( 0.002 \), each. Thus:

Area = 0.660 = K

so graphical integration verifies the equation

within \( \frac{664-660}{664} = 0.62\%

Graphical integration based on Table 6.1
6.5 Verify eqn. (6.30)

\[ \frac{d}{dx} \left[ \delta \left( \frac{y}{\delta} \right) \frac{u}{u_{\infty}} \left( \frac{u}{u_{\infty}} - 1 \right) \right] = -\frac{2\delta}{u_{\infty} \delta} \frac{\partial (u/u_{\infty})}{\partial (y/\delta)} \left. \right|_{y=0} \]

and substitute \( \frac{u}{u_{\infty}} = \frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \left( \frac{y}{\delta} \right)^3 \). Let us call \( \xi = y/\delta \). Then:

\[ \frac{d}{dx} \left[ \delta \left( \frac{3}{2} \xi - \frac{1}{2} \xi^3 \right) \left( \frac{3}{2} \xi - \frac{1}{2} \xi^3 - 1 \right) \frac{d\xi}{\delta} \right] = -\frac{3\delta}{u_{\infty} \delta} \left( \frac{3}{2} \right) \]

So:

\[ \frac{3\delta}{260} \frac{d\delta}{dx} = -\frac{3\delta}{2u_{\infty} \delta} \]

6.6 Derive \( \tau_w \) using the momentum integral method.

\[ \tau_w = \left( \mu u_{\infty}/\delta \right) \frac{\partial (u/u_{\infty})}{\partial (y/\delta)} = \frac{\mu u_{\infty}}{\delta} \frac{3}{2} \]

Substitute eqn. (7.31), \( \delta = 4.64 \sqrt{\frac{\mu u_{\infty}}{k}} \).

\[ \tau_w = \frac{3/2}{4.64} \frac{u_{\infty}}{\delta} \frac{\mu u_{\infty}}{\delta} = 0.3233 \frac{\mu u_{\infty}}{x} \text{Re}_x^{1/2} \]

This is only 3% below the exact value, eqn. (6.32)

6.7 Find \( \delta \) and \( \tau_w \) using the momentum integral method and assuming \( u/u_{\infty} = y/\delta \) for the velocity profile.

Using eqn. (7.25):

\[ \frac{d}{dx} \left[ \delta \left( \frac{y}{\delta} \right) \frac{u}{u_{\infty}} \left( \frac{u}{u_{\infty}} - 1 \right) \right] = -\frac{2\delta}{u_{\infty} \delta} \left. \right|_{y=0} \]

So:

\[ \int_0^\delta d\xi = -\int_0^{12\delta} \frac{\xi}{u_{\infty} \delta} \frac{d\xi}{\delta} \text{ or } \delta = \sqrt{12 \frac{x}{\text{Re}_x}} = 3.46 \sqrt{\frac{x}{\text{Re}_x}} \]

And:

\[ \tau_w = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0} = \frac{\mu u_{\infty}}{\delta} \frac{\partial (u/u_{\infty})}{\partial (y/\delta)} \left. \right|_{y=0} \]

\[ = 1 \]

So:

\[ C_f = \frac{\tau_w}{\frac{1}{2} \rho u_{\infty}^2} = \frac{2\delta}{\delta u_{\infty}} = \frac{2}{12} \delta u_{\infty} \sqrt{\frac{u_{\infty}}{2x}} = \frac{0.577}{\sqrt{\text{Re}_x}} \]

The use of this extremely crude approximation to \( u/u_{\infty} \) yields a value of \( x/\delta \) that is low by only 30% and a \( C_f \) that is low by only 13%.
6.8 The b.l. thickness for a particular water flow (plotted below) is given by \( \delta m = 0.005 \Delta x \) \( \text{m} \). Add to the plot: \( \delta_t \) for the water flow; and \( \delta_t \) and \( \delta_{t, \text{air}} \) for air at the same temperature and velocity.

\[
\delta_{t, \text{H}_2\text{O}} = \frac{\delta}{r_{\text{H}_2\text{O}}} = \frac{5}{4.34^{-\frac{1}{3}}} = 0.613 \delta; \quad \delta_{\text{air}} = \sqrt{\frac{\delta_{\text{air}}^2}{r_{\text{air}}}} \delta_{\text{H}_2\text{O}} = \frac{1.7 \times 10^{-6}}{0.658 \times 10^{-4}} \delta_{\text{H}_2\text{O}}
\]

\[
\delta_{t, \text{air}} = \frac{\delta}{r_{\text{air}}} = \frac{5.083 \delta_{\text{H}_2\text{O}}}{0.709^{-\frac{1}{3}}} = 5.714 \delta_{\text{H}_2\text{O}}
\]

6.9 A liquid film flows down a plate as shown, at its terminal velocity, with a thickness of \( \delta_0 \). Derive an expression for \( u(y) \).

\[
\begin{align*}
0 \quad & \quad \frac{\partial u}{\partial x} + \frac{v}{w} \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{dP}{dx} - g + \frac{\partial}{\partial y} \frac{\partial}{\partial y} u \\
0 \quad & \quad \delta_0
\end{align*}
\]

So

\[
\frac{d^2 u}{dy^2} = \frac{g}{v} \quad \Rightarrow \quad u = \frac{g y^2}{2v^2} + C_1 y + C_2 \quad \text{w/ b.c.'s} \quad u(y = 0) = 0 \Rightarrow C_2 = 0
\]

\[
\frac{\partial u}{\partial y} \bigg|_{y = 0} = 0 \Rightarrow C_1 = -\frac{g \delta_0}{2v}
\]

Then:

\[
u = \frac{g y^2}{2v^2} - \frac{g \delta_0^2}{2v} \frac{y}{\delta_0}
\]

or:

\[
\frac{U}{g \delta_0^2} = \frac{1}{2} \left( \frac{y}{\delta_0} \right)^2 - \frac{y}{\delta_0} \quad (= \text{negative since it is downward})
\]
6.10 Evaluate $\text{Nu}_L$ for laminar flow over an isothermal flat plate.

We know that $\text{Nu}_x = A \text{Re}_x^{1/2}$, where

$$A = \frac{0.3387 \text{Pr}^{1/3}}{[1 + 0.0469/\text{Pr}^{2/3}]^{1/4}} \quad \text{from eqn. (6.63)}$$

so:

$$q(x) = \frac{k \Delta T}{x} \text{Nu}_x \quad \text{and} \quad \bar{q} = \frac{k \Delta T}{L} \int_0^L \frac{u_0 x}{K} \, dx = 2 \frac{k \Delta T}{L} \sqrt{\frac{u_0 L}{K}}$$

Then:

$$\text{Nu}_L = \frac{\bar{q} L}{k \Delta T} = A \text{Re}_L^{1/2} = 0.6774 \text{Pr}^{1/3} \text{Re}_L^{1/2} \left[1 + 0.0469/\text{Pr}^{2/3}\right]^{1/4}$$

6.11 Use an integral method to predict $\text{Nu}_x$ for a flat plate with $q_w = \text{constant}$, and find $\Delta T$ at the leading edge of the plate.

\[
\rho c_w \frac{d}{dx} \left[ \delta_k \Delta T \int_0^1 \left( \frac{u}{u_w} \right) \left( \frac{T - T_m}{T_m - T_0} \right) d \left( \frac{y}{\delta_k} \right) \right] = q_w
\]

\[
\frac{3}{20} \frac{\delta_k}{\alpha} - \frac{3}{280} \delta_k^2 = \frac{3}{20} \delta_k
\]

\[
\rho c_w (\delta_k \Delta T)(\frac{3}{20} \delta_k) = q_w x
\]

\[
\frac{3}{20} \frac{u_w}{\alpha} \delta_k \left( \frac{\delta_k}{\delta} \right)^2 = \frac{q_w x}{\Delta T L} = \text{Nu}_x
\]

But $\text{Nu}_x \approx \frac{3}{2} \frac{x}{\delta} \left( \frac{\delta}{\delta_k} \right)$ so

\[
\frac{3}{20} \frac{u_w}{\alpha} \left( \frac{4.64 x}{\text{Re}_x} \right)^2 \left( \frac{\delta_k}{\delta} \right)^3 = \frac{3}{2} \frac{x}{\delta}
\]

\[
\frac{\delta_k}{\delta} = \left( \frac{4.64^2}{10} \right)^{1/3} \text{Pr}^{1/3} = 1.291 \text{Pr}^{1/3}
\]

so

\[
\text{Nu}_x = \frac{3}{2} \frac{1.291}{4.64} \text{Pr}^{1/3} \text{Re}_x^{1/2} = 0.417 \text{Pr}^{1/3} \text{Re}_x^{1/2}
\]

and, since $h \sim \text{Re}_x^{1/2} / x \sim 1 / \sqrt{x}$

$$\Delta T = q_w / h \sim \sqrt{x}$$

we conclude that $\Delta T \to 0$ at the leading edge of the heater.
**Problem 6.12**  (a) Verify that eqn. (6.120) follows from eqn. (6.119). (b) Derive an equation for liquids that is analogous to eqn. (6.119).

**Solution**

a) Beginning with

\[
\bar{h} = \frac{1}{L \Delta T} \int_0^L q_w \, dx = \frac{1}{L} \left[ \int_0^{x_l} h_{\text{laminar}} \, dx + \int_{x_l}^{x_u} h_{\text{trans}} \, dx + \int_{x_u}^L h_{\text{turbulent}} \, dx \right]
\]

we may evaluate each integral separately. For a uniform temperature surface, the Nusselt numbers are given by these equations:

\[
\begin{align*}
\text{Nu}_{\text{laminar}} &= 0.332 \text{Re}_{x}^{1/2} \text{Pr}^{1/3} \\
\text{Nu}_{\text{trans}} &= \text{Nu}_{\text{laminar}} \left( \frac{\text{Re}_{x}}{\text{Re}_l} \right)^c \\
\text{Nu}_{\text{turbulent}} &= 0.0296 \text{Re}_{x}^{0.8} \text{Pr}^{0.6} \quad \text{for gases}
\end{align*}
\]

The three integrals are thus

\[
\begin{align*}
\frac{1}{L} \int_0^{x_l} h_{\text{laminar}} \, dx &= \frac{0.332 \text{Pr}^{1/3}}{L} \int_0^{x_l} \frac{u_\infty}{v} \, dx = \frac{0.332 \text{Pr}^{1/3}}{L} 2 \sqrt{\frac{u_\infty x_l}{v}} - \frac{k}{L} 0.664 \text{Re}_l^{1/2} \text{Pr} \\
\frac{1}{L} \int_{x_l}^{x_u} h_{\text{trans}} \, dx &= \frac{k}{L} \frac{\text{Nu}_{\text{laminar}} \left( \frac{\text{Re}_{x}}{\text{Re}_l} \right)^c}{\text{Pr}^{1/3}} \int_{x_l}^{x_u} x^{c-1} \, dx = \frac{k}{L} \frac{\text{Nu}_{\text{laminar}} \left( \frac{\text{Re}_{x}}{\text{Re}_l} \right)^c}{\text{Pr}^{1/3}} \frac{1}{c} \left( x_u^c - x_l^c \right) \\
\frac{1}{L} \int_{x_u}^L h_{\text{turbulent}} \, dx &= \frac{0.0296 \text{Pr}^{0.6}}{L} \left( \frac{u_\infty}{v} \right)^{0.8} \int_{x_u}^L x^{-0.2} \, dx = \frac{0.0296 \text{Pr}^{0.6}}{(0.8)L} \left( \text{Re}_L^{0.8} - \text{Re}_u^{0.8} \right) \\
\end{align*}
\]

where the last step follows because eqn. (6.114b) intersects \( \text{Nu}_{\text{turbulent}} \) at \( \text{Re}_x \), and

\[
\frac{1}{L} \int_{x_u}^L h_{\text{turbulent}} \, dx = \frac{k}{L} 0.037 \text{Pr}^{0.6} \left( \text{Re}_L^{0.8} - \text{Re}_u^{0.8} \right)
\]

Collecting these terms, we find:

\[
\overline{\text{Nu}}_L \equiv \frac{\bar{h} L}{k} = 0.037 \text{Pr}^{0.6} \left( \text{Re}_L^{0.8} - \text{Re}_u^{0.8} \right) + 0.664 \text{Re}_l^{1/2} \text{Pr}^{1/3}
\]

b) For a liquid flow, the turbulent correlation should be eqn. (6.113):

\[
\text{Nu}_{\text{turbulent}} = 0.032 \text{Re}_{x}^{0.8} \text{Pr}^{0.43} \quad \text{for nonmetallic liquids}
\]

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and the integral in the turbulent range changes to

\[
\frac{1}{L} \int_{s_u}^{L} h_{\text{turb}} \, dx = \frac{0.032}{L} k \Pr^{0.43} \left( \frac{u_{\infty}}{\nu} \right)^{0.8} \int_{s_u}^{L} x^{-0.2} \, dx = \frac{0.032}{(0.8)L} k \Pr^{0.43} \left( \text{Re}_L^{0.8} - \text{Re}_u^{0.8} \right)
\]

\[
= k \int \frac{0.040}{L} \Pr^{0.43} \left( \text{Re}_L^{0.8} - \text{Re}_u^{0.8} \right)
\]

Collecting these terms, we find:

\[
\overline{\text{Nu}}_L \equiv \frac{\overline{h}L}{k} = 0.040 \Pr^{0.43} \left( \text{Re}_L^{0.8} - \text{Re}_u^{0.8} \right) + 0.664 \Re_L^{1/2} \Pr^{1/3}
\]

\[
+ \frac{1}{c} \left( 0.032 \Re_u^{0.8} \Pr^{0.43} - 0.332 \Re_L^{1/2} \Pr^{1/3} \right)
\]

for nonmetallic liquids

\[
\text{contribution of transition region}
\]

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6.14 Do the differentiation in eqn. (6.24)

Leibnitz rule says that:
\[ \frac{d}{dx} \left( \int_{a(x)}^{b(x)} f(x,y) \, dy \right) = \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x,y) \, dy + \int_{a(x)}^{b(x)} \left( \frac{db}{dx} f(x,y) - \frac{da}{dx} f(x,y) \right) \, dy \]

Thus:
\[ \frac{d}{dx} \left( \int_{t_0}^{t_1} u(t) \, dt \right) = \int_{t_0}^{t_1} \frac{\partial}{\partial x} [u(t)] \, dt + u(t) \frac{\partial}{\partial x} [u(t)] \, dt - \frac{u(t)}{u_\infty} + \frac{u_\infty}{u_\infty} \frac{\partial}{\partial x} [u(t)] \, dt = 0 \]

and this gives eqn. above eqn. (6.24).

6.15 Glycerin or water flows over a flat plate at 2 m/s. \( T_\infty = 23^\circ C \)

Find: \( h(x = 0.12) \) and compare the drag forces in each case.

Evaluating properties at \( T_{avg} = (57 + 23)/2 = 40^\circ C \), we get:

**Water:**
- \( k = 0.627 \text{ W/m} \cdot \text{K} \)
- \( \lambda = 0.657 \times 10^{-4} \text{ W/s} \)
- \( \rho = 992 \text{ kg/m}^3 \)
- \( \beta = 4.36 \text{ C/m} \)
- \( Re_L = 0.12x_{0.57}/10 = 365,300 \)
- \( Nu_L = 0.332 \sqrt{365,300} \times (4.36)^{0.8} = 328 \)
- \( h(x = 0.12) = \frac{k}{\lambda} Nu_L = 1715 \text{ W/m}^2 \cdot \text{K} \)

**Glycerin:**
- \( k = 0.295 \text{ W/m} \cdot \text{K} \)
- \( \lambda = 0.000222 \text{ m/s} \)
- \( \rho = 1249 \text{ kg/m}^3 \)
- \( \beta = 2541 \text{ C/m} \)
- \( Re_L = 0.12x/0.000222 = 1057 \)
- \( Nu_L = 0.332 \sqrt{1057} (2541)^{0.8} = 146 \)
- \( h(x = 0.12) = \frac{k}{\lambda} Nu_L = 346 \text{ W/m}^2 \cdot \text{K} \)

Thus the water is a significantly better coolant. Furthermore, the ratio of drag forces is:
\[ \frac{F_D_{glyc}}{F_D_{water}} = \frac{(T_w)_{glyc} A}{(T_w)_{water} A} = \frac{\beta_T C_t}{C_t} = \frac{1249}{992} \sqrt{\frac{Re_L}{Re_{glyc}}} = 2.34 \]

So the glycerin exerts **23.4 times** as much drag on the plate as the water does.
**Problem 6.16** Air at \(-10{\degree}C\) flows over a smooth, sharp-edged, almost-flat, aerodynamic surface at 240 km/hr. The surface is at \(10{\degree}C\). Turbulent transition begins at \(Re_t = 140,000\) and ends at \(Re_u = 315,000\). Find: (a) the \(x\)-coordinates within which laminar-to-turbulent transition occurs; (b) \(h\) for a 2 m long surface; (c) \(h\) at the trailing edge for a 2 m surface; and (d) \(\delta\) and \(h\) at \(x_l\).

**Solution**

a) We evaluate physical properties at the film temperature, \(T_f = (\frac{-10 + 10}{2}) = 0{\degree}C\): \(\nu = 1.332 \times 10^{-5}\) m\(^2\)/s, \(Pr = 0.711\), and \(k = 0.244\) W/m·K. Also, \(u_\infty = 240\)(1000)/(3600) = 66.7 m/s. Then:

\[
x_l = \frac{Re_t \nu}{u_\infty} = \frac{(140000)(1.332 \times 10^{-5})}{(66.7)} = 0.0280\ m
\]

\[
x_u = \frac{Re_u \nu}{u_\infty} = \frac{(315000)(1.332 \times 10^{-5})}{(66.7)} = 0.0629\ m
\]

Observe that the flow is fully turbulent over \(1.937/2.00 = 96.9\%\) of its length.

b) First, we need \(Re_L\):

\[
Re_L = \frac{u_\infty L}{\nu} = \frac{(66.7)(2)}{1.332 \times 10^{-5}} = 1.00 \times 10^7
\]

Then we get \(c\) from eqn. (6.115):

\[
c = 0.9922 \log_{10}(140,000) - 3.013 = 2.09
\]

Now we may use eqn. (6.120):

\[
\overline{Nu}_L = 0.037(0.711)^{0.6} \left[(1.00 \times 10^7)^{0.8} - (3.15 \times 10^5)^{0.8}\right]
+ 0.664 (1.40 \times 10^5)^{1/2} (0.711)^{1/3}
+ \frac{1}{2.09} \left[0.0296(3.15 \times 10^5)^{0.8} (0.711)^{0.6} - 0.332 (1.40 \times 10^5)^{1/2} (0.711)^{1/3}\right]
= 11248.9 + 221.8 + 236.0 = 1.171 \times 10^4
\]

Thus

\[
\overline{h} = \frac{k}{L} \overline{Nu}_L = \frac{(0.0244)(1.171 \times 10^4)}{2} = 143\ W/m^2K
\]

c) With eqn. (6.112),

\[
Nu_L = 0.0296\ Re_L^{0.8} Pr^{0.6} = 0.0296 (1.00 \times 10^7)^{0.8} (0.711)^{0.6} = 9603
\]

so

\[
h(L) = \frac{k}{L} Nu_L = \frac{(0.0244)(9603)}{2} = 117\ W/m^2K
\]

d) The flow is laminar here. From eqn (6.58):

\[
Nu_{x_l} = 0.332\ Re_t^{1/2} Pr^{1/3} = 0.332(1.40 \times 10^5)^{1/2} (0.711)^{1/3} = 110.9
\]

so

\[
h(x_l) = \frac{k}{x_l} Nu_{x_l} = \frac{(0.0244)(110.9)}{0.0280} = 96.6\ W/m^2K
\]
With eqn (6.2), we find that the boundary layer here is very thin:

$$\delta = \frac{4.92 \chi}{\sqrt{\text{Re}_{x_l}}} = \frac{4.92(0.0280)}{\sqrt{1.4 \times 10^5}} = 0.000368 \text{ m} = 0.37 \text{ mm}$$
**Problem 6.17** Find $\bar{h}$ in Example 6.9 using eqn. (6.120) with $Re_l = 80,000$. Compare with the value in the example and discuss the implication of your result. *Hint:* See Example 6.10.

**Solution** Equation (6.120) is

$$\overline{Nu}_L \equiv \frac{\bar{h}L}{k} = 0.037 \Pr^{0.6} \left( \text{Re}_L^{0.8} - \text{Re}_u^{0.8} \right) + 0.664 \text{Re}_i^{1/2} \Pr^{1/3}$$

$$+ \frac{1}{c} \left( 0.0296 \text{Re}_u^{0.8} \Pr^{0.6} - 0.332 \text{Re}_i^{1/2} \Pr^{1/3} \right)$$

(6.120)

From Example 6.9, we have $Re_L = 1.270 \times 10^6$ and $Pr = 0.708$. We may find $c$ from eqn. (6.115):

$$c = 0.9922 \log_{10}(80,000) - 3.013 = 1.85$$

We also need $Re_u$, which we can find following Example 6.10:

$$Re_u^{1.85−0.8} = \frac{0.0296(0.708)^{0.6}(80,000)^{1.85}}{0.332(80,000)^{1/2}(0.708)^{1/3}}$$

Solving, $Re_u = 184,500$. Substituting all this into eqn. (6.120):

$$\overline{Nu}_L = 0.037(0.708)^{0.6}\left[ (1.270 \times 10^6)^{0.8} - (1.845 \times 10^5)^{0.8} \right] + 0.664 (8.0 \times 10^4)^{1/2}(0.708)^{1/3}$$

$$+ \frac{1}{1.85} \left[ 0.0296(1.845 \times 10^5)^{0.8}(0.708)^{0.6} - 0.332 (8.0 \times 10^4)^{1/2}(0.708)^{1/3} \right]$$

Evaluating, we find the contributions of the turbulent, laminar, and transition regions:

$$\overline{Nu}_L = \underbrace{1806.6}_{\text{turb.}} + \underbrace{167.4}_{\text{lam.}} + \underbrace{167.1}_{\text{trans.}} = 2,141$$

The transition region contributes 7.8% of the total. The average heat transfer coefficient is

$$\bar{h} = \frac{2141(0.0264)}{2.0} = 28.26 \text{ W/m}^2\text{K}$$

and the convective heat loss from the plate is

$$Q = (2.0)(1.0)(28.26)(310 - 290) = 1130 \text{ W}$$

The earlier transition to turbulence increases the heat removal by $\left[ \frac{(1130+22)}{(756+22)} - 1 \right] \times 100 = 48\%$. 

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6.19 Mercury flows at 25°C and 0.7 m/s over a 4 cm long plate at 60°C. Find \( h, \quad T_w, \quad h(x = 0.04 \text{ m}), \) and \( S(x = 0.04 \text{ m}). \)

**Solution**
Evaluate properties at \((25+60)/2 = 42.5°C = 315.5 K\)
\(
\alpha = 1.14(10)^{-2} \text{ m}^2/\text{s}, \quad Pr = 0.0248, \quad k = 7.39, \quad Re_h = \frac{0.14(0.04)}{1.14(10)^{-2}} = 245,600
\)
so:
\[
Nu_L = \frac{1.13\sqrt{245,600(0.0248)}}{88.2}; \quad h = \frac{88.2}{0.09} = 16,293 \frac{W}{\text{m}^2\cdot\text{K}}
\]
\[
\frac{S}{\sqrt{245,600}} = 0.004 \text{ m} = 0.4 \text{ mm} \quad \text{(pretty thin)}
\]
\[
h_L = \frac{1}{2} h = \frac{8,147 \frac{W}{\text{m}^2\cdot\text{K}}}{2}
\]
\[
T_w = \frac{1}{2} \rho C_f \frac{u^2}{\sqrt{245,600}} = \frac{13.573(0.7)^2 \cdot 1.328}{\sqrt{245,600}} = 8.91 \frac{\text{N}}{\text{m}^2}
\]

6.20 A plate is at rest in water at 15°C. It is suddenly translated parallel with itself at 1.5 m/s. Evaluate the liquid velocity, \( u, \) 0.015 m from the plate at \( t = 1, \) 10, and 1000 s.

Pose the problem:
\[
\frac{\partial^2 u}{\partial y^2} = \frac{1}{\beta} \frac{\partial u}{\partial t}
\]
with b.c.s:
\[
\begin{align*}
&u(y=0) = 1.5 \text{ m/s}, \quad t > 0 \\
&u(t=0) = 0
\end{align*}
\]

Compare this with the semi-infinite region solution in Sect 5.6
\[
\frac{\partial^2 (T-T_i)}{\alpha \partial x^2} = \frac{1}{\alpha} \frac{\partial (T-T_i)}{\partial t}
\]
with b.c.s:
\[
(T(T_i)_x = 0 = T(T_i)_t = 0)
\]
These problems are identical if:
\[
T(T_i) \Leftrightarrow u, \quad x \Leftrightarrow y, \quad \alpha \Leftrightarrow \beta
\]
and \((T(T_i)_t) \Leftrightarrow 1.5\)

Thus, its solution, \( \frac{T-T(T_i)}{T(T_i)} = \text{erf} \left( \frac{y}{2\sqrt{\beta t}} \right) \) is the solution to our problem once we make these changes. So:
\[
\frac{u-1.5}{-1.5} = \text{erf} \left( \frac{y}{2\sqrt{\beta t}} \right)
\]
where \( \beta = 1.184 \times 10^{-6} \text{ m}^2/\text{s} \)

Therefore: \( u \) at \( y = 0.015 \) m is given by
\[
u = 1.5(1 - \text{erf} \left( \frac{6.89}{\sqrt{t}} \right))
\]

Then at \( t = 1 \text{ sec}, \quad u = 0 \text{ m/s} \)
\( t = 10 \text{ sec}, \quad u = 0.003 \text{ m/s} \)
\( t = 1000 \text{ sec}, \quad u = 1.14 \text{ m/s} \)
6.21 Use the fact that, when Pr is very large, \( u/u_\infty = (3/2)(y/\delta) \) inside the thermal b.l., to create an expression for \( \text{Nu}_x \) during the flow of a High Pr fluid over a flat isothermal plate.

We begin with the integrated energy equation in the form of eqn. (6.51).

\[
\begin{align*}
\frac{u_\infty \Delta T}{\delta_t} \frac{d}{dx} \left[ \delta_t \left( \int_0^1 \frac{u}{u_\infty} \frac{(T-T_\infty)}{\Delta T} d\left( \frac{y}{\delta_t} \right) \right) \right] &= -\frac{\alpha \Delta T}{\delta_t} \left( \frac{\partial T}{\partial y} \right) \bigg|_{y=0} - \frac{\alpha \Delta T}{\delta_t} \left( \frac{\partial T}{\partial y} \right) \bigg|_{y=\delta_t} \\
&= -\frac{3}{2} \frac{\alpha \Delta T}{\delta_t} \\
\end{align*}
\]

or

\[
\begin{align*}
\frac{u_\infty \Delta T}{\delta_t} \frac{d}{dx} \left[ \delta_t \left( \int_0^1 \left( \frac{3}{2} x - \frac{3}{4} x^2 + \frac{3}{2} \phi x^4 \right) dx \right) \right] &= \frac{3}{2} \frac{\Delta T}{\delta_t} \alpha \\
\left( \frac{3}{4} - \frac{3}{4} + \frac{3}{2} \phi \right) \phi &= \frac{3}{20} \phi \\
\end{align*}
\]

or

\[
\frac{1}{10} \frac{1}{2} \frac{d}{dx} \frac{\delta_t^2}{\delta_t} = \frac{\alpha}{u_\infty \phi} \quad \text{or} \quad \delta_t = \sqrt{\frac{5 \alpha x}{u_\infty \phi}}
\]

Now we introduce \( \delta = 8 \phi \):

\[
\delta = \sqrt{\frac{5 \alpha x}{u_\infty \phi}}
\]

Then using eqn. (6.51a) for \( \delta \):

\[
\delta = \sqrt{\frac{5 \alpha x}{u_\infty \phi}}
\]

We equate the two expressions for \( \delta \) and solve for \( \phi \):

\[
\phi = \frac{3}{\sqrt{14}} \frac{1}{\sqrt{Pr}}
\]

But, in accordance with eqn. (6.57), \( h = \frac{3}{2} \frac{k}{\delta_t} = \frac{3}{2} \frac{k}{8 \phi} \)

Thus:

\[
\text{Nu}_x = \frac{3}{2} \frac{\text{Re}_x^{1/2} \text{Pr}^{1/3}}{\sqrt{\text{Re}_x \left( \frac{13}{14} \right) \frac{1}{\text{Pr}^{1/3}}}} = 0.3314 \text{ Re}_x^{1/2} \text{Pr}^{1/3}
\]

(The integral approx. in the text gives exactly this for \( \phi \to 0 \))
6.23 Water at 7°C flows at 0.38 m/s across the top of a 0.207m long, thin copper plate. Methanol at 87°C flows across the bottom of the same plate, at the same speed, but in the opposite direction. Make the obvious first-guess as to the temperature at which to evaluate physical properties. Then plot the plate temperature as a function of position. Do not bother to correct the physical properties in this problem but note Problem 6.24.

We shall first guess that the plate is at the mean temperature of \((7+87)/2 = 47°C\). Evaluate meth props. at \((47+87)/2 = 67°C\), --- water props. at \((7+87)/2 = 27°C\):

\[
Re_{\text{meth max}} = \frac{0.38(0.207)}{0.44(10)^{-6}} = 178,773, \quad Re_{H_2O} = \frac{0.38(0.207)}{0.826(10)^{-6}} = 95,230
\]

Both are laminar. Use \(h = 0.332 \times \text{Pr}^{1/3} \times \text{Re}_x^{1/2} / x\), so

\[
h_{\text{meth}} = 0.332(0.1308)(4.5)^{1/3}(0.38/0.44(10)^{-6})^{1/2} = \frac{100}{v \times m}
\]

\[
h_{H_2O} = 0.332(0.6084)(5.65)^{1/3}(0.38/0.826(10)^{-6})^{1/2}/\sqrt{0.207-x} = \frac{2.44}{\sqrt{0.207-x}}
\]
6.23 (continued)

\[ U = \frac{1}{\left[ \frac{1}{\frac{h_{\text{meth}}}{W/m^2\cdot\text{C}}} + \frac{1}{\frac{h_{\text{H}_2\text{O}}}{W/m^2\cdot\text{C}}} \right]} \quad \text{and} \quad U(T_{\text{meth}} - T_{\text{H}_2\text{O}}) = h_{\text{meth}}(T_{\text{meth}} - T_{\text{Cu}}) \]

<table>
<thead>
<tr>
<th>( x \text{ m} )</th>
<th>( h_{\text{meth}} \text{ W/m}^2\cdot\text{C} )</th>
<th>( h_{\text{H}_2\text{O}} \text{ W/m}^2\cdot\text{C} )</th>
<th>( T_{\text{Cu}} = 87^\circ \text{C} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>22.0</td>
<td>( \infty )</td>
<td>7</td>
</tr>
<tr>
<td>0.05</td>
<td>25.2</td>
<td>1091</td>
<td>2.05</td>
</tr>
<tr>
<td>0.10</td>
<td>30.6</td>
<td>772</td>
<td>2.19</td>
</tr>
<tr>
<td>0.15</td>
<td>41.9</td>
<td>630</td>
<td>2.52</td>
</tr>
<tr>
<td>0.20</td>
<td>1195</td>
<td>546</td>
<td>3.75</td>
</tr>
<tr>
<td>0.207</td>
<td>( \infty )</td>
<td>536</td>
<td>8.7</td>
</tr>
</tbody>
</table>

6.24 Work Problem 6.23 taking full account of property variations.

To do this, we regard the previous solution as a first iteration. Now we rework the problem using the plate temperatures above.

<table>
<thead>
<tr>
<th>( x \text{ m} )</th>
<th>( T_{\text{plate}} \text{ }^\circ \text{C} )</th>
<th>( T_{\text{f,lim}} = \frac{T_{\text{H}<em>2\text{O}} - T</em>{\text{Cu}}}{z} \text{ W/m}^2 \cdot \text{C} )</th>
<th>( z \text{ m} )</th>
<th>( k \text{ W/m} \cdot \text{C} )</th>
<th>( P_r )</th>
<th>( h_{\text{meth}} \text{ W/m}^2 \cdot \text{C} )</th>
<th>( h_{\text{H}_2\text{O}} \text{ W/m}^2 \cdot \text{C} )</th>
<th>( T_{\text{Cu}} \text{ }^\circ \text{C} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7</td>
<td>47</td>
<td>7</td>
<td>0.6</td>
<td>1.922</td>
<td>1.865</td>
<td>0.589</td>
<td>1.947</td>
</tr>
<tr>
<td>0.05</td>
<td>22</td>
<td>54.5</td>
<td>14.5</td>
<td>0.54</td>
<td>1.20</td>
<td>0.944</td>
<td>0.591</td>
<td>1.794</td>
</tr>
<tr>
<td>0.10</td>
<td>30</td>
<td>58.5</td>
<td>18.5</td>
<td>0.51</td>
<td>1.08</td>
<td>0.932</td>
<td>0.591</td>
<td>1.765</td>
</tr>
<tr>
<td>0.15</td>
<td>39</td>
<td>63.0</td>
<td>23</td>
<td>0.47</td>
<td>0.95</td>
<td>0.919</td>
<td>0.603</td>
<td>1.730</td>
</tr>
<tr>
<td>0.20</td>
<td>62</td>
<td>74.5</td>
<td>34.5</td>
<td>0.40</td>
<td>0.719</td>
<td>0.887</td>
<td>0.610</td>
<td>1.672</td>
</tr>
<tr>
<td>0.207</td>
<td>87</td>
<td>87</td>
<td>47</td>
<td>0.36</td>
<td>0.566</td>
<td>0.861</td>
<td>0.671</td>
<td>1.624</td>
</tr>
</tbody>
</table>

No temperature changed more than 0.7\( ^\circ \text{C} \). We can terminate the calculation. Furthermore, the plot above will stand. No point will move by more than a pencil-width.

Better property data have become available since we first worked this problem. Consequently, the numbers will change a bit. But the solution remains essentially correct.

6.25 If in Example 6.6 (with a constant \( q_w = 420 \text{ W/m}^2 \)) the wall temperature were instead held constant at its average value of 76\( ^\circ \text{C} \), what would the average wall heat flux be?

\[
\overline{Nu_L} = 0.664 \frac{Re_L^{1/2} Pr^{1/3}}{1.97 (0.6) \cdot 0.712^{1/3}} = 147.4
\]

where we have evaluated air properties at \( 76 + 15 \)/2 = 45.5\( ^\circ \text{C} \).

Then:

\[
\overline{h} = \frac{\overline{Nu_L} k_L}{L} = 147.4 \frac{(0.02482)}{0.6} = 6.10 \text{ W/m}^2 \cdot \text{C}
\]

\[
\overline{q_w} = \overline{h} \overline{\Delta T} = 6.10 (76 - 15) = 372 \text{ W/m}^2
\]

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A two foot square slab of mild steel leaves a forging operation 0.25 in.
thick at 1000°C. It is laid flat on an insulating bed and 27°C air is
blown over it at 30 m/s. How long will it take to cool to 200°C.
Assume the flow is laminar and state your assumptions about
property evaluation.

Compute \( \overline{h} \) based on air at \( \frac{1}{2} \left[ \frac{1000 + 27}{2} + \frac{200 + 27}{2} \right] = 313.5 \, ^\circ C \).
(This is pretty coarse. The value of \( z \) drops by a factor of four
over the range of film temperature involved.) Then:
\[
\overline{h} = 4.96 \times 10^{-5}, \quad Pr = 0.698, \quad k = 0.0418
\]
And \( \rho c_{\text{steel}} = 3.64 \times 10^6 \, J/m^2\cdot K \), \( k_{\text{steel}} = 35 \) at 600°C
\[
Re_L = \frac{2(0.3098)30}{4.96 \times 10^{-5}} = 368,710
\]
\[
Nu_L = 0.6649 Re_L^{1/2} Pr^{1/3} = 357.7 \quad \overline{h} = 357.7 \frac{0.0418}{2(0.3098)} = 26.3 \frac{W}{m^2\cdot K}
\]
Can we use lumped capacity? Check \( Bi = \frac{\overline{h} L}{k} = \frac{26.3(0.0259 \times 1/4)}{35} = 0.048 \ll 1 \)
Lumped capacity is fine. Then:
\[
T = \frac{\rho c \text{ thickness}}{\overline{h}} = \frac{3.64 \times 10^6(0.0259 \times 1/4)}{26.3} = 879 \text{ sec.}
\]
Finally:
\[
\frac{T-T_\infty}{T_i-T_\infty} = e^{-t/T} = \frac{100-27}{1000-27} = 0.1778 = e^{-t/879}
\]
So:
\[
t = 1.519 \text{ sec} = 25.3 \text{ minutes}
\]
This is a long time. Air is an ineffective coolant,
6.28 Do Problem 6.27 numerically, recalculating properties at successive points. If you did Problem 6.27, compare results.

First compute \( \dot{h} \) at

\[
\frac{\Delta T}{\alpha} = \frac{L_{c}^{2}}{K} \frac{1}{q} \frac{1}{P_{r}} \frac{1}{P_{r}} \frac{1}{\Delta T^{2}}
\]

\[
\overline{\dot{h}} = 0.669 \frac{Re \nu}{\sqrt{P_{r}}}
\]

\[
\dot{h} = 0.465 \frac{8 \sqrt{P_{r}} \nu}{\Delta T^{1/2}}
\]

Now compute \( \frac{\Delta T}{\alpha} \) thickness \( \frac{\dot{h}}{q} \text{ at } 23.114 \) use \( \frac{I-I_{0}}{I_{0}} = e^{-2/\alpha} \) to advance 100°C per step. Thus

\[
\begin{array}{c|c|c|c|c}
T \text{ from previous } & T_{0} \text{ sec} & T_{0-77} & T_{0-77} & t_{0-77} \\
900°C & 851 & 0.897 & 96.6 & 96.6 \\
800 & 889 & 0.885 & 108.2 & 204.8 \\
700 & 826 & 0.806 & 122.8 & 327.6 \\
600 & 884 & 0.874 & 142.2 & 469.8 \\
500 & 830 & 0.823 & 168.7 & 638.8 \\
400 & 786 & 0.788 & 208.1 & 846.6 \\
300 & 872 & 0.139 & 272.2 & 1118.8 \\
200 & 867 & 0.039 & 395.5 & 1514.3 \\
\end{array}
\]

We conclude that, since this is within 4.7 sec or 0.31 percent of the approximate result, the averaging that was used in Problem 6.27 is quite good in this case. The high variation of \( \nu \) is compensated in its influence on \( \dot{h} \) by the variation of \( k \). Furthermore the initial under-estimate of \( \frac{\Delta T}{\alpha} \) is compensated by the subsequent over-estimate of \( \frac{\Delta T}{\alpha} \).
6.31 A thin metal sheet separates air at 44°C, flowing at 48 m/s, from water at 4°C, flowing at 0.2 m/s. Both fluids start at a leading edge and move in the same direction. Plot $T_{\text{plate}}$ and $q$ as a function of $x$ up to $x = 0.1$ m.

Make first calculation evaluating properties on the basis of $T_{\text{plate}} = 10^\circ C$ (so $T_{\text{air}} = 27^\circ C$ and $T_{\text{H}_2O} = 5^\circ C$). Then:

\begin{align*}
\delta_{\text{air}} &= 1.566 (10)^{-5} \\
k_{\text{air}} &= 0.02614 \\
\delta_{\text{H}_2O} &= 1.422 (10)^{-6} \\
k_{\text{H}_2O} &= 0.6084
\end{align*}

So:

\begin{align*}
Re_{\text{air}} &= \frac{U x}{v_{\text{air}}} = \frac{48 x}{1.566 \times 10^{\circ} C} = 3.046 \times 10^6 x \\
Re_{\text{H}_2O} &= \frac{U x}{v_{\text{H}_2O}} = \frac{0.2 x}{1.422 \times 10^{\circ} C} = 140.650 x
\end{align*}

Thus the flows should be laminar: $h = k x = 0.322 Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}}$

\begin{align*}
h_{\text{air}} &= \frac{0.02614}{x} \left[ \frac{1}{3046 (10)^6 x} \right]^{\frac{1}{2}} \frac{0.711}{10^3} = 13.15 \times \sqrt{x} \\
h_{\text{H}_2O} &= \frac{0.6084}{x} \left[ \frac{1}{140.650 x} \right]^{\frac{1}{2}} \frac{10.26}{10^3} = 160.0 \times \sqrt{x}
\end{align*}

Then:

\[ U = \frac{1}{h_{\text{air}}} + \frac{1}{h_{\text{H}_2O}} = \frac{1}{13.15} + \frac{1}{160} \]

\[ q = \frac{UA \Delta T}{\frac{1}{13.15} (44 - 9)} = \frac{486}{\sqrt{x}} \]

and since $q = h_{\text{air}} (44 - T_{\text{plate}})$ so $T_{\text{plate}} = 44 - \frac{486}{13.15 \sqrt{x}} = 704^\circ C$
A mixture of 60% glycerin and 40% water flows over a 1m long flat plate.

The glycerin is at 20°C and the plate is at 40°C. A thermocouple, 1mm above the trailing edge records 35°C. What is \( u_\infty \), and what \( u \) at the thermocouple?

Using the momentum integral result (eqn. 6.50) we have:

\[
\frac{35-20}{40-20} = 1 - \frac{3}{2} \frac{y}{\delta_t} + \frac{1}{2} \left( \frac{y}{\delta_t} \right)^3
\]

from which we obtain, by trial and error, \( \frac{y}{\delta_t} = 0.1678 \)

Thus, at \( y = 0.001 \text{ m} \), \( \delta_t = 0.00596 \text{ m} \).

And (eqn. 7.55) \( \delta = Pr^{1/3} \delta_t = 49.3^{1/3}(0.00596) = 0.02185 \text{ m} \)

Finally from eqn. (7.2) \( \frac{\delta}{x} = \frac{4.92}{Re_{\text{h}}} \) ; \( \delta = \frac{4.92(1)}{\sqrt{\frac{U_\infty}{689(0.06)}}} = 0.02185 \text{ m} \)

So \( U_\infty = 0.349 \text{ m/s} \)

(Is the flow really laminar? Check it: \( Re_{\text{h}} = \frac{0.349(1)}{499} = 0.0007 \). Yes)

Then, at the thermocouple: \( \frac{u}{\delta} = 0.0458 \), so \( \frac{u}{u_\infty} = \frac{3}{2}(0.0458) - \frac{1}{2}(0.0458)^3 \)

\( = 0.0686, \quad u = 0.0239 \text{ m/s} \)

Thus the thermocouple is fairly deep into the thermal b.l. and very deep into the flow b.l. Note that slightly greater accuracy would have resulted from the consistent use of Table 6.1 in place of the integral method approximations.
6.33 What is the maximum $h$ that can be achieved in laminar flow over a 5m plate, based on data from Table A.3? What physical circumstances give this result?

$$h = 0.664 k P_r^{1/3} \sqrt{\frac{u_{\infty}}{L}} \text{, but } \frac{u_{\infty}}{D} \ll 3.5 \times 10^{-5} \text{ so:}$$

$$h_{\text{max}} = 0.664 k P_r^{1/3} \sqrt{3.5 \times 10^{-5} / 5^2} = 78.56 k P_r^{1/3}$$

The largest $k P_r^{1/3}$ is $12.3$, for glycerin at $0^\circ C$ (a viscous oil might will be better if we had $k$ data.) The gives:

$$h_{\text{max}} = 966 \text{ W/m}^2\text{.}$$

This corresponds with $u_{\infty} = 3.5 \times 10^{-5} (0.0053)/5 = 581 \text{ m/s}$. This looks high for a real system. It would require a Herculean pump.

6.34 A $17^\circ C$ sheet of water, $\Delta_1 \text{m}$ thick and moving at a constant speed $u_{\infty} \text{m/s}$, impacts a horizontal plate at $45^\circ$, turns, and flows along it. Develop a dimensionless equation for the thickness $\Delta_2$ at a distance $L$ from the point of impact. Assume that $\delta \ll \Delta_2$. Evaluate the result for $u_{\infty} = 1$, $\Delta_1 = 0.01$, $L = 0.1 \text{ m}$, in water at $27^\circ C$.

\[ \text{Mass balance:} \int_0^{\Delta_1} \rho u_{\infty} \Delta_1 = \int_0^{\Delta_2} \rho u_\theta dy + \int_0^{\Delta_2} \rho u_{\infty} dy \]

\[ 1 = \frac{\delta}{\Delta_1} \int_0^{\Delta_2} \left( \frac{3}{2} \frac{f}{\delta} - \frac{1}{2} f^3 \right) df + \frac{\Delta_2 - \delta}{\Delta_1} \]

\[ \frac{3}{4} - \frac{1}{8} = \frac{5}{8} \]

So:

\[ \frac{\Delta_2}{\Delta_1} = \frac{1 + \frac{3\delta}{8\Delta_1}}{\frac{5}{8}} \]

The momentum eqn. is already summarized in:

\[ \frac{\delta}{L} = \frac{4.92}{\sqrt{Re_L}} \]

\[ \text{so:} \frac{\Delta_2}{\Delta_1} = 1 + \frac{3}{8} \frac{4.92}{\sqrt{Re_L}} \frac{L}{\Delta_1} = 1 + \frac{1.845}{\sqrt{Re_L}} \frac{L}{\Delta_1} \]

In the case in point: $Re_L = \frac{0.1 \times 1}{0.926 (10)^{0.8}} = 121,065 \text{ so}$

\[ \frac{\Delta_2}{\Delta_1} = 1 + \frac{1.845}{\sqrt{121,065}} = 1.053 \]

(at this point $\delta = 0.00141 \text{ m which is } \ll \Delta_1$) Thus, contrary to the sketch above, the sheet swells to accommodate the reduced speed near the wall.)

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6.35 A good approximation to the temperature dependence of \( \mu \) in gases is given by the Sutherland formula: 

\[
\frac{\mu}{\mu_{\text{ref}}} = \left( \frac{T}{T_{\text{ref}}} \right)^{1.5} T_{\text{ref}}^{-1 \frac{S}{T-S}},
\]

where the reference state can be chosen anywhere. Use data for air at two points to evaluate \( S \) for air. Use this value to predict a third point. (\( T \) and \( T_{\text{ref}} \) are expressed in °K.)

Students might use any points from Table A-6. Let us do the problem for air using a value of \( T_{\text{ref}} = 300^\circ \text{K} \) and a temperature, \( T \), of interest equal to \( 500^\circ \text{K} \).

First we calculate \( S \) based on the known values of

\[
\mu(T = T_{\text{ref}} = 300^\circ \text{K}) = 1.857(10)^{-5} \text{ kg/m-s}
\]

\[
\mu(T = 400^\circ \text{K}) = 2.310(10)^{-5} \text{ kg/m-s}
\]

Using these values in Sutherland's formula, we get

\[
S = 120.7^\circ \text{K}
\]

Then, using this \( S \) and \( \mu_{\text{ref}}(T_{\text{ref}}) \) in Sutherland's formula, we obtain:

\[
\mu(T = 500^\circ \text{K}) = 2.71(10)^{-5} \text{ kg/m-s}
\]

which is exactly the tabled value to three decimal places.
6.36 We have derived a steady-state continuity equation in Section 6.3. Derive the time-dependent three-dimensional version of the equation:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0
\]

To do this, paraphrase the development of equation (2.14) requiring that mass be conserved instead of energy.

Now we use Gauss' theorem, \( \int_S (\rho \mathbf{u}) \cdot \hat{n} \, dS = \int_R \nabla \cdot \rho \mathbf{u} \, dR \) to get:

\[
\int_R (\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{u}) \, dR = 0
\]

(where we have used Leibnitz' rule as we did in the context of eqn. (6.24.)

Finally, since the integral must vanish identically, we obtain:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{u} = 0
\]

Notice that, since \( \nabla \cdot \rho \mathbf{u} = \rho \nabla \cdot \mathbf{u} + \mathbf{u} \cdot \nabla \rho \), this can be rewritten as:

\[
\frac{D \rho}{D t} + \rho \nabla \cdot \mathbf{u} = 0
\]

where \( \frac{D}{Dt} \) is the substantial derivative, \( (\frac{\partial }{\partial t} + \mathbf{u} \cdot \nabla )\).
6.37 Various considerations show that the smallest scale motions in a turbulent flow have no preferred spatial orientation at large enough values of Re. Moreover, these small eddies are responsible for most of the viscous dissipation of kinetic energy. The dissipation rate, $\varepsilon \, (W/kg)$, may be regarded as given information about the small scale motion, since it is set by the larger scale motion. Both $\varepsilon$ and $\nu$ are governing parameters of the small scale motion.

a.) Find the characteristic length and velocity scales of the small scale motion. These are called the Kolmogorov scales of the flow.

b.) Compute Re for the small scale motion, and interpret the result.

c.) The Kolmogorov length scale characterizes the smallest motions found in a turbulent flow. If $\varepsilon$ is 10 W/kg and the mean free path is $7(10^{-6})$ m, for air at standard conditions, show that turbulent motion is a continuum phenomenon and it is thus properly governed by the equations of this chapter.

\[ \begin{align*}
\text{a.) } & \text{Length } \eta \text{ and velocity scales, } \nu \text{, can be formed from } \varepsilon = \frac{W}{kg} = \frac{m^2}{s^3} \\
& \text{and } \nu = \frac{m^2}{s} \text{. We get:} \\
& \text{length scale, } \eta = \left( \frac{s^3}{\varepsilon} \right)^{1/4} \\
& \text{velocity scale, } \nu = \left( \frac{2 \eta}{\varepsilon} \right)^{1/4} \\
\text{b.) } & \text{Re} = \frac{\nu \eta}{\nu} = \frac{1}{2} \left( \frac{s^3}{\varepsilon} \right)^{1/4} \left( \frac{2 \eta}{\varepsilon} \right)^{1/4} = \frac{1}{2} \quad \text{since viscosity balances inertia, the small scales are extremely viscous.} \\
\text{c.) } & \text{For air at } 300^\circ \text{K, } \nu = 1.566 (10^{-5}) \text{ m}^2/\text{s} \text{, we get} \\
& \eta = \left( 1.566 (10^{-5}) \right)^{3/4} / 10^{1/4} = 0.00014 \text{ m} \\
\text{This is far larger than the mean free path. Therefore turbulent motion is a continuum phenomenon.} 
\end{align*} \]
**Problem 6.46** Two power laws are available for the skin friction coefficient in turbulent flow: \( C_f(x) = 0.027 \frac{1}{\sqrt{\text{Re}_x}} \) and \( C_f(x) = 0.059 \frac{1}{\sqrt[5]{\text{Re}_x}} \). The former is due to White and the latter to Prandtl [6.4]. Equation (6.102) is more accurate and wide ranging than either. Plot all three expressions on semi-log coordinates for \( 10^5 \leq \text{Re}_x \leq 10^9 \). Over what range are the power laws in reasonable agreement with eqn. (6.102)? Also plot the laminar equation (6.33) on same graph for \( \text{Re}_x \leq 10^6 \). Comment on all your results.

**Solution** The figure shows the two power laws and the mentioned turbulent and laminar expressions:

\[
C_f = \frac{0.455}{[\ln(0.06 \text{Re}_x)]^2} \quad (6.102)
\]

\[
C_f = \frac{0.664}{\sqrt{\text{Re}_x}} \quad (6.33)
\]

The \( \frac{1}{7} \) power law is within 5% of eqn. (6.102) for \( 3.5 \times 10^5 \leq \text{Re}_x \leq 10^9 \), while the \( \frac{1}{5} \) power law is within 5% for \( 10^5 \leq \text{Re}_x \leq 5 \times 10^7 \). We also observe that skin friction in laminar flow is far less than in turbulent flow.
**Problem 6.47** Reynolds et al. [6.27] provide the following measurements for air flowing over a flat plate at 127 ft/s with $T_{so} = 86$ °F and $T_w = 63$ °F. Plot these data on log-log coordinates as $Nu_x$ vs. $Re_x$, and fit a power law to them. How does your fit compare to eqn. (6.112)?

<table>
<thead>
<tr>
<th>$Re_x \times 10^6$</th>
<th>$St \times 10^3$</th>
<th>$Re_x \times 10^6$</th>
<th>$St \times 10^3$</th>
<th>$Re_x \times 10^6$</th>
<th>$St \times 10^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.255</td>
<td>2.73</td>
<td>1.353</td>
<td>2.01</td>
<td>2.44</td>
<td>1.74</td>
</tr>
<tr>
<td>0.423</td>
<td>2.41</td>
<td>1.507</td>
<td>1.85</td>
<td>2.60</td>
<td>1.75</td>
</tr>
<tr>
<td>0.580</td>
<td>2.13</td>
<td>1.661</td>
<td>1.79</td>
<td>2.75</td>
<td>1.72</td>
</tr>
<tr>
<td>0.736</td>
<td>2.11</td>
<td>1.823</td>
<td>1.84</td>
<td>2.90</td>
<td>1.68</td>
</tr>
<tr>
<td>0.889</td>
<td>2.06</td>
<td>1.970</td>
<td>1.78</td>
<td>3.05</td>
<td>1.73</td>
</tr>
<tr>
<td>1.045</td>
<td>2.02</td>
<td>2.13</td>
<td>1.79</td>
<td>3.18</td>
<td>1.67</td>
</tr>
<tr>
<td>1.196</td>
<td>1.97</td>
<td>2.28</td>
<td>1.73</td>
<td>3.36</td>
<td>1.54</td>
</tr>
</tbody>
</table>

**Solution**

The film temperature is $T_f = (63 + 86)/2 = 74.5$ °F = 23.6 °C = 296.8 K. At this temperature, Table A.6 gives $Pr = 0.707$. We can convert the given data to $Nu_x = St Re_x Pr$ using a spreadsheet.

To make a fit, we must recognize that $Pr$ does not vary. We have no basis for fitting a $Pr$ exponent. So, we can fit to

$$Nu_x = A Re_x^b$$

This fit may be done by linear regression if we first take the logarithm:

$$\ln Nu_x = \ln A + b \ln Re_x$$

Using a spreadsheet, we can calculate the logarithms and perform the linear regression to find $A = 0.0187$ and $b = 0.814$ ($r^2 = 0.9978$), or

$$Nu_x = 0.0187 \, Re_x^{0.814}$$

The fit is plotted with the equation, and the agreement is excellent.

With some additional effort, we may use the spreadsheet to find that the standard deviation of the data with respect to the fit is $s_x = 2.81\%$, which provides a 95% confidence interval (two-sided $t$-statistic for 21 points, $\pm 2.08s_x$) of $\pm 5.8\%$.

Equation (6.112) for $Pr = 0.712$,

$$Nu_x = 0.0296 \, Re_x^{0.8} Pr^{0.6} = 0.0240 \, Re_x^{0.8}$$

(6.112)

is also plotted in the figure, but it is systematically higher than this data set and our fit. (Reynolds et al. had 7 other data sets and reported an overall $s_x = 4.5\%$ for a $\pm 9\%$ uncertainty at 95% confidence.)
Pr = 0.707 (air)

T\textsubscript{w} = constant

Reynolds number, \textit{Re}\textsubscript{x}

Nusselt number, \textit{Nu}\textsubscript{x} = 0.0296 \textit{Re}\textsubscript{x}\textsuperscript{0.8} Pr\textsuperscript{0.6} = 0.0240 Re\textsubscript{x}\textsuperscript{0.8}

My fit, \textit{Nu}\textsubscript{x} = 0.0187 Re\textsubscript{x}\textsuperscript{0.814}
**Problem 6.48** Blair and Werle [6.36] reported the b.l. data below. Their experiment had a uniform wall heat flux with a 4.29 cm unheated starting length, \( u_\infty = 30.2 \text{ m/s} \), and \( T_\infty = 20.5^\circ\text{C} \).

a) Plot these data as \( \text{Nu}_x \) versus \( \text{Re}_x \) on log-log coordinates. Identify the regions likely to be laminar, transitional, and turbulent flow.

b) Plot the appropriate theoretical equation for \( \text{Nu}_x \) in laminar flow on this graph. Does the equation agree with the data?

c) Plot eqn. (6.112) for \( \text{Nu}_x \) in turbulent flow on this graph. How well do the data and the equation agree?

d) At what \( \text{Re}_x \) does transition begin? Find values of \( c \) and \( \text{Re}_t \) that fit eqn. (6.116b) to these data, and plot the fit on this graph.

e) Plot eqn. (6.117) through the entire range of \( \text{Re}_x \).

<table>
<thead>
<tr>
<th>( \text{Re}_x \times 10^{-6} )</th>
<th>( \text{St} \times 10^3 )</th>
<th>( \text{Re}_x \times 10^{-6} )</th>
<th>( \text{St} \times 10^3 )</th>
<th>( \text{Re}_x \times 10^{-6} )</th>
<th>( \text{St} \times 10^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.112</td>
<td>2.94</td>
<td>0.362</td>
<td>1.07</td>
<td>1.27</td>
<td>2.09</td>
</tr>
<tr>
<td>0.137</td>
<td>2.23</td>
<td>0.411</td>
<td>1.05</td>
<td>1.46</td>
<td>2.02</td>
</tr>
<tr>
<td>0.162</td>
<td>1.96</td>
<td>0.460</td>
<td>1.01</td>
<td>1.67</td>
<td>1.96</td>
</tr>
<tr>
<td>0.183</td>
<td>1.68</td>
<td>0.505</td>
<td>1.05</td>
<td>2.06</td>
<td>1.84</td>
</tr>
<tr>
<td>0.212</td>
<td>1.56</td>
<td>0.561</td>
<td>1.07</td>
<td>2.32</td>
<td>1.86</td>
</tr>
<tr>
<td>0.237</td>
<td>1.45</td>
<td>0.665</td>
<td>1.34</td>
<td>2.97</td>
<td>1.74</td>
</tr>
<tr>
<td>0.262</td>
<td>1.33</td>
<td>0.767</td>
<td>1.74</td>
<td>3.54</td>
<td>1.66</td>
</tr>
<tr>
<td>0.289</td>
<td>1.23</td>
<td>0.865</td>
<td>1.99</td>
<td>4.23</td>
<td>1.65</td>
</tr>
<tr>
<td>0.312</td>
<td>1.17</td>
<td>0.961</td>
<td>2.15</td>
<td>4.60</td>
<td>1.62</td>
</tr>
<tr>
<td>0.338</td>
<td>1.14</td>
<td>1.06</td>
<td>2.24</td>
<td>4.83</td>
<td>1.62</td>
</tr>
</tbody>
</table>

**Solution**

a) Calculate the Nusselt number from the values of Stanton number using \( \text{Nu}_x = \text{St} \cdot \text{Pr} \cdot \text{Re}_x \).

This is easily done with software (or by hand if you are patient) using \( \text{Pr} = 0.71 \). The results are plotted on the next page. The regions can be identified from the changes in slope and curvature (part b makes the laminar regime more obvious).

b) The appropriate formula is eqn. (6.116) for a laminar b.l. with an unheated starting length:

\[
\text{Nu}_{\text{lam}} = \frac{0.4587 \text{Re}_x^{1/2} \text{Pr}^{1/3}}{\left[1 - (x_0/x)^{3/4}\right]^{1/3}}
\]  
(6.116)

We have only \( \text{Re}_x \), not \( x \). However,

\[
\frac{x_0}{x} = \frac{\text{Re}_x}{\text{Re}_x} \quad \text{and} \quad \text{Re}_x = \frac{u_\infty x_0}{\nu} = \frac{(30.2)(0.0429)}{1.516 \times 10^{-5}} = 8.546 \times 10^4
\]

With this, the expression can be plotted. The agreement is pretty good. (Equation (6.71) is shown for comparison.)

c) The equation,

\[
\text{Nu}_{\text{turb}} = 0.0296 \text{Re}_x^{0.8} \text{Pr}^{0.6}
\]  
(6.112)

is plotted in the figure, with excellent agreement.

d) To use eqn. (6.114b), we can start by visualizing a straight line through the transitional data on the log-log plot to determine the slope, \( c \). This slope can be determined iteratively if using
software, or by drawing the line if working by hand. The slope is well fit by $c = 2.5$. Once the slope is found, we find the point at which this line intersects the laminar, unheated starting length curve. That point is well represented by $Re_t = 500,000$ and $Nu_{lam}(Re_t, Pr) = 321$. Hence,

$$Nu_{trans} = Nu_{lam}(Re_t, Pr) \left( \frac{Re_x}{Re_t} \right)^c = 321 \left( \frac{Re_x}{500,000} \right)^{2.5}$$  \hspace{1cm} (6.114b)

This equation is plotted in the figure, with very good agreement. Note that slightly different values of $Re_t$ and $Nu_{lam}$ may produce a good fit, if they lie on the same line. The best approach is to find $Re_t$ and then calculate $Nu_{lam}$ from eqn. (6.116).

e) Equation (6.117) uses the laminar, transitional, and turbulent Nusselt numbers from parts (b), (c), and (d):

$$Nu_x(Re_x, Pr) = \left[ Nu_{x,lam}^{5} + \left( Nu_{x,trans}^{-10} + Nu_{x,turb}^{-10} \right)^{-1/2} \right]^{1/5}$$  \hspace{1cm} (6.117)

This equation is plotted in the figure as well, with very good agreement.
**Problem 6.49**  Figure 6.21 shows a fit to the following air data from Kestin et al. [6.29] using eqn. (6.117). The plate temperature was 100 °C (over its entire length) and the free-stream temperature varied between 20 and 30 °C. Follow the steps used in Problem 6.48 to reproduce that fit and plot it with these data.

<table>
<thead>
<tr>
<th>Re (_x)×10^{-3}</th>
<th>Nu (_x)</th>
<th>Re (_x)×10^{-3}</th>
<th>Nu (_x)</th>
<th>Re (_x)×10^{-3}</th>
<th>Nu (_x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60.4</td>
<td>42.9</td>
<td>445.3</td>
<td>208.0</td>
<td>336.5</td>
<td>153.0</td>
</tr>
<tr>
<td>76.6</td>
<td>66.3</td>
<td>580.7</td>
<td>289.0</td>
<td>403.2</td>
<td>203.0</td>
</tr>
<tr>
<td>133.4</td>
<td>85.3</td>
<td>105.2</td>
<td>71.1</td>
<td>509.4</td>
<td>256.0</td>
</tr>
<tr>
<td>187.8</td>
<td>105.0</td>
<td>154.2</td>
<td>95.1</td>
<td>907.5</td>
<td>522.0</td>
</tr>
<tr>
<td>284.5</td>
<td>134.0</td>
<td>242.9</td>
<td>123.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Solution**

a) The results are plotted on the next page. The regions can be identified from the changes in slope.

b) The appropriate formula is eqn. (6.58) for a laminar b.i. on a uniform temperature plate:

\[
\text{Nu}_{\text{lam}} = 0.332 \frac{\text{Re}_x^{1/2}}{\text{Pr}^{1/3}}
\]  

(6.58)

The film temperature is between 60 and 65 °C, so Pr = 0.703. This equation is plotted on the figure. Only two data points touch the line, but they are in excellent agreement.

c) The appropriate equation,

\[
\text{Nu}_{\text{turb}} = 0.0296 \frac{\text{Re}_x^{0.8}}{\text{Pr}^{0.6}}
\]  

(6.112)

is plotted in the figure, with very good agreement.

d) To use eqn. (6.114b), we can start by visualizing a straight line through the transitional data on the log-log plot to determine the slope, c. The slope is well fit by c = 1.7. Once the slope is found, we find the point at which this line intersects the laminar, unheated starting length curve. That point is well represented by Re\(_l\) = 60,000 and \(\text{Nu}_{\text{lam}}(\text{Re}_l, \text{Pr})\) = 72.3. Hence,

\[
\text{Nu}_{\text{trans}} = \text{Nu}_{\text{lam}}(\text{Re}_l, \text{Pr}) \left( \frac{\text{Re}_x}{\text{Re}_l} \right)^c = 72.3 \left( \frac{\text{Re}_x}{60000} \right)^{1.7}
\]  

(6.114b)

This equation is plotted in the figure, with good agreement. Note that the most consistent approach is to find Re\(_l\) and then calculate \(\text{Nu}_{\text{lam}}\) from eqn. (6.58).

e) Equation (6.117) uses the laminar, transitional, and turbulent Nusselt numbers from parts (b), (c), and (d):

\[
\text{Nu}_x(\text{Re}_x, \text{Pr}) = \left[ \text{Nu}_{x,\text{lam}}^5 + \left( \text{Nu}_{x,\text{trans}}^{-10} + \text{Nu}_{x,\text{turb}}^{-10} \right)^{-1/2} \right]^ {1/5}
\]  

(6.117)

This equation is plotted in the figure as well, with very good agreement in the turbulent and transitional ranges. The laminar fit looks good with one data point, but not the other one. The data themselves make a sharp leap between Re\(_x\) of 66,300 and 85,300. (Kestin et al. varied the Reynolds number between these data by increasing the air speed, \(u_{\infty}\)—these data are not from spatially sequential points (unlike the data of Blair in Problem 6.48). The onset of turbulence is an instability, and the change in flow conditions may well have affected the transition.)
Pr = 0.703 (air)
Tw = constant

Eqn. (6.58), 0.332 Re^{1/2}Pr^{1/3}
Eqn. (6.114b), c = 1.7, Re_l = 60,000
Eqn. (6.112), 0.0296 Re^{0.8}Pr^{0.6}
Eqn. (6.117), c = 1.7, Re_l = 60,000
Kestin et al. (1961)
**Problem 6.50** A study of the kinetic theory of gases shows that the mean free path of a molecule in air at one atmosphere and 20 °C is 67 nm and that its mean speed is 467 m/s. Use eqns. (6.45) obtain $C_1$ and $C_2$ from the known physical properties of air. We have asserted that these constants should be on the order of 1. Are they?

**Solution** We had found that

$$\mu = C_1 \left( \rho \bar{C} \ell \right) \quad (6.45c)$$

and

$$k = C_2 \left( \rho c_v \bar{C} \ell \right) \quad (6.45d)$$

We may interpolate the physical properties of air from Table A.6: $\mu = 1.82 \times 10^{-5}$ kg/m·s, $k = 0.0259$ W/m·K, $\rho = 1.21$ kg/m$^3$, and $c_p = 1006$ J/kg·K. In addition, the specific heat capacity ratio for air is $\gamma = c_p / c_v = 1.4$.

Rearranging:

$$C_1 = \frac{\mu}{\rho \bar{C} \ell} = \frac{1.82 \times 10^{-5}}{(1.21)(467)(67 \times 10^{-9})} = 0.481$$

and

$$C_2 = \frac{k \gamma}{\rho c_p \bar{C} \ell} = \frac{(0.0259)(1.4)}{(1.21)(1006)(467)(67 \times 10^{-9})} = 0.952$$

The constants are indeed $\mathcal{O}(1)$. 

206e
7.1 Relate $u_{avg}$ to $dp/dx$ in laminar pipe flow.

$$
\int_0^{L/2} D^2 = \rho \int_0^{L/2} u(r) \frac{D}{Dx} \frac{dr}{2\pi} \int_0^{L/2} \left[ 1 - \frac{(2r^2)}{D^2} \right] r dr \left( \frac{\rho}{4\mu} \int_0^{L/2} \left( -\frac{dp}{dx} \right) \frac{dr}{2\pi} \right)
$$

so

$$
u_{avg} = \frac{-\frac{dp}{dx}}{\frac{32\mu}{D^2}} = \frac{-\frac{dp}{dx}}{\frac{8\mu}{D}}$$

7.2 Consider the air flow shown: After $x = 0$ either:

(a) $T_w = 68.4^\circ C$

or (b) $q_w = 378 \text{ W/m}^2$

Plot $T_w$, $q_w$, and $T_b$ vs. $x$ in each case.

First evaluate $\frac{2x}{6x} = \frac{2x}{u_{avg}D^3/\alpha} = \frac{2x}{2(x)(0.01)T/2.205(10^{-5})} = 0.2205 x$

And

$$
Nu_D = \frac{\frac{q_w D}{(T_w-T_b)k}} = \frac{0.01}{0.02614} \frac{q_w}{\Delta T} = 0.383 \frac{q_w}{\Delta T}
$$

From eqn. (7.4)

$$
q_w = \frac{\rho c u_{avg} D}{A} \frac{dT_b}{dx} = \frac{1193(100)(2)(0.01)}{4} \frac{dT_b}{dx}
$$

or

$$
q_w = 5.93 \frac{dT_b}{dx}
$$

a) $T_w = 68.4^\circ C = \text{ constant}$

$$
\frac{dT_b}{dx} = \frac{T_{bi+1} - T_{bi}}{\Delta x} = 0.169 \frac{dT_b}{dx} = 0.169 \frac{T_w - T_{bi+1}}{\Delta x}
$$

so

$$
T_{bi+1} = T_{bi} + 0.441 \Delta x \frac{Nu_D}{0.383}
$$

or

$$
T_{bi+1} = \frac{T_{bi} + 30.16 \Delta x \frac{Nu_D}{1 + 0.441 \Delta x \frac{Nu_D}}}{1 + 0.441 \Delta x \frac{Nu_D}{}
$$

Now we pick an $x = 0$ where $Nu_D$ will equal $\alpha$. This gives $T_{bi+1} = 27.0$. Then we advance by $\Delta x = 0.02$, where we can get a new $Nu_D$ from Fig. 8.4. Then we march forward as shown in the table below.

(b) $\frac{dT_b}{dx} = 0.169(378) = 63.88 \text{ so } T_b = 27 + 63.88 A$
Problem 7.2: Added Note

Equation (7.30) expresses \( \text{Nu}_D \) as a function of the local Graetz Number, for a constant wall heat flux. We could thus use it, with the help of a spreadsheet, rather than reading from Fig. 7.4.

For a constant wall temperature, we can use equation (7.57) to find \( T_b \). And equation (7.29) gives the overall heat transfer coefficient. We can then let \( L \) be the local value of \( x \), and use these two equations to calculate \( T_b \) and the overall heat transfer coefficient at points along the tube. Once again, a spreadsheet would allow us to carry out the calculations and to plot the graph.
7.3 Prove that $C_f = 16/\text{Re}_D$ in laminar pipe flow.

$$C_f = -\frac{\tau_w}{\rho \bar{u}^2/2}, \text{ but } \tau_w = |\mu \frac{\partial u}{\partial r}|_{r=R} = \mu |2\bar{u}(0-2\frac{r}{R})|_{r=R} = -4\frac{\mu \bar{u}}{R} \quad (\text{eqn. (8,8)})$$

So:

$$C_f = \frac{2}{\rho \bar{u}^2} \left( \frac{4\mu \bar{u}}{R} \right) = \frac{8\mu \bar{u}}{\bar{u} R} = \frac{16}{\bar{u} \partial \bar{u}/\partial s} = \frac{16}{\text{Re}_D} \quad \left[ \frac{1}{f} = 4C_f = 64/\text{Re}_D \right]$$
7.4 (a) Find \( \overline{h} \) for the flow shown:

\[
\text{air} \quad \text{cylinder} \quad \begin{array}{c}
\begin{aligned}
&\text{200}^\circ \text{C} \\
&240 \\
&4 \text{ m/s}
\end{aligned}
\end{array} \quad 0.03 \text{ m}
\]

(b) If the flow were H_2O at 200\(^\circ\)C, what velocity would give the same \( \overline{h} \), the same \( \overline{N}u_D \), and same \( \text{Re}_D \)? Evaluate properties at 220\(^\circ\)C.

\[
\text{a) } \text{Re}_D = \frac{u_0 D}{v} = \frac{4(0.03)}{3.106 \times 10^{-5}} = 3863 \text{ so from Fig. 7.14 } \overline{N}u_D = 0.3 \left[ 1 + \left( \frac{\alpha}{\alpha_D} \right) \right]^{1/4}
\]

but \( \alpha = 0.698 \) so we calculate \( \overline{N}u_D = 30.64 \)

\[
\text{or: } \overline{h} = \frac{k \overline{N}u_D}{D} = \frac{0.0355(30.64)}{0.03} = 36.7 \text{ W/m}^2\text{K}
\]

\[
\text{b) For H}_2\text{O, } \text{Re}_D = \frac{u_0 (0.03)}{1.444 \times 10^{-7}} = 3863, \text{ so } u_0 = \text{ only } 0.0186 \text{ m/s}
\]

At \( \overline{N}u_D = 30.64 \),

\[
\frac{30.34}{(0.811)^5} \left[ 1 + \left( \frac{0.9}{0.811} \right)^{1/3} \right]^{1/4} = 35.7
\]

where we read from Fig. 7.14 \( \text{Re}_D = 3250 = \frac{u_0 (0.03)}{1.444 \times 10^{-7}} \).

\[
\text{so } u_0 = 0.0156 \text{ m/s}
\]

\[
\text{If } \overline{h} = 36.7:
\]

\[
\frac{36.7(0.03) - 0.3}{0.691(0.811)^{1/5}} \left[ 1 + \left( \frac{0.9}{0.811} \right)^{1/3} \right]^{1/4} = 1.67
\]

so we use eqn. (7.66) to calculate \( \text{Re} = (1.47/0.62)^2 = 7.255 \).

\[
\text{This gives } 7.255 = \frac{u_0 (0.03)}{1.444 (10)^{-7}} \text{ so } u_0 = 0.000035 \text{ m/s}
\]

which is absurdly slow!

\[
\text{C) } \overline{N}u_D = f(\text{Re}_D, \text{Pr}_D) \quad \text{In hot water, } \text{Pr}_D \approx \text{Pr}_D \text{ air closely enough to make a good approximation.}
\]

This can be made equal to \( \text{Re}_D \) for an air flow, if \( u_0 \) is kept very low.

Thus we can model the air flow approximately in water.
**Problem 7.5** Compare the $h$ value computed in Example 7.3 with values predicted by the Dittus-Boelter, Colburn, McAdams, and Sieder-Tate equations. Comment on this comparison.

**Solution:** Taking values of components from Example 7.3, we get:

\[
\begin{align*}
h_{DB} &= (k/D)(0.0243)(Pr)^{0.4}(Re_D)^{0.8} \\
&= (0.661/0.12)(0.0243)(3.61)^{0.4}(412,300)^{0.8} = 6747 \text{ W/m}^2\text{-K}
\end{align*}
\]

\[
\begin{align*}
h_{Colburn} &= (k/D)(0.023)(Pr)^{1/3}(Re_D)^{0.8} \\
&= (0.661/0.12)(0.023)(3.61)^{1/3}(412,300)^{0.8} = 6193 \text{ W/m}^2\text{-K}
\end{align*}
\]

\[
\begin{align*}
h_{Mcdams} &= (k/D)(0.0225)(Pr)^{0.4}(Re_D)^{0.8} = (0.0225/0.0243)h_{DB} \\
&= 6247 \text{ W/m}^2\text{-K}
\end{align*}
\]

\[
\begin{align*}
h_{ST} &= h_{Colburn}(\mu_b/\mu_w)^{0.14} = 6193(1.75)^{0.14} = 6193(1.081) \\
&= 6698 \text{ W/m}^2\text{-K}
\end{align*}
\]

The more accurate Gnielinski equation gives $h = 8400 \text{ W/m}^2\text{-K}$. Therefore, these old equations are low by roughly 20%, 26%, 26%, and 25%, respectively.

Why such consistently large deviations? It is because the old correlations represent much more limited data sets than Gnielinski’s correlation. In this case, $Re_D = 412,000$ was a good deal higher than the $Re_D$ values used to build the old correlations.
7.8 If \( u_\infty \) and \( T_W \) vary in Example 7.4, but all other conditions remain the same, plot \( u_\infty \) against \( T_W \).

With reference to the Example, we write:

\[
\frac{u_\infty}{D} = \frac{1.596 \times 10^{-5}}{10^{-4}} \left[ \frac{Q}{D} \frac{D}{k} \left( \frac{T_w - T_a}{0.62 (Pr)^{\frac{1}{3}}} \right) \left[ 1 + \left( \frac{0.4}{Pr} \right)^{2/3} \right]^{\frac{1}{4}} \right]^2
\]

\[
U_\infty = \frac{1.596 \times 10^{-5}}{10^{-4}} \left[ \frac{17.8}{\pi (T_w - 20)(0.0299)} - 0.3 \left( \frac{0.4}{0.71} \right)^{2/3} \right]^{\frac{1}{4}} U_\infty
\]

So

\[
U_\infty = 0.677 \left( \frac{189.4}{T_w - 20} - 0.3 \right)^2
\]

<table>
<thead>
<tr>
<th>( T_w ) °C</th>
<th>( U_\infty ) m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>∞</td>
</tr>
<tr>
<td>25</td>
<td>956</td>
</tr>
<tr>
<td>30</td>
<td>235</td>
</tr>
<tr>
<td>45</td>
<td>35.8</td>
</tr>
<tr>
<td>50</td>
<td>24.5</td>
</tr>
<tr>
<td>60</td>
<td>13.3</td>
</tr>
<tr>
<td>80</td>
<td>5.52</td>
</tr>
<tr>
<td>100</td>
<td>2.89</td>
</tr>
<tr>
<td>150</td>
<td>0.91</td>
</tr>
</tbody>
</table>

(When \( u_\infty \gtrsim 100 \) m/s, the incompressible assumption breaks down and the prediction of \( q \) is more difficult.)
7.10 NAK flows full-developed at 8 m/s and 395°C in a 0.05 m I.D. tube. What is h if Tw is 403°C?

\[ k = 26.7 \text{ W/m \cdot °C} \text{, } Pr = 0.0069 \text{, } \delta = 2.67 \times 10^{-7} \text{ m/s at } \bar{T} = 399°C. \]

\[ Re_D = \frac{8(0.05)}{2.67(10^{-7})} = 1.498 \times 10^6 \text{ so the flow is turbulent.} \]

Then eqn. (8.32) gives: \( Nu_0 = 0.625(1.498 \times 10^6 \times 0.0069)^{0.4} = 25.07 \)

\[ \dot{q} : h = \frac{26.7}{0.05} 25.07 = 13.385 \left( \frac{W}{m^2 \cdot °C} \right) \]

7.11 Water enters a 0.07 m diam., 73°C, pipe at 50°C. u avg = 0.86 m/s. Plot Tb vs. x, neglecting entry conditions. Assume the pipe is smooth. Thus:

\[ Nu_0 = \left( \frac{f}{8} \right) \frac{Pr}{1.67 + 12.7 \sqrt{\frac{f}{8}}} \left( \frac{M_w}{M_w} \right)^{0.11} \]

\[ f = \frac{1}{(1.824 \times 10^3 \times Re_D - 1.69)} \]

If we move down the pipes in increments of \( \delta_x \),

\[ T_{b,x+\delta_x} = T_{b,x} + \frac{\bar{h} \pi \delta_x (T_{w,T_{b,x}})}{\frac{C_p \pi (D/2)^2 \sqrt{u_{avg}}}} = T_{b,x} + \frac{4 \pi \delta_x (T_{w,T_{b,x}})}{\frac{C_p \pi (D/2)^2 \sqrt{u_{avg}}}} \]

<table>
<thead>
<tr>
<th>x (m)</th>
<th>Tb°C</th>
<th>( \frac{T_{b,x} - Tw°C}{2} )</th>
<th>( \frac{\delta \times 10^6}{Pr} )</th>
<th>Re_D</th>
<th>f</th>
<th>Nu_0</th>
<th>h = \frac{k}{D} \frac{Nu_0}{T_{b,x} - Tw°C}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>312</td>
<td>0.47 0.6853 4.14 4.48</td>
<td>88.851</td>
<td>1.157</td>
<td>0.0023</td>
<td>413</td>
<td>368.5 13.0</td>
</tr>
<tr>
<td>2</td>
<td>316</td>
<td>0.618 0.693 4.48</td>
<td>97.411</td>
<td>1.135</td>
<td>0.0023</td>
<td>453</td>
<td>408.3 20.5</td>
</tr>
<tr>
<td>4</td>
<td>320</td>
<td>0.546 0.679 4.13 4.37</td>
<td>106.360</td>
<td>1.122</td>
<td>0.00222</td>
<td>508</td>
<td>461.8 32.5</td>
</tr>
<tr>
<td>7</td>
<td>326</td>
<td>0.532 0.679 4.12</td>
<td>113.158</td>
<td>1.074</td>
<td>0.00219</td>
<td>494</td>
<td>453.6 44.3</td>
</tr>
<tr>
<td>11</td>
<td>331.6</td>
<td>0.482 0.679 4.12 3.35</td>
<td>124.996</td>
<td>1.048</td>
<td>0.00219</td>
<td>498</td>
<td>462.0 55.0</td>
</tr>
<tr>
<td>16</td>
<td>337</td>
<td>0.490 0.679 4.10 2.19</td>
<td>136.818</td>
<td>1.03</td>
<td>0.00210</td>
<td>512</td>
<td>443.1 65.3</td>
</tr>
<tr>
<td>24</td>
<td>342</td>
<td>0.441 0.679 4.10 2.55</td>
<td>146.472</td>
<td>1.01</td>
<td>0.00207</td>
<td>497</td>
<td>466.7 72.3</td>
</tr>
</tbody>
</table>

(continued ...)
7.11 (continued)

Plot the results

This might be converging too quickly owing to the large steps taken later in the calculation (cf. Problem 7.13)
7.14 This problem can have hundreds of solutions. It has the value of putting the student in an active (attack) mode.

7.15 Water at 24°C flows at $u_{avg} = 0.8 \text{ m/s}$ in a 0.015 m smooth tube which is kept at 30°C. The flow could be either laminar or turbulent. Calculate $h_{turb}/h_{laminar}$ if the flow is fully developed.

The properties at $T = \frac{24 + 30}{2} = 27°C$ or 300°C are:

\[ \delta = 0.826 \times 10^{-6} \]
\[ k = 0.608 \]
\[ \rho c = 4.161 \times 10^6 \]
\[ Pr = 5.65 \]

\[ Re_D = \frac{u_{avg} D}{v} = 14,528 \]

For laminar flow:

\[ Nu_D = 3.658, \quad h = \frac{0.6084}{0.015} \times 3.658 = 148.4 \text{ W/m}^2\text{°C} \]

For turbulent flow:

\[ \frac{f}{8} = \frac{1}{8(1.72 \log_{10} 14,528 - 1.64)^2} = 0.00355 \]
7.15 (continued)

$$N_u = \frac{f \cdot \frac{Re \cdot Pr}{1.07 + 12.7 \cdot \frac{Re}{D} \cdot (Pr^{0.7})}}{H \cdot \mu_w^{0.11}}$$

but \(\mu_b/\mu_w^{0.11} = (5_b/5_w)^{0.11} = \left( \frac{0.915}{0.787} \right)^{0.11} = 1.017\), so:

$$N_u = 109.2, \quad h = \frac{0.6084}{0.015} \cdot 109.2 = 44.29 \frac{W}{m^2 \cdot K}$$

Thus:

$$h_{turbulent}/h_{laminar} = \frac{44.29}{14.84} = 29.8$$

Turbulent flow (in this case) gives 30 times the heat transfer in laminar flow.

7.16 Laboratory observations of heat transfer during the forced flow of air at \(27^\circ C\) over a bluff body, 12 cm wide, kept at \(77^\circ C\), yield

\(q = 646 \ W/m^2\) when the air moves 2 m/s and 3590 \(W/m^2\) when it moves 18 m/s. In another test, everything else is the same, but now \(17^\circ C\) water flowing 0.4 m/s yields 131,000 \(W/m^2\). The correlations in Chapt. 7 suggest that, with such limited data, we can probably create a fairly good correlation in the form: \(N_u = CRe^aPr^b\). Estimate the constants \(C, a,\) and \(b,\) by cross-plotting the data on log-log paper.

For the air case:

$$Re_L = \frac{(2 \ or \ 18)(0.12)}{1.809 (10)^{-5}} = 13,267 \ or \ 119,400$$

and \(Pr = 0.709\).

$$N_u = \frac{q \cdot L}{AT \cdot k} = \frac{(646 \ or \ 3590)(0.12)}{(77-27)(0.02192)} = 55.5 \ or \ 308.6$$

For water:

$$Re_L = \frac{0.4(0.12)}{0.566 (10)^{-5}} = 84,806 \ , \ Pr = 3.67,$$

and

$$N_u = \frac{131,000 (0.12)}{(77-17)(0.6367)} = 411.5$$

(over)
This curve suggests that $\overline{Nu_L} = C \overline{Re_L}^{3/4} Pr^{1/3}$.

Use $411.5 = C (89,806)^{3/4} (3.67)^{1/3}$ to get $C = 0.0537$

Then:

$\overline{Nu_L} = 0.0537 \overline{Re_L}^{3/4} Pr^{1/3}$

This curve suggests that $\overline{Nu_L} = C \overline{Re_L}^{3/4} Pr^{1/3}$.

Pseudo-point for use in the adjacent Pr plot.

Δ H2O data point
○ air data points
**Problem 7.17** Air at 1.38 MPa (200 psia) flows at 12 m/s in an 11 cm I.D. duct. At one location, the bulk temperature is 40 °C and the pipe wall is at 268 °C. Evaluate \( h \) if \( \varepsilon/D = 0.002 \).

**Solution** We evaluate the bulk properties at 40°C = 313.15 K. Since the pressure is elevated, we must use the ideal gas law to find the density of air with the universal gas constant, \( R^o \), and the molar mass of air, \( M \):

\[
\rho = \frac{pM}{R^o T} = \frac{(1.38 \times 10^6)(28.97)}{(8314.5)(313.15)} = 15.36 \text{ kg/m}^3
\]

The dynamic viscosity, conductivity, and Prandtl number of a gas depend primarily upon temperature. At 313 K, \( \mu = 1.917 \times 10^{-5} \text{ kg/m} \cdot \text{s} \), \( k = 0.0274 \text{ W/m} \cdot \text{K} \), and \( Pr = 0.706 \). Hence,

\[
\text{Re}_D = \frac{\rho u_{av} D}{\mu} = \frac{(15.36)(12)(0.11)}{1.917 \times 10^{-5}} = 1.058 \times 10^6
\]

The friction factor may be calculated with Haaland’s equation, (7.50):

\[
f = \left( 1.8 \log_{10} \left[ \frac{6.9}{1.058 \times 10^6} + \left( \frac{0.002}{3.7} \right)^{1.11} \right] \right)^2 = 0.02362
\]

We can see from Fig. 7.6 that this condition lies in the fully rough regime, as confirmed by eqns. (7.48):

\[
\text{Re}_e = \frac{u^* \varepsilon}{\nu} = \frac{\text{Re}_D \nu}{D} \sqrt{\frac{f}{8}} = (1.058 \times 10^6)(0.002) \sqrt{\frac{0.02362}{8}} = 114.9 > 70
\]

Next, we may compute the Nusselt number from eqn. (7.49):

\[
\text{Nu}_D = \frac{(f/8) \text{ Re}_D \text{ Pr}}{1 + \sqrt{f/8(4.5 \text{ Re}_e^{0.2} \text{ Pr}^{0.5} - 8.48)}} = \frac{(0.02362/8)(1.058 \times 10^6)(0.706)}{1 + \sqrt{0.02362/8(4.5(114.9)^{0.2}(0.706)^{0.5} - 8.48)}} = 2061
\]

The temperature difference is quite large, so we should correct for variable properties using eqn. (7.45):

\[
\text{Nu}_D = \text{Nu}_D \left| \frac{T_b}{T_w} \right|^{0.47} = (2061) \left( \frac{313.15}{541.15} \right)^{0.47} = 1594
\]

Finally,

\[
h = \frac{k}{D} \text{ Nu}_D = \frac{0.0274}{0.11} (1594) = 397 \text{ W/m}^2 \text{K}
\]
7.18 How does $\bar{h}$ vary with the heater diameter during crossflow over a cylindrical heater when $Re_D$ is very large.

From eqn. (8.34) \[
\lim_{\text{large } Re_D} \frac{Nu_D}{Re_D} = 0.62 \frac{u_\infty}{\sqrt{Pr}} \left[ 1 + \frac{0.4}{Pr} \right]^{1/3} \frac{Re_D}{292,000} \left( \frac{Re_D}{292,000} \right)^{4/5} = \left(\frac{Re_D}{292,000}\right)^{4/5}
\]

Therefore:
\[
\bar{h} = k \frac{u_\infty}{\bar{h}} \left(\frac{Re_D}{292,000}\right)^{4/5} \text{ which is independent of } D
\]

We encounter this size-independence again in natural convection when the size is large. See Problem 8.31.
7.20 Write $Re_D$ in terms of $\dot{m}$ in pipe flow and explain why this representation could be particularly useful in dealing with compressible pipe flows.

$$Re_D = \frac{\rho U D}{\mu} = \frac{\rho U A 4D}{\mu \kappa D^2} = \frac{4\dot{m}}{\kappa U D}$$

$\dot{m}$ must remain constant in a compressible pipe flow while both $P$ and $u$ vary. In an isothermal gas flow with a pressure drop, we see that $Re_D$ actually stays constant -- a fact that is not clear in the conventional form.

7.21 NAK at 394°C flows at 0.57 m/s across a 1.82 m length of 0.036 m O.D. tube. The tube is kept at 404°C. Find $h$ and the heat removal rate from the tube.

Evaluate the properties a $(94 + 404)/2 = 399°C$:

$$Re_D = \frac{U D}{v} = \frac{0.57(0.036)}{2.67 \times 10^{-7}} = 78,854, \quad Pr = 0.0068, \quad Re_b = Re_pr = 523$$

So we use eqn. (8.35)

$$Nu_D = 0.3 + 0.62(78,854)^{1/2}(0.0068)^{1/4} = 16.55$$

$$h = Nu_D \frac{k}{D} = 16.55 \frac{26.7}{0.036}$$

And:

$$Q = h A \Delta T = 12,725[1.82 \pi (0.036)](404-394) = 25,266\text{W}$$

7.23 Check the value of $h$ given in Example 7.3 by using Reynold's analogy directly to calculate it. Which $h$ do you deem to be in error, and by what percent.

Direct use of Reynold's analogy yields the Colburn equation. We have already made this comparison in Problem 7.5. The resulting deviation from the far more accurate Gnielinski equation was 26%.
7.26 Report the maximum percent scatter of data in Fig 7.14. What is happening in the fluid flow when the scatter is worst?

We identify the distance \( \downarrow \) between the highest and lowest points at \( \text{Re}_0 \approx 30000 \) and compare it with the log scale (as we see here:)

The error is such that \( 58(1+\text{scatter})(1+\text{scatter}) = 100 \).

So: \( \text{scatter} = \pm 0.31 = \pm 31\% \)

The error, while not generally this bad, is still high in the range: \( 20,000 < \text{Re}_0 < 300,000 \). Figure 7.11 tells us that in this range, the conventional vortex street is gradually breaking down and becoming three-dimensional. When the b.l. on the cyl. finally becomes turbulent (and vortex shedding becomes unclear — see Fig. 7.12), then the scatter reduces to about \( \pm 8\% \).
7.28 Freshly painted aluminum rods, 0.02 m in diameter are withdrawn
from a drying oven at 150°C and cooled in a 3 m/s crossflow of air
at 23°C. How long will it take to cool them to 40°C, so they can
be handled?

We shall evaluate air properties at an average, average temp-
perature of \( \frac{1}{2} \left[ \frac{150 + 23}{2} + \frac{40 + 23}{2} \right] = 57.25 \quad C = 330.4 \quad K \)

\( \Delta = 1.982 \times 10^{-5} \), \( \kappa = 0.0282 \), \( \rho = 0.708 \)

For aluminum, \( \rho c_p = 2707(905) = 2.45 \times 10^6 \quad J/m^2K \), \( k = 240 \quad W/m°C \)

Then:
\[
Re_D = \frac{0.02 (3)}{1.982 (10)^{-5}} = 3027 \quad \text{so we use eqn. (7.66)}
\]

\[
\frac{Nu_D}{D} = 0.3 + \frac{0.62 \sqrt{3027 \times 0.708}}{\left[1 + (0.4/0.708)^{2/3}\right]^{1/4}} = 27.6
\]

And:
\[
\frac{h}{D} = \frac{Nu_D k}{\rho c_p} = 27.6 (0.0282)/0.02 = 39.2 \quad W/m^2°C
\]

Next, we calculate \( B_i = \frac{39.2 (0.02)}{240} = 0.0033 \ll 1 \), so we
can assume lumped capacity,

\[
\frac{\rho c V}{hA} = \frac{\rho c D}{4h} = \frac{2.45 \times 10^6 \times 0.02}{4 (39.2)} = 308 \quad sec
\]

Then:
\[
\frac{T - T_0}{T_{w} - T_0} = \frac{40 - 23}{150 - 23} = 0.134 = e^{-t/310}
\]

Thus it will take \( t = 623s = 10min, 23s \) to cool the rods.
At what speed, \( u_\infty \), must 20°C air flow across an insulated tube before the insulation on it will do any good? The tube is at 60°C and 6 mm in diameter. The insulation is 12 mm in diameter with \( k = 0.08 \, \text{W/m} \cdot \text{°C} \).

(Notice that we do not ask for the \( u_\infty \) for which the insulation will do the most harm.)

With reference to Fig. 2.14, we require that the sum of the thermal resistances of the insulated tube must exceed the thermal resistance around the uninsulated tube, so:

\[
R_{t_{\text{ins}}} + R_{t_{\text{conv. for ins. tube}}} > R_{t_{\text{conv. for unins. tube}}}
\]

\[
\frac{\ln r_0/r_i}{2\pi k_{\text{ins}}} + \frac{1}{2\pi r_0 h_{\text{ins}}} > \frac{1}{2\pi r_i h_{\text{unins}}}
\]

or:

\[
\frac{\ln 2}{2\pi (0.08)} + \frac{1}{2\pi (0.004) h_{\text{ins}}} > \frac{1}{2\pi (0.003) h_{\text{unins}}} \quad 1.379 + \frac{26.53}{h_{\text{ins}}} > \frac{53.05}{h_{\text{unins}}}
\]

To calculate \( h_{\text{ins}} \) we shall evaluate properties at \( T = 27°C \) \( (T_{\text{ins}} = 34°C) \) and correct later if we must. \( (\gamma = 1.566 \times 10^5, \quad k = 0.0261, \quad Pr = 0.711) \)

To calculate \( h_{\text{unins.}} \), we evaluate at \((60+20)/2 = 40°C \) \( (\gamma = 1.69 \times 10^5, \quad k = 0.0270, \quad Pr = 0.710) \). Then using eqn. (7.68) we get:

\[
\frac{\overline{Nu}_{0_{\text{unins.}}}} = 0.3 + \frac{0.62 (0.006/1.69005)^{1/2} 0.710^{1/3} 1}{1 + (0.9/0.710) 0.667^{1/4}} [1 + (0.003/1.69005)^{1/2} \sqrt{\overline{u}_\infty}]
\]

\[
= 0.3 + 9.15 \overline{u}_\infty (1 + 0.0793 \overline{u}_\infty)
\]

\[
\overline{Nu}_{0_{\text{ins.}}^2} = 0.3 + \frac{0.62 (1.566 \times 10^5)^{1/2} 0.711^{1/3} 1}{1 + (0.9/0.711) 0.647^{1/4}} [1 + (1.566/0.282)^{1/2} \sqrt{\overline{u}_\infty}]
\]

\[
= 0.3 + 13.45 \overline{u}_\infty (1 + 0.165 \overline{u}_\infty)
\]

Now solve by trial and error

\[
\begin{array}{c|c|c|c|c|c|c|c}
\overline{u}_\infty & 1 & 1.97 & 2.79 & 3.06 & 3.32 & 3.60 & 3.97 \\
\hline
\overline{Nu}_{0_{\text{ins.}}} & 10.18 & 45.9 & 1.156 & 2.142 & 1.648 & 2.494 & 2.986 \\
\overline{Nu}_{0_{\text{unins.}}} & 7.133 & 32.1 & 7.133 & 32.1 & 7.133 & 32.1 & 7.133 \\
\hline
\frac{\overline{Nu}_{0_{\text{ins.}}}}{\overline{Nu}_{0_{\text{unins.}}}} & 0.261 & 0.012 & 0.261 & 0.012 & 0.261 & 0.012 & 0.261 \\
\overline{h}_{\text{ins.}} & 53.05 & 26.53 & 53.05 & 26.53 & 53.05 & 26.53 & 53.05 \\
\end{array}
\]

Therefore, the velocity must be at least 5.0 cm/s if the insulation is to serve its function. (This gives \( T_{\text{outside}} = 48.8°C \) so properties would better have been evaluated at \((48.8 + 20)/2 = 34.4°C \) for \( h_{\text{insulation}} \).)
7.32 Evaluate \( \overline{N_u_D} \) using Giedt's data for air flowing over a cylinder at 
\( Re_D = 140,000 \). Compare your result with the appropriate correlation, 
and with Figure 7.13

\[
\overline{N_u_D} = \frac{1}{180} \int_0^{180} \frac{D}{k} \frac{d\Theta}{k} \cdot \Theta \left( \frac{Dh(\Theta)}{k} \right) \Theta = \frac{1}{180} \int_0^{180} \left( \frac{Dh(\Theta)}{k} \right) \Theta \cdot \Theta \cdot \Theta 
\]

Obtain data from Fig. 7.13

\[
\overline{N_u_D} = 3.54 \text{ from Giedt's data} \]

The appropriate correlation is eqn. (7.68). We don't know the temp. 
of the air, but the only property we need to evaluate is \( Pr \). It is very insensitive to temp. Use \( Pr = 0.711 \) (for \( T = 270^\circ C \)).

Then:

\[
\overline{N_u_D} = 0.3 + \frac{0.62(190,000)^{\frac{1}{2}}(0.711)^{\frac{1}{3}}}{[1 + (0.4/0.711)^{\frac{1}{3}}]^\frac{1}{4}} \left[ 1 + \left( \frac{190,000}{282,000} \right)^{\frac{1}{2}} \right] = 3.10
\]

(If one erroneously used eqn. (7.65), he'd get \( \overline{N_u_D} = 2.71 \) which is low.)

The correlation underpredicts the data by 12.4%.

From Fig. 7.13 we read: \( 330 < \left( \frac{N_u_D - 0.3}{1 + (0.4/0.711)^{\frac{1}{3}}} \right) < 460 \)

at \( Re_D = 140,000 \); Giedt's data point gives \( \left[ \ldots \right] = 451 \) which is high but in the range of the other data. The correlation passes through \( \left[ \ldots \right] = 355 \).

7.33 A 25 mph wind blows across a 0.25 in. telephone line. What is the pitch 
of the hum that it emits?

We don't know \( T_{air} \), but between 0 and 100\,°F \( 1.6(10)^{-5} < 2(10)^{-5} \)

And 25 mph = 36.67 ft/s = 11.18 m/s , \( 0.25\,\text{in} = 0.020844 = 0.00635\,\text{m} \)

Then \( Re_D = \frac{\overline{N_u_D} D}{\nu} \) so \( 2558 < Re_D < 4437 \).

In this range (see Fig. 7.12) \( Str \) is virtually constant at 0.21.

Therefore: \( 0.21 = \frac{f_D}{\nu} \), \( f_V = 0.21(36.67/0.0208) = 370 \text{ cycle/sec} \)

One half step in a tempered scale is a factor of \( 3(10)^{\frac{1}{12}} \) or 1.0595 in frequency. We note that

\( 440(1.0595)^3 = 370 \), therefore we are 3 half tones below a concert A. The pitch is an f\( ^{\frac{3}{2}} \).
7.35 Consider the situation described in Problem 4.38 but suppose you do not know \( \bar{h} \). Suppose instead that you know there is a 10 m/s crossflow of 27°C air over the rod. Then rework the problem.

With reference to the solution to Problem 4.38 we shall take the root temperature to be 122.4°C to evaluate properties. Then the average, average temp. on the rod is \( \left( \frac{122.4 + 27}{2} \right) = 70.1°C \). Let’s evaluate properties at 325°C for simplicity’s sake:

\[
\begin{align*}
\Delta \text{air} &= 1.814(10)^{15}, \quad k = 0.02792, \quad Pr = 0.709 \quad \text{and} \quad Re_D = \frac{0.005(10)^{15}}{1.814(10)^{15}} = 2756 \,
\end{align*}
\]

Then, with the help of eqn. (7.66)

\[
\begin{align*}
\overline{Nu_D} &= 0.3 + \frac{0.62 \sqrt{1755 (0.709)^{-3}}}{1+ (0.4/0.709)^{3/4}} = 25.8, \quad \bar{h} = 25.8 \frac{0.02792}{0.005} = 144 \frac{W}{m^2°C}
\end{align*}
\]

This is within 4% of the original assumption of 150, so no reiteration is needed. Then in accordance with Problem 4.38, solution:

\[
\Delta T = \frac{q_0}{km} = \frac{q_0}{km_{\text{previous}} \sqrt{h_{\text{previous}}}} = 954 \sqrt{150} = 97.9°C
\]

Thus: \( T_{\text{base}} = 97.9 + 27 = 124.4°C \)

7.36 A liquid whose properties are not known flows across a 40 cm diameter tube at 20 m/s. The measured heat transfer coefficient is 8000 W/m²°C.

We can be fairly confident that \( Re_D \) is very large indeed. What would \( \bar{h} \) be if \( D \) were 53 cm? What would \( \bar{h} \) be if \( u_\infty \) were 28 m/s?

At large \( Re_D \), eqn. (7.68) reduces to: \( \overline{Nu_D} = f_n(\text{physical properties}) Re_D \)

or: \( \bar{h} = f_n(\text{physical properties}) u_\infty \)

Therefore: A change of diameter will not influence \( \bar{h} \)

And since \( \bar{h} \sim u_\infty \), the new \( \bar{h} \) will be:

\[
\bar{h} = 8000 \frac{28}{20} = 11,200 \frac{W}{m^2°C}
\]
Glycerin is added to water in a mixing tank at 20°C. The mixture discharges through a 4 m length of 0.04 m ID tubing under a constant 3 m head. Plot the discharge rate in m³/hr as a function of composition.

Using eqn. (B.20), \[ f = \frac{3}{0.04} \frac{u_{av}^4}{u_{av}^2} = \frac{3 - 0.058 u_{av}^2}{5.1 u_{av}^2} = \frac{f(Re_p)}{Re_p} = \frac{0.64}{u_{av}} \]

(Note to instructors: How many students will forget to add the velocity head to 3 m? Perhaps you should remind them.)

<table>
<thead>
<tr>
<th>% glyc.</th>
<th>25 m/s</th>
<th>( u_{av} D/R_p )</th>
<th>The ( u_{av} ) consist with [ f(u_{av}) \text{ above, } &amp; \text{ Fig. 7.6} ]</th>
<th>flow rate ( \frac{f(0.04)^2 u_{av}^2}{3600} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.035(10)^{-6}</td>
<td>38647 ( u_{av} )</td>
<td>4.71</td>
<td>turbulent</td>
</tr>
<tr>
<td>20</td>
<td>1.681 &quot;</td>
<td>23795 ( u_{av} )</td>
<td>4.62</td>
<td>&quot;</td>
</tr>
<tr>
<td>40</td>
<td>3.467 &quot;</td>
<td>11537 ( u_{av} )</td>
<td>4.33</td>
<td>&quot;</td>
</tr>
<tr>
<td>60</td>
<td>5.36 &quot;</td>
<td>4274 ( u_{av} )</td>
<td>3.93</td>
<td>turbulent</td>
</tr>
<tr>
<td>80</td>
<td>4.97(10)^{-5}</td>
<td>805 ( u_{av} )</td>
<td>(3.4)</td>
<td>turbulent</td>
</tr>
<tr>
<td>100</td>
<td>0.00112</td>
<td>35.7 ( u_{av} )</td>
<td>0.0329</td>
<td>laminar</td>
</tr>
</tbody>
</table>

The \( u_{av} \) values have been obtained by trial & error using Fig. B.6 in the turbulent \& laminar range, except for \( f = 0.64/Re_p \) in the laminar range.
7.40 Rework Problem 5.40 without assuming the Bi to be very large.

we need $\bar{T}$, since $Bi$ should not be small we shall evaluate properties close to the gas temp. - at $277^\circ C$ or $550^K$. and use eqn. (7.68). Then
\[
25 = 4.955 \times (10)^{-5}, \quad k = 0.0426, \quad Pr = 0.698
\]
\[
Re_p = \frac{0.26(1)}{4.955(10)^{-5}} = 5835 \quad \frac{Nu_D}{\bar{T}} = 0.3 + \frac{0.62 + 5835(0.698)}{1.14} \left[ 1 + \left( \frac{5835}{221000} \right)^{0.795} \right] 
\]
\[
= 39.74
\]
\[
\bar{T} = 39.74 \quad \frac{0.0426}{0.26} = 6.51 \quad B_{c}^{-1} = \frac{0.68}{6.51(0.13)} = 0.803 \quad \text{(where we use} \quad k = k_{H_2O})
\]

This gives $Fo = 0.3$ so $t = 0.3 (0.13)^2/1.35(10)^7 = 37.555 \text{sec}$

Therefore the cooking time is considerably extended to $10.43 \text{ hrs}$

The cooking should actually commence at about 5:30AM

(When this is done in Utah, the pig is started around 7:00 or 8:00. It cooks more quickly than we predict because the flame also heats a bed of coals which radiate additional heat to the pig.)

7.41 Water enters a 0.5 cm ID pipe at $24^\circ C$. The pipe walls are held at $30^\circ C$. Plot $T_b$ against distance from entry, if $u_{av}$ is $0.27 \text{ m/s}$, neglecting entry behavior in your calculation. (Indicate the entry region on your graph, however).

At $27^\circ C : \Delta = 0.826(10)^{-6}, \rho_r = 5.65, \quad Re_p = \frac{0.005(0.27)}{0.826(10)^{-6}} = 1634 \quad (\text{laminar})$

Then from eqns. 7.57, 7.58, and 7.23,
\[
\frac{T_b - T_{bo}}{T_w - T_{bo}} = 1 - \exp \left[ - \frac{Nu_D}{Pr \cdot Re_p} \frac{4}{D} \times \right]
\]
\[
T_b = 24 + 6 \left[ 1 - \exp \left( -0.317x \right) \right]
\]
\[
x_e = 0.05 \quad Re_p = 0.405m
\]
Deive a numerical scheme that will allow you to be able to find the velocity distribution and friction factor in a square duct of side length a. Set up a square grid of size \( N \) by \( N \) and solve the difference equations by hand for \( N = 2, 3 \) and 4. Hint: First show that the velocity distribution is given by the solution to the equation

\[
\frac{2\bar{u}}{a^2} + \frac{2\bar{u}}{\alpha^2} = 1
\]

where \( \bar{u} = 0 \) on the sides of the square and \( \bar{u} = u \sqrt{\frac{a}{\mu}} \frac{d\bar{u}}{d\bar{x}} \), \( \bar{x} = \frac{x}{a} \) and \( \bar{y} = \frac{y}{a} \). Then show that the friction factor, \( f \) (equation (8.21)), is given by

\[
f = \frac{-2}{\mu u a^2} \int u du d\bar{x} d\bar{y}
\]

Note that the area integral can be evaluated as \( \bar{f} u/N^2 \).

**Solution:** By following the discussion preceding and following eqn. (8.6) we write the mom. eqn. for a square duct as:

\[
\frac{2d\bar{u}}{dx} + \frac{2d\bar{u}}{dy} = \frac{1}{\mu} \frac{d\bar{u}}{dx}
\]

or, using \( x = \frac{x}{a} \), \( y = \frac{y}{a} \), \( \bar{u} = \frac{u}{a^2} \frac{d\bar{u}}{d\bar{x}} \),

we get:

\[
\frac{2\bar{u}}{a^2} + \frac{2\bar{u}}{\alpha^2} = 1
\]

with b.c.s \( u = 0 \) on all sides.

Now, since the flow must be symmetrical about the diagonals & about the bisectors of the sides, we need only solve for flow in \( \theta \) of the duct to know the entire flow field. However:

having a square grid of size \( \Delta x \) & using central differences obtain a general point \( i, j \), we get:

\[
\bar{u}_{i-1,j} + \bar{u}_{i+1,j} - 2\bar{u}_{i,j} \Delta x^2 + \bar{u}_{i,j-1} + \bar{u}_{i,j+1} - 2\bar{u}_{i,j} \Delta y^2 = 1
\]
7.42 (Continued)

Now for good accuracy $\Delta x$ must be $<< 1$. However, we only want to show the method here so we use a very coarse grid: $\Delta x = 1/3$

\[
\overline{u}_1 = \frac{\overline{u}_2 + \overline{u}_3}{4} - \frac{1}{4(9)}
\]

\[
\overline{u}_2 = \frac{\overline{u}_1 + \overline{u}_4}{4} - \frac{1}{4(9)}
\]

etc.

By inspection we find that $\overline{u}_1 = \overline{u}_2 = \overline{u}_3 = \overline{u}_4 = -\frac{1}{10}$ in this case.

The friction factor (eqn. 7.34) is

\[
f = \frac{2a}{\rho u_0^2} \frac{dp}{dz}
\]

but $u_{av} = \frac{1}{N} \sum u \Delta x$ so:

\[
f = \frac{\rho u_{av}^2}{\mu} = \frac{2N^2}{\sum u}
\]

\[
f \text{Re}_a = \frac{2 \cdot (3)(3)}{4 \cdot (\frac{1}{10})} = 81
\]

This computation can be redone for larger $N$'s (or smaller $\Delta x$'s.) $\Delta x = \frac{1}{4}$ can in fact still be done by hand with the result that $f \text{Re} = 69.424$.

The correct answer, given in [1.20], is 56.8.

It looks like $\Delta x$ should be less than $1/5$.
Problem 7.51  Consider the water-cooled annular resistor of Problem 2.49 (Fig. 2.24). The resistor is 1 m long and dissipates 9.4 kW. Water enters the inner pipe at 47 °C with a mass flow rate of 0.39 kg/s. The water passes through the inner pipe, then reverses direction and flows through the outer annular passage, counter to the inside stream.

a) Determine the bulk temperature of water leaving the outer passage.

b) Solve Problem 2.49 if you have not already done so. Compare the thermal resistances between the resistor and each water stream, $R_i$ and $R_o$.

c) Use the thermal resistances to form differential equations for the streamwise (x-direction) variation of the inside and outside bulk temperatures ($T_{b,i}$ and $T_{b,o}$) and an equation the local resistor temperature. Use your equations to obtain an equation for $T_{b,o} - T_{b,i}$ as a function of $x$.

d) Sketch qualitatively the distributions of bulk temperature for both passages and for the resistor. Discuss the size of: the difference between the resistor and the bulk temperatures; and overall temperature rise of each stream. Does the resistor temperature change much from one end to the other?

e) Your boss suggests roughening the inside surface of the pipe to an equivalent sand-grain roughness of 500 µm. Would this change lower the resistor temperature significantly?

f) If the outlet water pressure is 1 bar, will the water boil? *Hint:* See Problem 2.48.

g) Solve your equations from part (c) to find $T_{b,i}(x)$ and $T_r(x)$. Arrange your results in terms of $NTU_i = 1/(\dot{m}c_p R_i)$ and $NTU_o = 1/(\dot{m}c_p R_o)$. Considering the size of these parameters, assess the approximation that $T_r$ is constant in $x$.

Solution

a) The answer follows directly from the 1st Law, $Q = \dot{m}c_p (T_{b,\text{out}} - T_{b,\text{in}})$:

$$\Delta T_b = Q/(\dot{m}c_p) = 9400/(0.39 \cdot 4180) = 5.77 \ {^\circ C}$$

so $T_{b,\text{out}} = 47 + 5.77 = 52.8 \ {^\circ C}$.

b) The inside thermal resistance, $R_i = 3.69 \times 10^{-2} \ \text{K/W}$, is greater than the outside resistance, $R_o = 3.00 \times 10^{-2} \ \text{K/W}$.

c) With eqn. (7.10), putting $(q_u P)_{\text{inside}} = (T_r - T_{b,i})/R_iL$ and $(q_u P)_{\text{outside}} = (T_r - T_{b,o})/R_oL$ where the tube length is $L = 1 \ \text{m}$:

$$\dot{m}c_p \frac{dT_{b,i}}{dx} = \frac{T_r - T_{b,i}}{R_iL} \quad (1)$$

$$\dot{m}c_p \frac{dT_{b,o}}{dx} = \frac{T_r - T_{b,o}}{R_oL} \quad (2)$$

Recalling the solution of Problem 4.29, we can divide the resistance equation by $L$ to obtain a local result (assuming that $h$ is equal to $\overline{h}$ along the entire passage):

$$\frac{T_r - T_{b,i}}{R_iL} + \frac{T_r - T_{b,o}}{R_oL} = \frac{Q}{L} = \text{constant} \quad (3)$$

Each of $T_{b,i}, T_{b,o}$, and $T_r$ are functions of $x$.

By adding eqn. (1) to eqn. (2), and then using eqn. (3),

$$-\dot{m}c_p \frac{d(T_{b,o} - T_{b,i})}{dx} = \frac{Q}{L}$$
and integrating (with $T_{b,o} = T_{b,i}$ at $x = L$), we find

$$T_{b,o} - T_{b,i} = \frac{Q}{mc_p}(1 - x/L)$$  \hspace{1cm} (4)

d) From working part (a) and Problem 2.49, we already know that the resistor will be much hotter than the water on either side (194 °C at the end where the water enters and exits). At any point, $T_r - T_b \gg T_{b,o} - T_{b,i}$, so that $T_r - T_{b,i} \approx T_r - T_{b,o} \approx$ constant, along the entire passage. From eqns. (1) and (2), then, the bulk temperature of each stream has a nearly straight line variation in $x$, but the outer passage temperature rises a bit faster because the thermal resistance on that side is lower. Similarly, eqn. (3) shows that the resistor temperature varies by no more than do the bulk temperatures.

e) Your solution to Problem 2.49 shows that the epoxy layers provide the dominant thermal resistance on each side. Roughness will make the convection resistance smaller, but convection resistance is only about 10% of the overall resistance. Your boss’s idea will add cost and pressure drop, but it won’t lower the resistor temperature much. (Suggestion: Find a diplomatic way to tell him that.)

f) The water will not boil if the highest temperature of the epoxy is below $T_{sat}$. The hottest point for the epoxy is in the outlet stream at the exit (where the bulk temperature is greatest). From the solution to Problem 2.49, using the voltage divider relation from Problem 2.48,

$$T_{epoxy} - T_{b,outlet} = (T_r - T_{b,outlet})\frac{R_{conv}}{R_{outside}} = (194 - 52.8)\frac{0.00307}{0.0300} = 14.4 \text{ K}$$

The water will not boil.

g) Rearranging eqn. (3) with eqn. (4):

$$T_r - T_{b,i} + (T_r - T_{b,i})\frac{R_i}{R_o} = QR_i - (T_{b,o} - T_{b,i})\frac{R_i}{R_o}$$

$$(T_r - T_{b,i})\left(1 + \frac{R_i}{R_o}\right) = QR_i - \frac{QR_i}{mc_p R_o} (1 - x/L)$$

$$T_r - T_{b,i} = (QR_i)\left(\frac{R_o}{R_o + R_i}\right) \left[1 - \frac{1}{mc_p R_o}(1 - x/L)\right]$$  \hspace{1cm} (5)

From eqn. (3), we may estimate that $QR_i \approx (T_r - T_{b,i})/2$; thus, we can see that the second term on the right is very small and could be neglected entirely.

Upon substituting eqn. (5) into eqn. (1) we have:

$$\dot{m}c_p \frac{dT_{b,i}}{dx} = \frac{Q}{L} \left(\frac{R_o}{R_o + R_i}\right) \left[1 - \frac{1}{mc_p R_o}(1 - x/L)\right]$$

Integration gives:

$$T_{b,i}(x) - T_{b,in} = \frac{Q}{mc_p} \left(\frac{R_o}{R_o + R_i}\right) \left[\frac{x}{L} - \frac{1}{mc_p R_o} \left(\frac{x}{L} - \frac{x^2}{2L^2}\right)\right]$$

Because the second term in the square brackets is small, we see that the bulk temperature has an essentially straight line variation.
More precisely, we may think of this arrangement as a heat exchanger, where \( UA = 1/R_o \) so that

\[
\text{NTU}_o = \frac{UA}{\dot{m}c_p} = \frac{1}{\dot{m}c_p R_o} = \frac{1}{(3.00 \times 10^{-2})(0.39)(4180)} = 0.020 \ll 1
\]

From Chapter 3, we recall that a heat exchanger with very low NTU causes very little change in the temperature of the streams, as is the case here. Putting our result in terms of the outside and inside NTUs:

\[
T_{b,i}(x) - T_{b,\text{in}} = (QR_i)\text{NTU}_i \left( \frac{R_o}{R_o + R_i} \right) \left[ \frac{x}{L} - \text{NTU}_o \left( \frac{x}{L} - \frac{x^2}{2L^2} \right) \right]
\]

Substituting eqn. (6) into eqn. (5):

\[
T_r - T_{b,\text{in}} = (QR_i) \left( \frac{R_o}{R_o + R_i} \right) \left[ 1 - \text{NTU}_o \left( 1 - \frac{x}{L} \right) - \text{NTU}_i \left[ \frac{x}{L} - \text{NTU}_o \left( \frac{x}{L} - \frac{x^2}{2L^2} \right) \right] \right]
\]

Since \( \text{NTU}_i \) has a similar value to \( \text{NTU}_o \), the resistor temperature is indeed nearly constant, with variations on the order of \( \text{NTU}_0 = 0.02 \).
8.1 Show that $T_4$ is equivalent to $PrRe^2/Ja$.

The velocity that the condensing film would reach in free fall through a characteristic length is $\sqrt{\frac{9(\alpha-\beta)\gamma l}{\mu k}}$. Let's call this $U_{ch}$. Then:

$$
T_4 = \frac{\beta \gamma h_f L^3}{\mu k (T_{sat} - T_w)} = \frac{\rho_f u_c^2 h_f L^3}{\rho c (T_{sat} - T_w)} = \frac{u_c^2 l^2}{\rho c} \frac{\gamma}{\sigma} \frac{h_f}{c_p(T_{sat} - T_w)} = \frac{Re^2 Pr}{Ja}
$$

8.2 For the figure shown, Plot

$\delta$ and $h$ vs. $x$

$T$ and $u$ vs. $y$

from eqns. (8.24) & (8.27)

$$
\delta = 4.87 \times 10^{-4} Re_{\alpha}^{1/4}, \quad Nu_k = 0.3773 Re_{\alpha}^{1/4}
$$

$$
Re_{\alpha} = 0.0098 \frac{Re_{\alpha}^{1/4}}{k}
$$

and $Re_{\alpha}^{1/4} = \frac{0.89(0.00033x)(14)}{(1.566 \times 10^{-5})^{2.711}} x^{3/4} = 1.91 x^{3/4}$

$$
\delta = 0.0255 x^{1/4}
$$

$$
\frac{h_c}{x} = 1.883 x^{1/4}
$$

$\delta = 0.2$

$u_c = \frac{y}{8} \left(1 - \frac{y}{8}\right)^2$

where $u_c = C_f \frac{\phi \delta T \dot{\theta}}{D}$

$$
= \frac{Pr}{3(0.552 + Pr)} \frac{\phi \delta T \dot{\theta}}{D} = 1.208 \text{ m/s}
$$

So $U = 1.208 \frac{y}{0.01104} \left(1 - \frac{y}{0.01104}\right)^2$

and $\Delta T = T - T_w = 14 \left(1 - \frac{y}{0.01104}\right)^2$

$\Delta T = T - 20$
8.3 Re-do the Squire-Eckert analysis neglecting inertia.

Omitting the inertial terms form the momentum equation, we reduce the equation after equation (8.20) to:

\[
0 = \frac{1}{3} - C_1 \quad \text{or} \quad C_1 = 1/3
\]

Eqn. (8.22) is unchanged, so we put this \( C_1 \) in it and get:

\[
\delta^4 = \frac{240 \alpha^2}{\beta g \Delta T Pr} x \quad \text{and} \quad \frac{\delta}{x} = 3.936 \; \text{Ra}_{x}^{-1/4}
\]

Then:

\[
\text{Nu}_x = 2 \frac{x}{\delta} = 0.508 \; \text{Ra}_{x}^{1/4}
\]

This is exactly the Squire-Eckert result for \( \text{Pr} \gg 1 \).

8.4 Predict \( \text{Nu}_x \), using an integral method and the assumed profiles shown:

Thus:

\[
\frac{1}{3} \frac{d}{dx} \left( 6 u_{\text{max}}^2 \right) = \frac{\alpha \Delta T \delta}{2} - \frac{3 \beta x}{\delta}
\]

so:

\[
\frac{1}{3} \frac{d}{dx} (6 u_{\text{max}}^2) = \frac{\alpha \Delta T \delta}{2} - \frac{3 \beta x}{\delta}
\]

and the energy equation gives:

\[
\frac{1}{3} \frac{d}{dx} \left( 6 u_{\text{max}} \Delta T \right) \left[ 6 \delta \frac{\Delta T}{\alpha} \left( \frac{x}{2} \right) d \left( \frac{x}{2} \right) + \frac{1}{3} \frac{\Delta T}{\alpha} \left( \frac{x}{2} \right)^2 d \left( \frac{x}{2} \right) \right] = \frac{\sigma \Delta T}{\delta}
\]

\[
= 15/64
\]
8.4 (continued)

\[ \frac{5}{16} \frac{d}{dx} (S_{\text{max}}) = \frac{\alpha}{\delta} \]

energy eqn.

But \( S_{\text{max}} = C_1 \delta^2 \) so the mom. eqn. eqns. become:

**momentum:**

\[ C_1 \frac{1}{16} \frac{d}{dx} (\delta^4) = \frac{9 \beta \lambda T}{2} - 3 \lambda \delta C_1 \]

or

\[ \delta^4 = \frac{24 \alpha T}{5} \frac{\alpha x}{C_1} \]

**energy:**

\[ C_1 \frac{5}{16} \frac{d}{dx} (\delta^4) = \alpha \]

or

\[ \delta^4 = \frac{24 \alpha x}{5} \frac{\alpha x}{C_1} \]

Equating these eqns. for \( \delta^4 \), we get:

\[ C_1 = \frac{9 \beta \lambda T}{\alpha \left( \frac{\lambda x}{C_1} \right)} \]

so:

\[ \delta^4 = \frac{24 \alpha x}{5} \frac{\alpha x}{\beta \lambda T} \left( \frac{\lambda x}{C_1} \right) \]

Equating these, we get:

\[ \delta^4 = \frac{24 \alpha x}{5} \frac{\alpha x}{\beta \lambda T} \left( \frac{\lambda x}{C_1} \right) \]

and

\[ \text{Nu}_x = -\frac{dL}{dT} \delta x = \frac{\delta x}{\delta} = 0.432 \left( \frac{Pr}{Pr + \frac{3}{5}} \right)^{\frac{1}{4}} \]

\[ \text{Re}_x \]

Triangular approximations above -- accurate within 15%.

8.5 Find \( \bar{h} \) and \( Q \) for

\( T_w = 35^\circ \text{C} \)

\( (T_{air} = 25^\circ \text{C}) \)

We know that \( \text{Nu}_c = 0.52 \frac{\text{Re}}{c^4} \) where \( c = 2(10) \text{cm} = 0.1 \text{m} \). Thus \( (T_{air} = 30^\circ \text{C}) \)

\[ \bar{h} = 0.52 \frac{k_{air}}{0.1} \left( \frac{9 \beta \lambda T_0}{25} \right)^{\frac{1}{4}} = 0.52 \frac{0.02635}{0.1} \left( \frac{9.9 \left( \frac{1}{25} \right)^{\frac{1}{3}} (10)(0.4)^{\frac{1}{3}}}{(1.596)(2.246)} \right)^{\frac{1}{4}} \]

or

\[ \bar{h} = 3.00 \frac{W}{m^2 \circ C} \]

and

\[ Q = \bar{h} \Delta T = 3.00 \frac{W}{m^2 \circ C} \]

\[ (4(\pi)^2) \bar{m}^2 \times 10^\circ C = 24.0 \frac{W}{m} \]
8.6 Heat heat flux from a 3 m high electrically heated panel in a wall is 75 W/m² in an 18°C room. What is the average temperature of the panel? What is the temperature at the top? -- at the bottom?

\[
q_w = 75 \text{ W/m}^2 \\
R_{oL} = \frac{q_w L^4}{k \Delta \alpha} = \frac{(9.8/273.15 \times 75)(3)^4}{0.6264 \times (15.64)(7203)} \times 10^6
\]

where we assume \((T_w + T_a)/2 \approx 300\). Then \(R_{oL} = 2.2 \times 10^{13}\)

\[
0.67 \frac{R_{oL}^{\frac{1}{4}}}{1 + (0.932)^{3/4}} = 0.67 \frac{(2182)}{1 + (0.341)^{3/4}} = 11.22 = \frac{\text{Nu}_L^{5/4}}{1 + 0.64 \text{Nu}_L^{1/4}}
\]

Then \(\overline{\Delta T} = \frac{q_w L}{\text{Nu}_L k} = \frac{75(3)}{276(0.02614)} = 31.2\)

This gives \(T_w = 319.5°C\) and we assumed 309°C. If we accept this as close enough \(T_w = 18 + 31.2 = 49.2°C\)

and since this "average" is a mid-point value we go to Fig. 8.9 and write:

\[
\frac{49.2}{C} = 0.8706 \quad C = 56.5
\]

So \(\overline{\Delta T} = 56.5(y/L)^{1/5}\)

at the leading edge \(\Delta T = 56.5 \times 0 = 0\), \(T_y = 18°C\)

at the top, \(\Delta T = 56.5°C\) \(T_x = 56.5°C\)
8.7 Find pipe diameters and wall temperatures for which the film condensation heat transfer coefficients given in Table 1.1 are valid.

We must make some assumptions here since there are many circumstances under which these values would be obtainable. Let us take \( p = 1 \) atm. Then:

**for water:**

\[
\begin{align*}
\text{\( h_f \)} & = 2,257,000 \text{ J/kg} \\
\text{\( \rho_f \)} & = 0.577 \text{ kg/m}^3 \\
\text{\( \rho_v \)} & = 957.2 \text{ kg/m}^3 \\
\text{\( \kappa_f \)} & = 0.6811 \text{ W/m°C} \\
\text{\( \mu_f \)} & = 0.000278 \text{ kg/m-s} \\
\text{\( \text{\( c_p \)}} & = 4.219 \times 10^5 \text{ kJ/kg} \\
\end{align*}
\]

**for benzene:**

\[
\text{\( T_{sat} \)} = 68°C \\
\text{\( h_f \)} = 401,000 \\
\text{\( \rho_f \)} = 1.02 \\
\text{\( \rho_v \)} = 927 \\
\text{\( \kappa_f \)} = 0.164 \\
\text{\( \mu_f \)} = 0.000365 \\
\text{\( \kappa_f \)} = 1.74 \times 10^5
\]

Note to instructor:

These properties are not available in the text. The student must go to conventional data sources to obtain them.

\[
\theta_f = 0.729 \left( \frac{\frac{\rho_f(\rho_v-\rho_f)}{\mu_f \kappa_f}}{h_f} \right)^{1/4} \frac{1}{(\Delta T)^{1/4}} \left( 1 + \frac{c_p}{h_f} \Delta T \right)^{1/4} 
\]

\[
\begin{align*}
15,000 & = 8979 \left( 1 + 0.00187 \Delta T \right)^{1/4} \quad \text{for water} \\
1700 & = 1746 \left( 1 + 0.0043 \Delta T \right)^{1/4} \quad \text{for benzene}
\end{align*}
\]

Some possible solutions:

**water:**

\[
\begin{align*}
\Delta T & = 3°C, \quad D = 0.043 \text{ m} \\
\Delta T & = 6°C, \quad D = 0.0216 \text{ m} \\
\Delta T & = 10°C, \quad D = 0.013 \text{ m}
\end{align*}
\]

**benzene:**

\[
\begin{align*}
\Delta T & = 10°C, \quad D = 0.116 \text{ m} \\
\Delta T & = 15°C, \quad D = 0.079 \text{ m} \\
\Delta T & = 25°C, \quad D = 0.049 \text{ m}
\end{align*}
\]
8.8 A 0.3 m high plate at 90°C condenses steam at 1 atm. Change the height or the temperature to values that will cause the laminar to turbulent transition to occur at the bottom.

From eqn. (8.72), turbulence occurs when:

$$\delta_{tu} = \sqrt[3]{[3\mu^2/\rho_f (\rho_f - \rho_g) g]450} = \sqrt[3]{(3v_f^2/g)450}$$

$$= \sqrt[3]{[3(0.294)^2 10^{-12}/9.8]450} = 0.0000228 \text{ m}$$

Then, using eqn. (8.56) we get,

$$\frac{\delta_{tu}}{\delta_{30}} = \left( \frac{x_{tu}}{0.3 \text{m}} \right) ^{1/4} ; \ x_{turb} = 0.3 \left( \frac{0.000228}{0.000103} \right)^4 = 7.28 \text{ m}$$

or

$$\frac{\delta_{tu}}{\delta_{30}} = \left( \frac{\Delta T_{turb}}{10^\circ C} \right)^{1/4} ; \ \Delta T_{turb} = 10 \left( \frac{0.000228}{0.000103} \right)^4 = 240^\circ C$$

We can't reach turbulence in a 0.3 cm length by cooling. The flow would freeze up first.

8.9 A cool plate spins in a synchronously rotating vapor, so $g(x) = \omega^2 x$. Find: $\text{Nu}_L$

$$q_{eff} = \frac{x \omega^{8/3} 4/3}{x^{2/3} \int_0^x x^{1/3} dx} = \frac{4 \omega^2}{3} x$$

so:

$$\text{Nu}_x = \left( \frac{\rho_f (\rho_f - \rho_g) 4 \omega^2 x^4 h_{fs}'}{4 \mu k \Delta T} \right)^{1/4}$$

and

$$h = \left( \frac{\rho_f - \rho g}{3 \mu \Delta T} \right)^{1/4} = \text{constant} = \bar{h}$$

Thus

$$\text{Nu}_L = 0.760 \left( \frac{\rho_f (\rho_f - \rho_g) \omega^2 h_{fs} L^3}{\mu k \Delta T} \right)^{1/4}$$
8.10 For the flow shown, calculate $\delta(x)$, $Nu_x$, and $Nu_L$

Eqn. (8.55) applies in this case. We rewrite it as follows:

$$\frac{d}{dx} \left( \frac{d \delta}{dx} \right) = \frac{k \mu \Delta T}{\beta \left( \frac{\rho_f}{\rho_l} \right) \phi \chi \mu}$$

And we integrate this using the b.c. $\delta(x=0) = \delta_0$:

$$\delta = \delta_0 = \frac{k \mu \Delta T x}{\beta \left( \frac{\rho_f}{\rho_l} \right) \phi \chi \mu}$$

So:

$$Nu_x = \frac{x}{\delta} = \left[ \frac{4k \mu \Delta T x}{\beta \left( \frac{\rho_f}{\rho_l} \right) \phi \chi \mu} \right]^{-1/4} + \left( \frac{\delta_0}{x} \right)^{-1/4}$$

and

$$Nu_L = \frac{1}{k} \int_0^L h(x) dx = \int_0^L \frac{dx}{\left[ \frac{4k \mu \Delta T x}{\beta \left( \frac{\rho_f}{\rho_l} \right) \phi \chi \mu} + \delta_0^2 \right]^{1/4}}$$

$$= \frac{4}{3} \left[ \frac{\rho_f \left( \frac{\rho_f}{\rho_l} \right) \phi \chi \mu}{4k \mu \Delta T} \left[ \frac{4k \mu \Delta T x}{\beta \left( \frac{\rho_f}{\rho_l} \right) \phi \chi \mu} + \delta_0^2 \right]^{1/4} \right]^{x=L}_{x=0}$$

$$Nu_L = \frac{4}{3} \left[ \left( \frac{\rho_f \left( \frac{\rho_f}{\rho_l} \right) \phi \chi \mu}{4k \mu \Delta T} \right)^{1/3} + \left( \frac{\rho_f \left( \frac{\rho_f}{\rho_l} \right) \phi \chi \mu \delta_0^2}{4k \mu \Delta T} \right)^{1/3} \right]^{2/3}$$

$$= \frac{4}{3} \left( \frac{\rho_f \left( \frac{\rho_f}{\rho_l} \right) \phi \chi \mu \delta_0^3}{3k \mu \Delta T} \right)$$
8.11 Prepare a table of formulas of the form:
\[ \overline{h} \text{ W/m}^2\text{-OC} = C[\Delta T \text{ OC/L m}]^{1/4} \]

for natural convection at normal gravity in air and in water at \( T_w = 27^\circ\text{C} \). Assume that \( T_w \) is close to \( 27^\circ\text{C} \). Your table should include results for vertical plates, horizontal cylinders, spheres, and possibly additional geometries. Do not include your calculations.

\[
Ra_{H_2O} = \left( \frac{3.08}{(\Delta T)} \right) \Delta T L^3 = \left( \frac{9.8(0.000275)}{1.962(0.262) \times 10^{-9}} \right) \Delta T L^3 = 2.23 \times 10^7 \Delta T L^3
\]

\[
Ra_{\text{air}} = \left( \frac{3.08}{1.566(2.203) \times 10^{-9}} \right) \Delta T L^3 = 9.47 \times 10^7 \Delta T L^3
\]

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Reference equation</th>
<th>Simplified formula for ( \overline{h} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical plate</td>
<td>eqn. (8.27)</td>
<td>153 (( \Delta T/L ))^{1/4}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.414 (( \Delta T/L ))^{1/4}</td>
</tr>
<tr>
<td>Horizontal cyl.</td>
<td>eqn. (8.28) (neglect lead const. restrict to larger values of ( Ra_D ))</td>
<td>109 (( \Delta T/D ))^{1/4}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.01 (( \Delta T/D ))^{1/4}</td>
</tr>
<tr>
<td>Sphere</td>
<td>eqn. (8.32) (neglect lead const. restrict to larger ( Ra_D )'s.)</td>
<td>101 (( \Delta T/D ))^{1/4}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.11 (( \Delta T/D ))^{1/4}</td>
</tr>
<tr>
<td>Other situation where</td>
<td>( Nu_D = C \overline{Ra_D}^{1/4} )</td>
<td>( C(235)(\Delta T/D)^{1/4} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( C(2.58)(\Delta T/D)^{1/4} )</td>
</tr>
</tbody>
</table>

8.12 For what value of the Prandtl number is the condition:

\[ \frac{2u}{v^2} \bigg|_{y=0} = \frac{\rho g \Delta T}{\nu} \]

satisfied exactly in the Squire-Eckert b.l. solution?

In the context of eqn. (8.19) we saw that \( C_1 \) must be \( 1/4 \); but eqn. (8.23) tells us that:

\[ C_1 = \frac{Pr}{3(20/21 + Pr)} = \frac{1}{4} \]

so:

\[ \frac{20}{21} + Pr = \frac{4}{3} Pr \]

Solving this, we obtain:

\[ Pr = 2.86 \]
**Problem 8.13**  The side wall of a house is 10 m in height. The overall heat transfer coefficient between the interior air and the exterior surface is 2.5 W/m²K. On a cold, still winter night \( T_{\text{outside}} = -30 \, ^\circ\text{C} \) and \( T_{\text{inside air}} = 25 \, ^\circ\text{C} \). What is \( \overline{h}_{\text{conv}} \) on the exterior wall of the house if \( \epsilon = 0.9 \)? Is external convection laminar or turbulent?

**Solution**  The exterior wall is cooled by both natural convection and thermal radiation. Both heat transfer coefficients depend on the wall temperature, which is unknown. We may solve iteratively, starting with a guess for \( T_w \). We might assume (arbitrarily) that \( \frac{2}{3} \) of the temperature difference occurs across the wall and interior, with \( \frac{1}{3} \) outside, so that \( T_w \approx (25 + 30)/3 - 30 = -11.7 \, ^\circ\text{C} = 261.45 \, \text{K} \). We may take properties of air at \( T_j \approx 250 \, \text{K} \), to avoid interpolating Table A.6:

<table>
<thead>
<tr>
<th>Properties of Air at 250 K</th>
<th>( k )</th>
<th>0.0226</th>
<th>W/m·K</th>
</tr>
</thead>
<tbody>
<tr>
<td>thermal conductivity</td>
<td>( k )</td>
<td>0.0226</td>
<td>W/m·K</td>
</tr>
<tr>
<td>thermal diffusivity</td>
<td>( \alpha )</td>
<td>1.59 \times 10^{-5}</td>
<td>m²/s</td>
</tr>
<tr>
<td>kinematic viscosity</td>
<td>( \nu )</td>
<td>1.135 \times 10^{-5}</td>
<td>m²/s</td>
</tr>
<tr>
<td>Prandtl number</td>
<td>( \text{Pr} )</td>
<td>0.715</td>
<td></td>
</tr>
</tbody>
</table>

The next step is to find the Rayleigh number so that we may determine whether to use a correlation for laminar or turbulent flow. With \( \beta = 1/T_f = 1/(250) \, \text{K}^{-1} \):

\[
Ra_L = \frac{g \beta (T_w - T_{\text{outside}}) L^3}{\nu \alpha} = \frac{(9.806)(-11.7 + 30)(10^3)}{(250)(1.59)(1.135)(10^{-10})} = 3.98 \times 10^{12}
\]

Since, \( Ra_L > 10^9 \), we use eqn. (8.13b) to find \( \overline{N\nu}_L \):

\[
\overline{N\nu}_L = \left\{ 0.825 + \frac{0.387 \, Ra_L^{1/6}}{1 + (0.492/\text{Pr})^{9/16}} \right\}^{2/7} = 1738
\]

Hence

\[
\overline{h}_{\text{conv}} = \frac{1738 \times 0.0226}{10} = 3.927 \, \text{W/m}^2\text{K}
\]

The radiation heat transfer coefficient, for \( T_m = (261.45 + 243.15)/2 = 252.30 \, \text{K} \), is

\[
h_{\text{rad}} = 4\epsilon \sigma T_m^3 = 4(0.9)(5.6704 \times 10^{-8})(252.30)^3 = 3.278 \, \text{W/m}^2\text{K}
\]

The revised estimate of the wall temperature is found by equating the heat loss through the wall to the heat loss by convection and radiation outside:

\[
(2.5)(25 - T_w) = (3.927 + 3.278)(T_w + 30)
\]

so that \( T_w = -15.8 \, ^\circ\text{C} \), which is somewhat lower than our estimate. We may repeat the calculations with this new value (without changing the property data) finding \( Ra_L = 3.09 \times 10^{12} \), \( \overline{N\nu}_L = 1799 \), \( \overline{h}_{\text{conv}} = 4.065 \, \text{W/m}^2\text{K} \), \( T_m = 250.3 \, \text{K} \), and \( h_{\text{rad}} = 3.201 \, \text{W/m}^2\text{K} \). Then

\[
(2.5)(25 - T_w) = (4.065 + 3.201)(T_w + 30)
\]

so that \( T_w = -15.9 \, ^\circ\text{C} \). Further iteration is not needed. Since the film temperature is very close to 250 K, we do not need to update the property data.

To summarize the final answer, \( \overline{h}_{\text{conv}} = 4.07 \, \text{W/m}^2\text{K} \) and most of the boundary layer is turbulent.
8.14 Plot $T_{sheet}$ vs. time for Ex. 8.2, if the sheets are 1% carbon, 2 m long and 8 mm thick ($w = 0.003$ m). The bath is water at 60°C and the sheets are introduced at 18°C. Compare the result with exponential response.

With reference to Example 8.2, with properties evaluated at $(60 + 80)/2$, or 35°C:

$$B = 0.678 \frac{0.6253}{2} \left[ \frac{A_{16}}{0.952 + 0.16} \right]^{1/4} \left[ \frac{2.2 (0.000371) 2^3}{1.505 (0.67) 10^{-13}} \right]^{1/4} = 14.8 \frac{W}{m^2 \cdot K}$$

Then:

$$T = 60 - \frac{1}{A_{1/4} + \frac{B}{4(780)(473)(0.003)}} \left( \frac{B}{4(780)(473)(0.003)} \right)^{-1/4} = 60 - \frac{1}{[0.393 + 0.00334t]^{1/4}}$$

The exponential response is:

$$\frac{T_T}{T_T - T_0} = e^{-t/T}$$

where

$$T = \frac{\rho CV}{kA} = \frac{7801(473)(w)}{B(39-18)A} = 31.94 \text{ sec}$$

$$T = 60 - 42 \exp(-0.0314t)$$

(Continued, next page)
range in which $h$ is overpredicted by the exponential approximation. (The predicted warming is too fast)
Problem 8.15  In eqn. (8.7), we linearized the temperature dependence of the density difference. Suppose that a wall at temperature \(T_w\) sits in water at \(T_\infty = 7\) °C. Use the data in Table A.3 to plot \(|\rho_w - \rho_\infty|\) and \(\rho_f \beta_f (T_w - T_\infty)\) for \(7\) °C \(\leq T_w \leq 100\) °C, where \((..)_f\) is a value at the film temperature. How well does the linearization work?

Solution  With values from Table A.3, we may perform the indicated calculations and make the plot. The linearization is accurate to within 10% for temperature differences up to \(40\) °C, and within 13% over the entire range.

**Properties of water from Table A.3**

<table>
<thead>
<tr>
<th>(T) [°C]</th>
<th>(\rho) [kg/m(^3)]</th>
<th>(\beta) [K(^{-1})]</th>
<th>((\rho_w - \rho_\infty))</th>
<th>(-\rho_f \beta_f (T_w - T_\infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>999.9</td>
<td>0.0000436</td>
<td>0.0</td>
<td>0.000</td>
</tr>
<tr>
<td>12</td>
<td>999.5</td>
<td>0.000112</td>
<td>-0.4</td>
<td>-0.389</td>
</tr>
<tr>
<td>17</td>
<td>998.8</td>
<td>0.000172</td>
<td>-1.1</td>
<td>-1.08</td>
</tr>
<tr>
<td>22</td>
<td>997.8</td>
<td>0.000226</td>
<td>-2.1</td>
<td>-2.02</td>
</tr>
<tr>
<td>27</td>
<td>996.5</td>
<td>0.000275</td>
<td>-3.4</td>
<td>-3.18</td>
</tr>
<tr>
<td>32</td>
<td>995.0</td>
<td>0.000319</td>
<td>-4.9</td>
<td>-4.52</td>
</tr>
<tr>
<td>37</td>
<td>993.3</td>
<td>0.000361</td>
<td>-6.6</td>
<td>-6.05</td>
</tr>
<tr>
<td>47</td>
<td>989.3</td>
<td>0.000436</td>
<td>-10.6</td>
<td>-9.54</td>
</tr>
<tr>
<td>67</td>
<td>979.5</td>
<td>0.000565</td>
<td>-20.4</td>
<td>-18.1</td>
</tr>
<tr>
<td>87</td>
<td>967.4</td>
<td>0.000679</td>
<td>-32.5</td>
<td>-28.4</td>
</tr>
<tr>
<td>100</td>
<td>958.3</td>
<td>0.000751</td>
<td>-41.6</td>
<td>-36.2</td>
</tr>
</tbody>
</table>
8.16 A 77°C vertical wall heats air at 27°C. Find $Ra_L$, $S_{top}/L$, and L, where the line in Fig. 8.3 ceases to be straight. Comment on the implications of your results.

The line in Fig. 8.3 begins to deviate from straightness, and flatten out, when:

$$Ra_L \left[1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right]^{-1.7719} = 10^3$$

But for $T = 325^\circ$K, $Pr = 0.7085$, so $Ra_L = 2884$.

(Thus result could reasonably range from $10^3$ to $10^4$.)

But $Ra_L = 2884 = \frac{9 \beta \Delta T L^3}{2\sigma} = \frac{9.8(\frac{1}{360})50 L^3}{1.014(2561)10^{10}}$

$L = 0.00936 \text{m} = 0.936 \text{cm}$

Find $\frac{S_{top}}{L}$:

$$Nu_K \bigg|_{x=L} = 2 \frac{L}{b} = \frac{3}{4} Nu_L$$

$$\frac{S_{top}}{L} = \frac{8}{3Nu_L}$$

But $Nu_L = 0.68 + 0.67 \frac{Ra_L^{1/4}}{\left[1 + (0.492)^{9/16}\right]^{1/9}} = 0.68 + 0.67 (10^{3})^{1/4} = 4.45$
8.16 (continued)

Thus: \[ x = 0.56 \text{ cm} \]

\[ \Delta \theta = 0.60 \] 

The b.l. looks something like this -- quite thick. Thus the deviation from the linear relationship reflects the breakdown of the b.l. assumptions.

8.17 A horizontal 0.08 m diameter pipe, at 150°C on the inside, has 85% magnesia insulation with a 0.11 O.D. What is the heat loss if \( T_\infty = 17°C \)?

First we have to guess the outside temperature to evaluate properties. \( h \) on the outside should be low -- around 6 W/m²·°C so we go to equation (2.25) and calculate

\[
Q = \frac{\Delta T}{\frac{1}{2\pi h r_0} + \frac{\ln r_e/r_0}{2\pi k}} = \frac{150 - 17}{\frac{1}{2\pi (6)(0.055)} + \frac{\ln(0.055/0.04)}{2\pi (0.071)}} = 111 \text{ W/m}
\]

for 80% mag.

Therefore \( \Delta T \) across \( h = \frac{Q}{h(10)} = \frac{111}{6(10)(0.11)} = 53.5 \), \( T_{\text{outside mag.}} = 70.5°C \)

we then evaluate properties at \( \frac{70.5 + 17}{2} = 44°C \) or 317°C

Then:

\[
\frac{1}{4} \left[ \frac{9.9 \frac{1}{2} (70.5 - 17)(0.11)^3}{1.735(2.447) 10^{-10}} \right]^{1/4} = 36.9
\]

\[
\overline{Nu}_D = 0.36 + 0.518(36.9) = 19.5 \quad \overline{h} = 19.5 \frac{0.02614}{0.11} = 4.63 \text{ W/m²·°C}
\]

This gives an outside temp. of \( \frac{99.3}{4.63(0.11)} = 62°C \). It should suffice to correct the Rayleigh no. by a factor of \( \left( \frac{62.0 - 17}{70.5 - 17} \right)^{1/4} \) or 0.958. Then \( \overline{h} \) will become 4.44 W/m²·°C which we shall use:

\[
Q = \frac{150 - 17}{\frac{1}{0.99(2\pi)(0.055)} + \frac{\ln(0.055/0.04)}{2\pi (0.071)}} = \frac{13.3}{0.652 + 0.114} = 97.3 \text{ W/m}
\]
8.18 How much heat is needed to keep a horizontal wire, 10⁻⁵ m in diam., at 40°C in 10°C water.

\[
\text{eqn. (8.28): } \overline{N_u}_p = 0.36 + \frac{0.54}{(1 + \frac{(0.333)}{Pr})^{0.54}} \begin{bmatrix} \frac{9 \beta \Delta T D^3}{\nu \alpha} \\ \frac{\nu}{\alpha} \end{bmatrix}^{0.411} \frac{9.8(0.00021)(30)10^{-4}}{1.159(0.885) \times 10^{-13}}
\]

where we evaluate properties at 298°C

\[
\overline{h} = \frac{k}{D \overline{N_u}_p} = 0.6057 \times 0.333 = 26,269 \frac{W}{m^2°C}
\]

So

\[
\overline{Q} = \overline{h} \pi D \Delta T = 26,269(\pi)10^5(30) = 24.8 \frac{W}{m}
\]

8.19 A 20 cm vertical run of 0.5 cm diam. tubing carries condensing vapor at 60°C. The air outside is at 27°C. What is the heat loss?

Neglect the resistance of the tubing and of \(h_{\text{cond.}}\). They aren't specified, and they would have to be small.

First treat the tube like a vertical wall, \(\pi(0.005 \text{ m})\) wide and 0.2 m high. From equation (8.13a), using \(T = 316.5°C\):

\[
\overline{N_u}_L = 0.68 + 0.67 \begin{bmatrix} \frac{9 \beta \Delta T L^3}{\nu \alpha} \\ \frac{\nu}{\alpha} \end{bmatrix}^{0.416} \left[ 1 + \left( \frac{0.492}{Pr} \right) \right]^{-1.4/9}
\]

\[
= 0.68 + 0.67 \begin{bmatrix} \frac{9.8(0.333)}{1.402(2.14) \times 10^{-10}} \times 33(0.2)^3 \\ 1.402(2.14) \times 10^{-10} \end{bmatrix}^{0.416} \left[ 1 + \left( \frac{0.492}{0.709} \right) \right]^{-1.4/9} = 37.11
\]

\[
\overline{h} = \frac{k}{L \overline{N_u}_L} = \frac{0.02731}{0.2} \times 37.11 = 5.06 \frac{W}{m^2 °C}
\]

Now we must correct \(\overline{h}\) for curvature using Fig. (8.7)

\[
\frac{R \pi}{(Ra/Pr)^{1/4}} \overline{h} = \frac{2 \sqrt{2}(0.2)}{77.22(0.005)} = 2.93 \text{, so } \overline{h}_{\text{actual}} = 1.8 \frac{W}{m^2 °C}
\]

The actual value of \(\overline{h}\) is 1.8 (5.06) = 9.11 \(\frac{W}{m^2 °C}\) and

the heat loss is:

\[
\overline{h} A \Delta T = 9.11 \pi(0.005)(0.2)(33) = 0.944 W
\]
8.20 How much heat is removed from the body shown.

\[
\overline{Nu}_e = 0.52 \sqrt{Ra} = 0.52 \left( \frac{3.8 \times 10^3 \times 20 \times (0.2693)^3}{1.437 \times (2.072 \times 10^{-10})} \right)^{1/4} \\
= 42.5 \\
\]

So:
\[
\overline{h} = 42.5 \frac{0.02536}{0.2693} = 4.00 \text{ W m}^{-2} \text{K}^{-1}
\]

The area of the cone is \(2 \times \frac{1}{2} \text{ base} \times \text{perimeter of base} \times \text{lateral edge} \)

\[
= \frac{0.2693}{2} \pi (0.1) = 0.0423 \text{ m}^2
\]

So:
\[
Q = \overline{h} A \Delta T = 4(0.0423)(20) = 3.38 \text{ W}
\]
Problem 8.22 You are asked to design a vertical wall panel heater, 1.5 m high, for a dwelling. What should the heat flux be if no part of the wall should exceed 33 °C? How much heat goes to the room if the panel is 7 m wide with ε = 0.7? Hint: Natural convection removes only about 200 W depending on what room temperature you assume.

Assume \( T_\infty = 23^\circ C \) (73.4°F) as a maximum value. Then \( \Delta T = 10^\circ C \). From Fig. 8.9, \( \Delta T_{\text{max}} = 10^\circ C \) gives \( \Delta T = 0.833 \times 10 = 8.33^\circ C \).

To get \( q_w \) we must now solve eqn. (8.43a) by trial and error:

\[
\left( \frac{q_w}{k(\Delta T)} \right)^{1/4} - 0.68 \left( \frac{q_w}{k(\Delta T)} \right)^{1/6} = \frac{0.67 \left[ \frac{9.8}{k\Delta T} q_w L^4 \right]^{1/4}}{1 + \left( \frac{0.492}{\left( \frac{q_w}{k(\Delta T)} \right)^{1/6}} \right)^{9/4}}
\]

Evaluate properties at \( 28^\circ C \approx 300^\circ K \) so:

\[
\left( \frac{q_w}{k(\Delta T)} \right)^{1/4} = 1.25 \left( \frac{0.0264}{0.333} \right) = 0.25 \frac{q_w}{k(\Delta T)} = \frac{0.67 \left[ \frac{9.8}{0.0264(1.566 \times 1.200 \times 10^7)} \right]^{1/4}}{1 + \left( \frac{0.492}{0.0264(1.566 \times 1.200 \times 10^7)} \right)^{9/4}}
\]

or

\[
11.16 q_w = 1.102 \times 1.25 q_w = 1.40 q_w
\]

or

\[
11.16 q_w = 191.1 \quad \text{so} \quad q_w = 17.12 \frac{W}{m^2}
\]

(This corresponds with \( \bar{n} = \frac{17.12}{8.33} \approx 2.05 \frac{W}{m^2 \cdot ^\circ C} \).)

\[
\bar{Q} = q_w A = 17.12 \times (1.5)(7) = 180 \text{ W}
\]

Since the wall temperature is known, the radiation loss can be computed separately because \( T_m \) does not change much along the length of the wall:

\[
T_m = \frac{(T_\infty + \bar{T}_{\text{wall}})}{2} = \frac{(23 + 23 + 8.33)}{2} + 273.15 = 300.3 \text{ K}
\]

and then

\[
q_{\text{rad}} = h_{\text{rad}} T_m^3 (\bar{T}_{\text{wall}} - T_\infty) = 4(0.7)(5.670 \times 10^{-8})(300.3)^3(8.33) = 35.8 \text{ W/m}^2
\]

Thus, radiation carries an additional \((35.8)(1.5)(7) = 376 \text{ W}\), for a total panel heating power of \(376 + 180 = 556 \text{ W}\).

8.23 A 0.14 cm high wall is heated by condensation of steam at one atm. What will happen to \( \bar{h} \) and \( Q \) if the steam is replaced with an organic vapor?

\[
\bar{h}_{\text{organic}} = \frac{k_o}{k_s} \left[ \frac{\rho_o \left( \rho_s - \rho_o \right) \rho_o h_{fg} \left( 1 + c_p(T_s - T_w) \right)}{h_{fg} \rho_o \left( 1 + c_p(T_s - T_w) \right) + \mu_s k_o}{\mu_s} \right]^{1/4}
\]

We can probably neglect \( \mu_s \) at 1 atm and the \( c_p(T_s - T_w)/h_{fg} \) terms will contribute little. Thus:

\[
\bar{h}_o \approx \left( \frac{k_o}{k_s} \right)^{3/4} \left[ \frac{(T_{s_o} - T_w)}{T_{s_o} - T_w} \frac{\rho_s}{\rho_o} \frac{h_{fg} \rho_o}{h_{fg} \rho_o} \right]^{1/4}
\]

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8.23 (continued) Finally, since \( Q = hA(T_s - T_w) \):

\[
\frac{Q_s}{Q_t} = \left( \frac{k_s}{k_t} \frac{T_s - T_w}{T_s - T_w} \right)^{3/4} \left( \frac{\rho_s \gamma_s}{\rho_t \gamma_t} \frac{h_{fg, s}}{h_{fg, t}} \right) \]

These expressions give the factors by which \( h \) and \( Q \) will change, once the instructor specifies the particular fluid. The student should remember that \( T_{sat} \), as well as the other thermal properties, will change when the fluid is changed.

8.24 A 0.01 m diam. tube, 0.27 m long, runs horizontally through saturated steam. Plot \( Q \) vs. \( T_{tube} \) for \( 50 < T_{tube} < 150^\circ C \).

For natural convection, evaluating properties at \( \frac{125+100}{2} = 112.5^\circ C \) = 386 K

\[
Re_0 = \frac{9.8(0.0029)0.01}{(2.239)(2.20)} \Delta T = 56.3 \Delta T
\]

so:

\[
N_u_0 = 0.36 + \frac{0.518 Re_0^{1/4}}{[1 + (0.553 \Delta T)^{1/9}]} = 0.36 + 1.12 \Delta T^{1/4}
\]

And for film condensation, \( h_{fg} = 2.257 \times 10^4 \left[ 1 + \left( 0.663 - \frac{0.228}{1.75} \right) \Delta T \right] = 2.257 \times 10^4 (1 - 0.001 \Delta T) \), so we can use an average value (for \( \Delta T = 25^\circ C \)) of 2,313,000

\[
N_u_D = 0.728 \left[ \frac{2 \rho c_p k_s D^3}{\mu} \right]^{1/4} = 0.728 \left[ \frac{958(9.8)(2.313)}{2.77 (0.681)10^4} \right]^{1/4}
\]

\[
\frac{Q}{\Delta T^{1/4}} = \frac{4.26}{\Delta T^{1/4}}
\]

\[
\begin{array}{|c|c|c|}
\hline
\Delta T & N_u_D & Q = \frac{k}{D}N_u_D A \Delta T \\
50 & 3.34 & 3.28 W \\
40 & 3.19 & 2.50 \\
30 & 2.99 & 1.76 \\
20 & 2.72 & 1.075 \\
10 & 2.37 & 0.495 \\
-10 & - & -1389 W \\
-20 & - & -2337 \\
-30 & - & -3167 \\
-40 & - & -3930 \\
-50 & - & -4636 \\
\hline
\end{array}
\]

Notice that:

\[ Q_{cond} \gg Q_{conv} \]

\[ Q_{cond} \approx \Delta T^{3/4} \]

\[ Q_{conv} \approx \Delta T^{5/4} \]
8.25 A plate, 2m high, condenses steam at 1 atm. Calculate $\Delta T$ at which: a) Nusselt's solution loses accuracy; b) The film becomes turbulent.

$$\Gamma_c = \frac{\rho_f (\rho_f - \rho_g) \gamma}{3 \mu^2} = \frac{\rho_f}{3 \nu^2} = 6 \text{ for (a) and 450 for (b)}$$

but $\nu = 0.29 \times 10^{-6}$ so $\delta = \left( \frac{3 \Gamma_c \nu^2}{\rho_f} \right)^{1/3} = 0.00008106 \text{ m (a)}$

$b) \ 0.000342 \text{ m (b)}$

$$\delta = \left( \frac{4k\Delta T \mu L}{\rho_f (\rho_f - \rho_q) g h_f} \right)^{1/4} \approx \left( \frac{4k\Delta T \nu L}{\rho_f g h_f} \right)^{1/4} \text{ We'll go back and use } h_f^* \text{ if } \Delta T \text{ is large.}$$

$$\Delta T^{1/4} = 0.0000782$$

(a) $0.00008106 = 0.0000782 \Delta T^{1/4}$; $\Delta T = 1\text{–}15^\circ C$

when Nusselt's solution loses accuracy.

(b) $0.000342m = 0.0000782 \Delta T^{1/4}$; $\Delta T = 366^\circ C$

The flow can never become turbulent. It will freeze first.

8.26 A reflux condenser has $a = 18^\circ$, $d = 0.8$, $D = 6$ cm. At 30$^\circ$C it condenses steam at 1 atm. What is $h$? (Evaluate properties at 65$^\circ$C)

To use Fig. 8.14, compute: $B = \frac{\rho_f - \rho_g}{c P} \frac{\Delta T}{h_f} \tan^{\gamma} = \frac{979.4 - 0.6}{979.4}$

$$= \frac{4186(70) \tan^{2.18}}{2,257,000(1.72)} = 0.008$$

$d/D = 0.8/6 = 0.1333$

So: $\frac{Nu_L}{\Gamma} = \left( \frac{h_f' \gamma (\cos \gamma)^3}{\nu k \Delta T} \right)^{1/4} = 0.727$

but $h_f' = h_{f,9} \left( 1 + \frac{0.683 + 0.728}{P_{\gamma}} \right) = h_{f,9} \left( 1 + \frac{0.683 + 0.728}{2.67} \right) = (1 + 0.71 J_9) h_{f,9}$

$$Nu_L = 0.727 \left[ \frac{978.8(9.8)2.257(10)}{0.435(10)^{-6}(0.6585)(100-30)} (0.008 \cos 18^\circ)^3 \right]^{1/4}$$

$$= 110$$

Then: $h = \frac{k}{d \cos \gamma} \frac{Nu_L}{0.6585(110) \cos 18^\circ} = 9512 \frac{W}{m^2 \cdot ^\circ C}$
A 0.05m helix of 0.005 m diam. tubing carries 15°C water through saturated steam at 1 atm. Specify \( \alpha \) and the number of coils if 6 kg/hr of steam are to be condensed. 
\[ h_{\text{inside}} = 600 \text{W/m}^2\cdot\text{°C}. \]

First establish an approximate \( T_{\text{wall}} \) assuming an ordinate from Fig. 8.14.

\[
h_{\text{cond}}(T_{\text{sat}}-T_{\text{w}}) = \frac{k}{d\cos\alpha} \left(\frac{g(\rho_f-\rho_g)h_{fg} g(\cos\alpha)^3}{\nu(\rho_f-\rho_g)h_{fg} g(\cos\alpha)^3}\right)^{1/4}(T_{\text{sat}}-T_{\text{w}})
\]

\[
= h_i (T_{\text{w}}-T_i)
\]

For openers, call \( h_{\text{fg}} = h_{\text{fg}} \) and \( \cos\alpha = 1 \). Then:

\[
\frac{0.6811}{0.005} (\cos\alpha)^{-1/4} \left[ \frac{958(2257000)9.8(0.005)}{0.29\times10^{-6}(0.6811)} \right]^{3/4}(100-T_{\text{w}}) = 600(T_{\text{w}}-15)
\]

\[
77.23(100-T_{\text{w}})^{3/4} = T_{\text{w}}-15; \quad T_{\text{w}} = 98.88 \text{°C}
\]

Now do an accurate computation based on this estimate. Base properties on \( T_{\text{w}} = 100 \text{°C} \):

\[
B = \frac{958.3-0.6}{958} \frac{4219(18)}{2,257,000} \frac{\tan^2\alpha}{1.72} = 0.002\tan^2\alpha = 0, \text{and} \frac{d}{D} = 0.1
\]

Then from Fig. 9.14 the lead const. = 0.727

\[
\frac{0.6811}{0.005} \left[ \frac{958(2,257,000)9.8(0.005)}{0.29(10)^{-6}(0.6811)} \right]^{1/4} \cos^{-1/4}\alpha = 600(83.88) \quad \text{A}
\]

\[
\cos^{-1/4}\alpha = 0.9997, \text{ not possible}
\]

Pick \( T_{\text{w}} = 1.07 \text{°C} \). Then:

\[
\cos^{-1/4}\alpha = 0.9997 \left[ \frac{83.93(1.116)}{83.88(1.07)} \right]^{3/4} = 28°
\]

\[
\tilde{h} = \left[ \text{LHS of } \text{A} \right] \left( \frac{1.07}{1.116} \right)^{3/4} = 47,625 \frac{\text{W}}{m^2\cdot\text{°C}}
\]

Then
\[
\tilde{h} = \frac{Q}{h_{\text{fg}}} = \tilde{h} \frac{\text{length}(\pi d)\Delta T}{h_{\text{fg}}} = \frac{6}{3600} \frac{\text{kg}}{\text{kg/s}}, \text{so length} = 0.783 \text{ m}
\]

Finally:
\[
\text{no. of coils} = \frac{\text{length}}{\pi D/\cos\alpha} = \frac{0.783\cos28°}{\pi(0.05)} = 4.4
\]

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8.29 What is the maximum speed of air in the natural convection b.l., in

Example 8.3?

First find where u maximizes in \( y/\delta \) using eqn. (8.18)

\[
\frac{d(u/\delta)}{d(y/\delta)} = 0 = 1 - 4\frac{y^3}{\delta^3} + 3\left(\frac{y}{\delta}\right)^2 \quad \text{or} \quad \left(\frac{y}{\delta}\right)^2 - \frac{4}{3}\left(\frac{y}{\delta}\right) + \frac{1}{3} = 0
\]

Thus:

\[
\frac{y}{\delta} = \frac{2}{3} \pm \sqrt{\frac{4}{9} - \frac{3}{3}} = 1 \text{ or } \frac{1}{3}; \quad \frac{y}{\delta} = \frac{1}{3} \text{ gives } u_{\text{max}}.
\]

Now using: \( U(x) = C_1 \frac{\beta g A T}{2\delta} \delta^2 \) and \( C_1 = Pr/3(\frac{2\rho}{\rho_1} + Pr) \)

we get:

\[
U_{\text{max}} = \frac{Pr}{3(\frac{2\rho}{\rho_1} + Pr)} \frac{\beta g A T}{2\delta} \delta^2 \left[\frac{y}{\delta}(1 - \frac{y}{\delta})\right] \quad \frac{y}{\delta} = \frac{1}{3}
\]

Using numbers from Example 8.3 we obtain:

\[
U_{\text{max}} = \frac{0.711}{3(0.952 + 0.71)} \frac{0.00348(2.0)(0.40 - 14)}{1.566 \times 10^{-5}} \left[\frac{y}{\delta}(1 - \frac{y}{\delta})\right] \quad \frac{y}{\delta} = \frac{1}{3}
\]

\[
= 0.354 \text{ m/s}
\]
8.31 A large industrial process requires that water be heated by a large cylindrical heater using natural convection. The water is at 27°C. The cylinder is 5 m in diameter and it is kept at 67°C. First find \( \bar{h} \). Then suppose \( D \) is doubled (D→10 m). What is the new \( \bar{h} \)? Explain the similarity of these answers in the turbulent natural convection regime.

At 47°C:

\[
\Pr = 3.67 \quad \text{and} \quad Ra_L = \frac{9.8 \times (0.000435)(67-27)6^3}{0.542 \times 1.541 \times 10^{-6}} = 2.44 \times 10^4
\]

So we use equation (8.29) and obtain:

\[
\bar{Nu}_L = \left[ 0.60 + 0.387 \left( \frac{2.44 \times (10)^{14}}{1 + \left( \frac{0.559}{3.67} \right)^{9/6}} \right)^{1/6} \right]^2 = 7951
\]

\[
\bar{h} = 7951 \frac{k}{L} = 7951 \frac{0.6367}{5} = 1012 \text{ W/m}^2\text{°C} \quad L=5 \text{ m}
\]

If \( L \) is doubled we have

\[
\bar{Nu}_L = \left[ 0.60 + 0.387 \left( 1.437 \times 10^{-4} \times 2 \right)^{1/6} \right]^2 = 15,840
\]

\[
h = 15,840 \frac{0.6367}{10} = 1009 \text{ W/m}^2\text{°C} \quad L=10 \text{ m}
\]

We note that at high \( Ra \), eqn. (8.29) reduces to:

\[
\bar{Nu}_L = \frac{\bar{h}L}{k} = \frac{0.387^2}{\left[ 1 + \left( \frac{0.559}{\Pr} \right)^{9/6} \right]^{16/9}} \quad Ra_L^{1/3}
\]

So

\[
\bar{h} = f_h(\Pr) \left( \frac{9 \beta \Delta T k}{25/\rho c_p} \right)^{1/3} \neq f_h(L)
\]

The 1/3 power dependence of \( \bar{Nu}_L \) on \( Ra_L \) that occurs in turbulent natural convection causes \( \bar{h} \) to be independent of length in this regime!
8.32 A vertical jet of liquid, of diameter, \( d \), and moving at velocity, \( u_m \), impinges on a horizontal disc rotating \( \omega \) rad/s. There is no heat transfer in the system. Develop an expression for \( \delta(r) \), where \( r \) is the radial coordinate on the disc. Contrast the \( r \) dependence of \( \delta \) with that of a condensing film on a rotating disc and explain the difference qualitatively.

Nusselt's expression for the mass flow rate in the film is valid:

\[
\dot{m} \frac{k_f}{\mu} = \frac{\rho_f (\rho_f - \rho_a)}{3} g \delta^3(r)
\]

However, in this case, \( \rho_f \) is the liquid density and \( \rho_a \) is the air density, \( \rho_a \). The "gravity" is now \( \omega^2 r \). Then the total mass flow is:

\[
\dot{M} = \rho_f u_m \frac{\pi}{4} d^2 = \dot{m}(2\pi r) = \frac{\pi \rho_f (\rho_f - \rho_a)}{3} \omega^2 r^2 \delta^3(r)
\]

we solve this for \( \delta(r) \):

\[
\delta(r) = \frac{v}{\sqrt{\frac{3}{8} \frac{\rho_f u_m d^2}{\omega^2 (\rho_f - \rho_a) \delta^3}}} \frac{1}{r^{2/3}}
\]

The film thickness is uniform during condensation on a rotating disc (see discussion following eqn. (8.70).) because condensation causes the film to accrete liquid. Thus \( \dot{m} \) increases as \( r^2 \), and this accretion just compensates the natural thinning that must occur as the sheet spreads.

But in this case, \( \delta \sim r^{-2/3} \) because no fluid is added as the film spreads out.

8.33 We have seen that, if properties are constant, \( h \sim \Delta T^{1/4} \) in natural convection. If we consider the variation of properties as \( T_w \) is increased over \( T_\infty \), will \( h \) depend more or less strongly on \( \Delta T \) in air? -- in water?

We see that \( h \) in natural convection varies as \( k/(\gamma \alpha)^{1/4} \). We then find that this quantity increases strongly in water -- especially at lower values of \( T_\infty \) -- so \( h \) depends more strongly than \( \Delta T^{1/4} \) on \( \Delta T \). In the case of air \( k/(\gamma \alpha)^{1/4} \) is a constant within \( \pm 13 \\% \) over the entire range of properties given in the book. It drops off only slightly with increasing temperature so the dependency of \( h \) on \( \Delta T \) is only a little less strong than \( \Delta T^{1/4} \). If \( T_w \) were less than \( T_\infty \), these trends would be reversed.
Equation (8.54) still applies, but the sign is reversed, thus:

\[
\frac{k}{\delta} \frac{d}{dx} \left( \frac{T_w - T_{sat}}{\delta} \right) = -h_f g \frac{d\delta}{dx} = -h_f g \frac{(\rho_f - \rho_g)}{\delta} \frac{d^2 \delta}{dx^2}
\]

so:

\[
\frac{4 k \Delta T \delta^5}{h_f g (\rho_f - \rho_g) g} = -\frac{d\delta^4}{dx}
\]

Integrating from \( \delta(x=0) = \delta_0 \) to \( \delta(x) \) we get:

\[
\frac{4 k \Delta T \delta x}{h_f g (\rho_f - \rho_g) g} = \delta_0^4 - \delta^4 \quad \text{or} \quad \delta(x) = \left[ \delta_0^4 - \frac{4 k \Delta T \delta x}{h_f g (\rho_f - \rho_g) g} \right]^{1/4}
\]

Then:

\[
Nu_x = \frac{x}{\delta(x)} = \left[ \left( \frac{x}{\delta(x)} \right)^4 - \frac{4 k \Delta T \delta x}{h_f g (\rho_f - \rho_g) g} \right]^{-1/4}
\]

and \( x_f \) is the value of \( x \) at which \( \delta(x) = 0 \)

\[
x_f = \frac{3(\rho_f - \rho_g) h_f g \delta_0^4}{4 k \Delta T}
\]

For the specified case we set \( h_f g \approx h_f g \) and get

\[
x_f = \frac{3.8(957.2 - 0.6)(2257000)(0.0001)^4}{4 (0.6811)(5)(0.290)10^{-6}}
\]

the plate will dry out when \( x_f = 0.5356 \) m.
In a particular solar collector, dyed water runs down a vertical plate in a laminar film, with thickness, \( \delta_0 \), at the top. The sun's rays pass through parallel glass plates (see Section 11.6) and deposit \( q_s \) \( \text{W/m}^2 \) in the flowing water film. Assume the water to be saturated at the inlet and the plate behind it to be insulated. Develop an expression for \( \delta(x) \) as the water evaporates. Develop an expression for the maximum length of wetted plate, and provide a criterion for the laminar solution to be valid.

Equation (8.54) applies to this problem, but we must replace \( k(T_w - T_{sat})/\delta \) with \(-q_w\). Thus:

\[
-q_w = h_{fg} \frac{d\delta}{dx} = \frac{h_{fg}(\rho_f - \rho_g)}{2} q_x^2 \frac{d\delta}{dx}
\]

So:

\[
\frac{3q_w \delta}{q_{fg}(\rho_f - \rho_g)} = -\frac{d\delta^3}{dx}
\]

Integrating from \( \delta(x=0) = \delta_0 \) to \( \delta(x) \) we get

\[
\frac{3q_w \delta x}{q_{fg}(\rho_f - \rho_g)} = \delta_0^3 - \delta^3 \quad \text{or} \quad \delta(x) = \left[ \frac{3q_w \delta x}{q_{fg}(\rho_f - \rho_g)} \right]^{1/3}
\]

The film will dry out at \( x = x_f \) when \( \delta(x_f) = 0 \),

\[
x_f = \frac{q_{fg}(\rho_f - \rho_g)}{3q_w} \delta_0^3
\]

and this solution will only be valid when \( \Gamma_c < 450 \). Thus we write at the top of the plate:

\[
\Gamma_c = \frac{(\rho_f - \rho_g)g \delta_0^3}{3q_w} < 450 \mu \quad \text{or} \quad \delta_0 < \sqrt{\frac{1350 \delta^2 \mu}{3(q_f - q_g)}}
\]
What heat removal flux can be achieved at the surface of a horizontal 0.01 mm diameter electrical resistance wire in still 27°C air if its melting point is 927°C?

Evaluate \( \beta \) at 27°C and the other properties at \((927+27)/2\), or 477°C = 750°C K: 
\[
\begin{align*}
\zeta &= 7.93(10)^{-5}, \quad \alpha = 10.57(10)^{-5}, \quad k = 0.059, \quad Pr = 0.703.
\end{align*}
\]

\[
Ra_0 = \frac{\beta B (y/300) 300 (0.000009)^5}{7.43 (10.57) 10^{-5} 5} = 3.744 \times 10^{-6}
\]

Using equation (8.29) -- applicable for \( Ra_0 \gg 10^7 -- we get:

\[
\overline{Nu}_D = \left\{ 0.16 + 0.387 \left[ \frac{3.744 \times 10^{-6}}{1 + (0.059/0.703)^{0.5675} / 0.5675} \right]^{1/2} \right\}^2 = 0.4096
\]

Then:

\[
\overline{h} = \overline{Nu}_D \frac{k}{\delta} = 0.4096 \times 0.059 \times 10^{-5} = 2212 \frac{W}{m^2 \cdot ^\circ C}
\]

Thus:

\[
\overline{q} = 2212(927-27) = 1,920,700 \frac{W}{m^2}
\]

This is an incredibly high heat flux. Natural convection, which normally inefficient, becomes remarkably effective when the diameter is very small.

**Problem 8.37** A 0.03 m O.D. vertical pipe, 3 m in length with \( \varepsilon = 0.7 \), carries refrigerant through a 24°C room at low humidity. How much heat does it absorb from the room if the pipe wall is at 10°C?

Evaluate properties at \((10°C + 24°C)/2 = 17°C\). \( \zeta = 1.477(10)^{-5}, \alpha = 2.207(10)^{-5} \)

Then:

\[
Ra_L = \frac{9.8 (1297)(24-10) 3^3}{2.207(1.477) 10^{-5} 5} = 3.826(10)^{10}
\]

eqn. (8.21)

\[
\overline{Nu}_L = 0.68 + 0.67 \left( \frac{3.826 \times 10^{-10} \delta}{1 + \left( \frac{0.492}{0.713} \right)^{0.5625}} \right) = 0.9441
\]

\[
\overline{h}_{flat \ plate} = 228 \frac{0.0254}{3} = 1.933 \frac{W}{m^2 \cdot ^\circ C}
\]

Correct for curvature using Fig. 8.7:

\[
\frac{Ra_L}{Pr} = \frac{2.28}{\left( Ra_L / Pr \right)^{1/4}} \approx \frac{2.28}{(3.826(10)^{10})^{1/4} 0.713} = \frac{3}{0.015} = 1.18
\]

so \( \overline{h}_{cyl} / \overline{h}_{plate} = 1.37 \) and

\[
\overline{h}_{cyl} = 1.37(1.933) = 2.65 \frac{W}{m^2 \cdot ^\circ C}
\]

\[
Q = \overline{h}_{cyl} \Delta T = 2.65(3)(0.03)(24-10) = 10.5W
\]

But \( h_{rad} = 4 \varepsilon \sigma T_m^3 = 4(0.7)(5.670 \times 10^{-8})(17 + 273)^3 = 3.88 \frac{W}{m^2 \cdot ^\circ C} \)

Adding the natural convection and thermal radiation heat transfer coefficients, we can compute \( Q \):

\[
Q = (\overline{h}_{cyl} + h_{rad}) \Delta T = (2.65 + 3.88)(3)(0.03)(24 - 10) = 25.8W
\]
8.38 A 1 cm OD tube at 50°C runs horizontally in 20°C air. What is the critical radius of 85% magnesium insulation on the tube?

From eqn. (2.27) we have: \( r_{cr1} = \frac{k_{mag}}{h} = \frac{k_{mag}}{k_{air}} \frac{2r_c}{\bar{h}} \)

\[ \bar{h} = \frac{0.516 + 0.518}{\left[ 1 + \left( \frac{0.565}{0.711} \right)^{\frac{4}{9}} \right]^4} \frac{9.8}{1.546(2.203) \times 10^4} \left[ \frac{\Delta T(r_c)}{r_c} \right]^{\frac{3}{4}} = 2 \sqrt{\frac{k_{mag}}{k_{air}}} \]

We'll evaluate properties at 27°C (\( T_{wall} = 34°C \)) & hope that we won't have to re-iterate.

\[ 0.36 + 38.89 \left( \frac{r_c}{\bar{h}} \Delta T \right)^{\frac{3}{4}} = 3.202 \]

So \( 2r_c = 0.03055/\Delta T^{\frac{3}{4}} \).

Furthermore: \( q = \frac{k_{mag} (30 - \Delta T)}{\ln (r_c/0.005)} = \bar{h} \Delta T \)

but: \( \bar{h} = \frac{k_{air}}{2r_c} \left( \frac{\bar{h}}{r_c} \right) \frac{k_{mag}}{k_{air}} = \frac{1}{r_c} \sqrt{\frac{k_{mag}}{k_{air}}} \)

so: \( \frac{30}{\Delta T} - 1 = \frac{\ln (r_c/0.005)}{r_c} \sqrt{\frac{k_{air}}{k_{mag}}} = \frac{30}{0.03055} \left( \frac{2r_c}{0.03055} \right)^{\frac{3}{2}} = \frac{\ln (r_c/0.005)}{r_c} \frac{0.625}{0.03055} \)

so we solve for \( r_c \) by trial & error: \( r_c = 0.01745 \)

This gives: \( \Delta T = (0.03055/2[0.01745])^3 = 0.6^\circ C \)

Thus, we evaluated properties at a temperature 6-1/2°C above the right value. Further calculation would not be worth the trouble.
A horizontal electrical resistance heater, 1 mm in diameter, releases 100 W/m in water at 17°C. What is the wire temperature?

We modify eqn. (8.28), using \( \text{Ra}_t = \text{Ra}_D / \overline{\text{Nu}}_D \), and get:

\[
\overline{\text{Nu}}_D^{5/4} - 0.36 \overline{\text{Nu}}_D^{1/4} = \frac{0.518}{\left[ 1 + \left( \frac{65.5}{P_r} \right)^{0.5625} \right]^{-1/6}} \left[ \frac{g \beta \varphi D^4}{k} \right]^{1/4}
\]

Guess \( T_w = 37°C \). Then, at 27°C, \( \Delta = 0.826 \times 10^{-3} \), \( \varphi = 1.462 \times 10^{-7} \), \( P_r = 5.65 \)  
and \( k = 0.6084 \), and \( q_w = 100 / \pi (0.001) = 3183 \) W/m²

Then:

\[
\overline{\text{Nu}}_D^{5/4} - 0.36 \overline{\text{Nu}}_D^{1/4} = 0.4654 \left[ \frac{0.8 \times 10^{-2}(31830)10^{-12}}{0.824(1.462)10^{-12}0.6084} \right]^{1/4} = 1.53
\]

By trial and error we get \( \overline{\text{Nu}}_D = 1.70 \), so

\[
\overline{h} = 1.70 \frac{k}{D} = 1.70 \frac{0.6084}{0.001} = 1039 \text{ W/m}^2\cdot{^\circ}C
\]

Then:

\[
Q = \overline{h} A \Delta T \quad 100 = 1039 \pi [0.001] \Delta T
\]

\[
\Delta T = 30.78 {^\circ}C
\]

So \( T_w = 17 + 30.78 = 47.78 {^\circ}C \)

The properties should have been evaluated at 32.4°C instead of at 27°C. This is not enough difference to warrant a recalculation. However, if we did the recalculation we'd get:

\( T_w = 46.64 {^\circ}C \)

Which is less than 1°C improvement.
8.41 Solve Problem 5.39 using the correct formula for the heat transfer coefficient.

We shall evaluate the properties of water at \((17+27)/2 = 37^\circ\text{C}\):

\[
\delta = 0.656 \times (10^{-6}), \quad \alpha = 1.502 \times (10^{-7}), \quad Pr = 4.66, \quad k = 0.6126, \quad \beta = 0.000355
\]

Then: \(Ra_D = \frac{0.76(0.000355)(0.03)^2 \Delta T}{0.698 (1.502) \times 10^{-13}} = 69,900 \Delta T\) so eqn. (8.29) yields:

\[
h = \frac{k \bar{N}u_0}{\bar{D}} = \left[2.733 + 10.95 \Delta T^{1/6}\right]^2
\]

This is exactly the value given in Problem 5.39. Therefore its solution applies here.
8.43 A 0.25 mm diameter platinum wire, 0.2 m long, is to be held horizontally at 1035°C. It is black. How much electric power is needed? Is it legitimate to treat it as a constant wall temperature heater, in calculating the convective part of the heat transfer? The surroundings are at 20°C and the surrounding room is virtually black.

\[ Q_{\text{rad}} + Q_{\text{conv}} = \pi (0.00025)(0.2) \left[ \sigma (1309 + 293) + \frac{k_{\text{air}}}{D} N_u D \Delta T \right] \]

Evaluate properties at (1035 + 20)/2 = 527.5°C \approx 800°K,
\[ \sigma = 5.67 \times 10^{-8}, \quad a = 1.73 \times 10^{-5}, \quad k = 0.0569, \quad \rho_r = 0.704 \]
\[ Ra_D = \frac{9.8(\frac{1}{0.00025})(1035-20)}{8.26(11.73)10^{-10}} = 0.12 \]
\[ Nu_D = 0.36 + 0.518(0.02) \frac{1}{\sqrt{\frac{a}{0.553}0.5625^{0.444}}} = 0.507 \]

So:
\[ Q = \left[ 165.596 + \frac{0.0569}{0.00025} \right] \pi (0.00025)(0.2) = 44.4 \text{W} \]

\[ B_{\text{conv}} = \frac{117129}{1035-20} \frac{0.00025}{6.9} = 0.000343 \]
\[ B_c_{\text{total}} = \frac{117129 + 166594}{1035-20} \frac{0.00025}{84} = 0.000829 \]

In either case \( B_c \ll 1 \), so \( T_w = \text{const.} 15^\circ \text{C} \) valid.

8.44 A vertical plate, 11.6 m long, condenses saturated steam at one atmosphere.

We want to be sure that the film stays laminar. What is the lowest allowable plate temperature and what is \( \bar{q} \) at this temperature?

Let us save work by adapting a result from Example 8.6:
\[ \delta = 0.000139 L^{1/4} \left( \frac{\Delta T}{10} \right)^{1/4} = 0.0001432 \Delta T^{1/4} = \left( \frac{3.2 \pi m}{(\rho_{\text{steam}} - \rho_{\text{vapor}})} \right)^{1/3} \]

Then using \( Re = 450 \), we get:
\[ \Delta T = \left[ \frac{1}{14.3L^{10/3}} \left( \frac{3.29(0.290)}{5} \right)^{2/3} 450 \right]^{1/3} = 6.23°C \quad T_w_{\text{lowest}} = 93.77°C \]

\[ \bar{q} = \frac{q}{3} \frac{k A}{b} = \frac{q}{3} \frac{0.621 L}{0.0001432} \frac{6.23}{0.0001432} = 25004 \text{ W/m}^2 \]
8.45  a) Show that $\Theta_{pp} = m^2 L^2 \Theta^{5/4}$ for a straight fin in natural convection.

b) Develop an iterative method to solve the equation assuming an insulated tip.

c) Solve the resulting difference equations.

a) With $Nu \sim Gr^{1/4}$, $\bar{h} = \bar{h}_0 [(T_0-T_m)/(T_0-T_m)]^{5/4}$ where $\bar{h}_0$ is $\bar{h}$ at $T=T_m$.

The given equation follows immediately from eqn. (4.30) and the subsequent non-dimensionalisation.

b) Using central differences for the second derivative, we have

$$\Theta_i^{k-1} - \left[2 + m^2 L^2 \delta x^2 \left(\Theta_i^{k-1}\right)^{1/4}\right] \Theta_i^k + \Theta_i^{k-1} = 0$$

$$\Theta_0^k = 1.0$$

$$2 \Theta_{n-1}^k - \left[2 + m^2 L^2 \delta x^2 \left(\Theta_{n-1}^{k-1}\right)^{1/4}\right] \Theta_{n-1}^k = 0$$

(To suppress natural convection, we change the exponent $\frac{1}{4}$ to zero)

c) Sample output from a BASIC program that solves these equations is given below. The program itself is on the next page.
8.45 (continued)
This old solution was carried out on an HP-85 calculator.
Today we would certainly use more a more modern means of
calculation.

10 ' "FIN WITH NATURAL CONVECTION HEAT EXCHANGE"
20 ' T0=tep from previous iteration; T1=tep from current iteration;
30 ' A, B, C, R are coefficients in the difference equations.
40 ' D is an intermediate value in the tridiagonal algorithm.
50 DIM T(200), T1(200), B(200)
60 DIM R(200), C(200), A(200), N(200)
70 ' N is the number of spatial units
80 DISP "ENTER (ml), N"
90 INPUT N, N
100 ' E1 = the min. sum of ABS(T(i)-T0(i))
110 E1=0
120 T(1)=N/10
130 D=(N+1)/2
140 I:=0
150 FOR I:=1 TO N
160 A(I)=0 @ B(I)=-(2*D) @ C(I)=1 @ R(I)=0
170 T(I)+1 @ T0(I)=1
180 NEXT I
190 N:=0
200 A(I)=0 @ C(I)=0 @ A(N)=2
210 R(I)=1
220 M=4*N+1
230 R(I)=1
240 FOR I:=2 TO N
250 R(I)=1
260 NEXT I
270 GOSUB 400
280 R:=0
290 FOR I:=1 TO N
300 R=R+A(S(T(I)-T0(I)))
310 ' I=0 suppresses natural convection
320 TO(I)=T1(I)
330 IF I=0 THEN GOTO 350
340 B(I)=-(2*A(I)*X1^2)*SQR(SQR(T1(I))))
350 : 
360 NEXT I
370 DISP "R=";R
380 IF R<0 THEN DISP "R=";R @ GOSUB 500
390 GOTO 200
400 ' Tridiagonal algorithm
410 FOR I:=0 TO N
420 T(I)=T0(I)
430 NEXT I
440 X(2)=X1
450 D(I)=D(I)/B(I)
460 R(I)=R(I)/B(I)
470 FOR I:=2 TO N
480 D(I)=D(I)/A(I)/D(I-1)
490 R(I)=R(I)/A(I)-R(I-1))/D(I)
500 D(I)=C(I)/A(I)/D(I)
510 NEXT I
520 TI(N)-R(N)
530 FOR I:=2 TO N
540 I:=I+1
550 T(I)=RI(I)-D(I)*T1(I+1)
560 NEXT I
570 RETURN
580 PRINT
590 PRINT "Fin with natural convection heat exchange"
600 IF I=0 THEN PRINT "Natural convection suppressed"
610 PRINT & PRINT "ml=ijX1"
620 PRINT "R=";R
630 PRINT "Number of iterations is";N
640 PRINT
650 PRINT "Step size is X1/L=";X1
660 F:=0
670 FOR I:=0 TO N
680 IF I=1 THEN F=F+T1(I)^1.25
690 IF I=0 THEN F=F+T1(I)
700 NEXT I
710 IF I=1 THEN F=F-(T1(I)^1.25+T1(N)^1.25)/2
720 IF I=0 THEN F=F-(T1(I)+T1(N))/2
730 F=X1
740 PRINT "Efficiency=";F
750 SCALE 0.1,0.1
760 XAXIS 0,1
770 YAXIS 0,1
780 MOVE 0,1
790 FOR I:=1 TO N
800 DRAW X1,T(I),T1(I+1)
810 NEXT I
820 GRAPH
830 IF I=0 THEN I:=1 @ GOTO 150
835 COPY
840 END

Note:
The difference equations are written as
\[ A_{i} \theta_{i-1} + B_{i} \theta_{i} + C_{i} \theta_{i+1} = R_{i} \]
Notes:
1) The maximum heat transfer occurs near the root where $h = h_0$. Hence the maximum reduction in $\gamma$ is only 6%.

2) $\delta^0 = 0.01$ is quite satisfactory for $mL^2 < 100$, and $\delta^0 = 0.005$ is not quite small enough at $mL^2 = 1000$.

3) $\Theta'' = mL^2 t^{\frac{5}{2}}$ can be solved exactly for $mL \rightarrow \infty$. Let $\delta = \frac{3}{mL}$. Then we have

$$\frac{d^2 \Theta}{d\delta^2} = \Theta^{\frac{5}{2}}$$

Multiply both sides by $2d\Theta/d\delta$ and integrate twice to get

$$\Theta = \left[ 1 + \frac{V_2 - \frac{1}{2}}{\delta^2} \right]^{-\frac{2}{5}}$$

This leads to $\lim_{mL \rightarrow \infty} \frac{1}{\gamma} = \frac{2V_2}{3} = 0.9428$.

The program for $mL^2 = 1000$, $\delta^0 = 0.005$ gives 0.9434.
8.46 Find the temperature of a black sphere in equilibrium with air at 20°C and surroundings at 1000°C.

Equation 8.32 gives

$$
\bar{h} = \frac{k}{D} \left[ 2 + 0.43 \, Ra^{1/4} \right]
$$

The equation we need to solve is

$$
T = T_\infty + \frac{Q \sigma}{\bar{h}} \left( T^4 - T_\infty^4 \right)
$$

We first guess that the properties in the expression for $\bar{h}$ can be evaluated at 500°C. This gives

$$
\frac{\sqrt{3}}{D} = \frac{9.8 \times (253)^{-1} (2 \times 10^{-5})}{3.75 \times 10^{-5} (5.78 \times 10^{-5})} = 129.6 \text{ (°C)}^{-1}
$$

$Ra$ will be $< 10^5$ if $\Delta T < 772$ °C, or $T < 1065$ °C.

This will always be the case, so

$$
\bar{h} = 3.55 + 2.87 \left( T - T_\infty \right)^{1/4}
$$

or

$$
T = 293 + \frac{0.5667 \times 10^{-8} \left( 10^{12} - T^4 \right)}{3.55 + 2.87 \left( T - 293 \right)^{1/4}}
$$

The solution to this equation is $601.43$ °C, so $T_{lim} = 601.43$ °C.

If the properties are evaluated at 150 °C, we have

$$
\bar{h} = 3.63 + 3.14 \left( T - T_\infty \right)^{1/4}
$$

and $T = 591.35$ °C. An additional iteration with $T_{lim} = 440$ °C gives $T = 594.54$ °C.

Note: The iteration process described in footnote 2 of chapter 6 diverges if the initial guess for $T$ is too close to either 293 or 1000.
**Problem 8.53** An inclined plate in a piece of process equipment is tilted $30^\circ$ above horizontal and is 20 cm long in the inclined plane and 25 cm wide in the horizontal plane. The plate is held at 280 K by a stream of liquid flowing past its bottom side; the liquid is cooled by a refrigeration system capable of removing 12 W. If the heat transfer from the plate to the stream exceeds 12 W, the temperature of both the liquid and the plate will begin to rise. The upper surface of the plate is in contact with ammonia vapor at 300 K and a varying pressure. An engineer suggests that any rise in the bulk temperature of the liquid will signal that the pressure has exceeded a level of about $p_{\text{crit}} = 551$ kPa.

a) Explain why the gas’s pressure will affect the heat transfer to the coolant.
b) Suppose that the pressure is 255.3 kPa. What is the heat transfer (in watts) from gas to the plate, if the plate temperature is $T_w = 280$ K? Will the coolant temperature rise?
c) Suppose that the pressure rises to 1062 kPa. What is the heat transfer to the plate if the plate is still at $T_w = 280$ K? Will the coolant temperature rise?

**Solution**
a) Sufficiently high pressures can cause condensation of the NH$_3$ vapor on the plate. In addition, before condensation occurs, pressure changes may cause significant properties variations in the NH$_3$ vapor.
b) At 255.3 kPa, the saturation temperature is $T_{\text{sat}} = 260$ K < 280 K; condensation will not occur. Replacing $g$ with an effective gravity $g \cos 60^\circ$, the Rayleigh number is

$$
\text{Ra}_L = \frac{g \cos 60^\circ \beta A T L^3}{\nu \alpha} = \frac{9.81 \times (1/2) \times 0.00345 \times 20 \times 0.2^3}{(5.242 \times 10^{-6})(5.690 \times 10^{-6})} \approx 9.07 \times 10^7
$$

The Nusselt number is

$$
\overline{\text{Nu}}_L = 0.68 + 0.67 \text{Ra}_L^{1/4} \left[ 1 + \left( \frac{0.492}{\text{Pr}} \right)^{9/16} \right]^{-4/9}
$$

$$
= 0.68 + 0.67 \times (9.07 \times 10^7)^{1/4} \left[ 1 + \left( \frac{0.492}{0.92} \right)^{9/16} \right]^{-4/9}
$$

$$
\approx 52.3
$$

Then,

$$
h = \overline{\text{Nu}}_L \frac{k}{L} = 52.3 \times \frac{0.0244}{0.2} = 6.38 \text{ W/m}^2\text{K}
$$

and the heat transfer is

$$
Q = hA(T_{\infty} - T_w) = 6.38 \times 0.2 \times 0.25 \times (300 - 280) \approx 6.38 \text{ W} < 12 \text{ W}
$$

and the plate and liquid temperatures will not rise.
c) At a pressure of 1062 kPa, the saturation temperature is $T_{\text{sat}} = 300$ K > 280 K; condensation occurs. The Nusselt number is

$$
\overline{\text{Nu}}_L = 0.9428 \left[ \frac{\rho_f(\rho_f - \rho_g)g \cos 60^\circ h'_f L^3}{\mu k(T_{\text{sat}} - T_w)} \right]^{1/4} = 1814
$$
The heat transfer coefficient is
\[ h = \frac{\text{Nu}_L k}{L} = 4353 \text{ W/m}^2\text{K} \]

The heat transfer rate is
\[ Q = hA(T_{\text{sat}} - T_w) = 4353 \text{ W} \gg 12 \text{ W} \]
and the plate and liquid temperatures will rise.
**Problem 8.54** A characteristic length scale for a falling liquid film is \( \ell = (y^2/g)^{1/3} \). If the Nusselt number for a laminar film condensing on plane wall is written as \( \text{Nu}_\ell \equiv h\ell/k \), derive an expression for \( \text{Nu}_\ell \) in terms of \( \text{Re}_c \). Show that, when \( \rho_f \gg \rho_g \), \( \text{Nu}_\ell = \left(3\text{Re}_c\right)^{-1/3} \).

**Solution** Starting with eqns. (8.58) and (8.72), we have

\[
\text{Nu}_x = \frac{hx}{k} = \frac{x}{\bar{\delta}} \quad (8.58)
\]

and

\[
\text{Re}_c = \frac{\rho_f(\rho_f - \rho_g)g\delta^3}{3\mu^2} = \frac{\rho_f\Delta\rho g\delta^3}{3\mu^2} \quad (8.72)
\]

Then, by replacing \( x \) by \( \ell \)

\[
\text{Nu}_\ell = \frac{h\ell}{k} = \frac{\ell}{\bar{\delta}}
\]

and, by rearranging \( \text{Re}_c \),

\[
\delta = \left(\frac{3\mu v}{g\Delta\rho \text{Re}_c}\right)^{1/3}
\]

So

\[
\text{Nu}_\ell = \left(\frac{y^2}{g}\right)^{1/3} \left(\frac{g\Delta\rho}{3\mu v}\right)^{1/3} \text{Re}_c^{-1/3} = \left(\frac{\Delta\rho}{3\rho_f}\right)^{1/3} \text{Re}_c^{-1/3}
\]

and when \( \rho_f \gg \rho_g \), \( \Delta\rho \approx \rho_f \) so

\[
\text{Nu}_\ell \approx \left(3\text{Re}_c\right)^{-1/3} \quad \text{for} \ \rho_f \gg \rho_g
\]
**Problem 8.59** Using data from Tables A.4 and A.5, plot $\beta$ for saturated ammonia vapor for $200 \, \text{K} \leq T \leq 380 \, \text{K}$, together with the ideal gas expression $\beta_{\text{IG}} = 1/T$. Also calculate $Z = P/\rho RT$. Is ammonia vapor more like an ideal gas near the triple point or critical point temperature?

**Solution**

With $p$ and $\rho$ from Table A.5, and using $R = R^o/M_{\text{NH}_3} = 8314.5/17.031 = 488.2 \, \text{J/kg-K}$, we find $Z$ as below. For an ideal gas, $Z = 1$.

<table>
<thead>
<tr>
<th>$T$ [°C]</th>
<th>$Z$</th>
<th>$T$ [°C]</th>
<th>$Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.9944</td>
<td>300</td>
<td>0.8788</td>
</tr>
<tr>
<td>220</td>
<td>0.9864</td>
<td>320</td>
<td>0.8263</td>
</tr>
<tr>
<td>240</td>
<td>0.9722</td>
<td>340</td>
<td>0.7606</td>
</tr>
<tr>
<td>260</td>
<td>0.9505</td>
<td>360</td>
<td>0.6784</td>
</tr>
<tr>
<td>280</td>
<td>0.9198</td>
<td>380</td>
<td>0.5716</td>
</tr>
</tbody>
</table>

Saturated ammonia vapor only behaves like an ideal gas for temperatures close the triple point temperature (195.5 K) and is highly non-ideal in the vicinity of the critical point temperature (405.4 K). This behavior underscores the importance of using data for $\beta$ when dealing with vapors near saturation conditions.
9.1 Water boils, according to the graphical relation in Fig. 9.2, on a 1.27 cm thick copper slab which starts out at 650°C. Plot $T_{slab}$ vs. time, indicating the regime of boiling and noting the temperature at which the cooling is most rapid.

$$B_i = \frac{hL}{k} \approx \frac{0.0127 \times 376}{376} < 1$$ as long as $h$ is less than $29,600 \frac{w}{m^2 \cdot K}$

from eqn. (1.19)

$$\frac{Q}{A} = \frac{q}{t} = \frac{\delta \left( \rho c V (T_{sat}) \right)}{\delta t}$$

or

$$\delta t = \rho c L \frac{\delta T}{q} = 43661 \frac{\delta T}{q}$$

Now we use this eqn and Fig 9.2 to calculate:

<table>
<thead>
<tr>
<th>$\Delta T_i$, °C</th>
<th>$\Delta T_f$, °C</th>
<th>$\frac{W}{A}$, mW/cm²</th>
<th>$\delta t$, sec</th>
<th>$t_{sat} + \delta t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>650</td>
<td>400</td>
<td>4.5 x 10⁻⁵</td>
<td>14.56</td>
<td>14.36</td>
</tr>
<tr>
<td>400</td>
<td>300</td>
<td>0.9 x 10⁻⁵</td>
<td>59.59</td>
<td>69.14</td>
</tr>
<tr>
<td>300</td>
<td>200</td>
<td>0.5 x 10⁻⁵</td>
<td>87.33</td>
<td>156.5</td>
</tr>
<tr>
<td>200</td>
<td>150</td>
<td>0.5 x 10⁻⁵</td>
<td>43.03</td>
<td>200.2</td>
</tr>
<tr>
<td>150</td>
<td>100</td>
<td>2.8 x 10⁻⁵</td>
<td>7.8</td>
<td>208.7</td>
</tr>
<tr>
<td>100</td>
<td>60</td>
<td>7.5 x 10⁻⁵</td>
<td>2.3</td>
<td>210.3</td>
</tr>
<tr>
<td>60</td>
<td>30</td>
<td>11.4 x 10⁻⁵</td>
<td>1.15</td>
<td>211.95</td>
</tr>
<tr>
<td>30</td>
<td>10</td>
<td>7.8 x 10⁻⁵</td>
<td>1.12</td>
<td>212.57</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>2.9 x 10⁻⁵</td>
<td>1.14</td>
<td>213.71</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>0.3 x 10⁻⁵</td>
<td>2.9</td>
<td>216.62</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.15 x 10⁻⁵</td>
<td>2.6</td>
<td>219.83</td>
</tr>
</tbody>
</table>

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9.2 Predict \( q_{\text{max}} \) for horizontal cylinders for the cases in Fig. 10.3b and indicate the fraction of \( q_{\text{max}} \) in each case.

(a) 0.0322 cm diam. in methanol with \( g = 98 \) m/s\(^2\)
(b) 0.164 cm diam. in benzene with \( g = 9.8 \) m/s\(^2\)

\[
\text{first find } R' : \quad R' = R \sqrt{\frac{g(p_1-p_2)}{\sigma}}
\]

\[
R'_{\text{meth.}} = \frac{0.332\sqrt{\frac{9800(0.813)}{19.7}}}{2} = 0.332
\]

\[
R'_{\text{benz.}} = \frac{0.164\sqrt{\frac{9800(0.814)}{21.3}}}{2} = 0.502
\]

\[
q_{\text{max, meth.}} = \frac{\pi}{24} (1.17 \text{ kg/m}^2)^{1/4} \frac{1}{1.192 \text{ kg}} \frac{1}{7 \text{ kg}} \sqrt{98 \text{ m}^2/\text{s}^2} \frac{0.0117 \text{ kg/s}}{38 \text{ (b) kg/m}^2} \frac{(0.813 \text{ kg/m}^2)}{21.3} = 980,000 \frac{\text{W}}{\text{m}^2}
\]

\[
q_{\text{max, benz.}} = \frac{\pi}{24} (2.94)^{1/4} 39(000) \sqrt{3.8(0.0213)(814)} = 283,600 \frac{\text{W}}{\text{m}^2}
\]

From Fig. 9.13, upper left-hand corner, we read:

\[
\left( \frac{q_{\text{max, meth.}}}{q_{\text{max, benz.}}} \right) = 1.2 \quad \text{So} \quad q_{\text{max, meth.}} = 1,170,000 \frac{\text{W}}{\text{m}^2}
\]

\[
\left( \frac{q_{\text{max, benz.}}}{q_{\text{max, meth.}}} \right) = 1.1 \quad \text{So} \quad q_{\text{max, benz.}} = 218,000 \frac{\text{W}}{\text{m}^2}
\]

Fig. 9.3b shows methanol at 1,040,000 W/m\(^2\) or 88.4 % of \( q_{\text{max}} \).

Fig. 9.3b shows benzene at 350,000 W/m\(^2\) or 90.7 % of \( q_{\text{max}} \).

9.3 Water at 70\(^\circ\)C is depressurized until it is subcooled 30\(^\circ\)C. Find the pressure at this point and the diameter of the critical nucleus.

Psat. at 70\(^\circ\)C = 31,170 N/m\(^2\),  
Psat at 40\(^\circ\)C = 7375 N/m\(^2\)

\[
r_c = \frac{2 \sigma \text{ at } 70^\circ \text{C}}{31,170 - 7375} = 2 \quad \frac{65.49 \text{ dyne/cm}}{23,795 \text{ N/m}^2}
\]

\[
\text{diameter of nucleus } = 2r_c = 1.1(10)^{-6} \text{ m}
\]
9.4 Plot \( r_c \) vs. liquid superheat for water at 1 atm.

\[
\frac{r_c}{\mu} = \frac{\sigma}{\mu_{\text{sat}} + T_{\text{sup}} - 10^5} ; \quad \sigma = 0.2358 \left( 1 - \frac{T_{\text{sat}}}{697.2} \right)^{1.256} \left[ 1 - 0.025 \left( \frac{T_{\text{sup}}}{697.2} \right) \right]^{1/2}
\]

<table>
<thead>
<tr>
<th>( T_{\text{sup}}, ^\circ\text{C} )</th>
<th>( \Delta T^\circ\text{C} = \frac{T_{\text{sup}} - 100}{\mu_{\text{sat}} + T_{\text{sup}}} )</th>
<th>( \sigma )</th>
<th>( \mu_{\text{sat}} + T_{\text{sup}} )</th>
<th>( r_c, \mu )</th>
<th>( r, \text{mm} )</th>
<th>( r, \text{Å} )</th>
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<tbody>
<tr>
<td>110/383</td>
<td>10</td>
<td>0.0482</td>
<td>1.933 \times 10^5</td>
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<td>0.00223</td>
<td></td>
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<tr>
<td>130/403</td>
<td>30</td>
<td>0.0423</td>
<td>2.701</td>
<td>0.50</td>
<td>0.00050</td>
<td></td>
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<tr>
<td>160/433</td>
<td>60</td>
<td>0.0342</td>
<td>6.18</td>
<td>0.132</td>
<td>0.000132</td>
<td>338</td>
</tr>
<tr>
<td>200/473</td>
<td>100</td>
<td>0.0246</td>
<td>15.55</td>
<td>0.038</td>
<td>0.000038</td>
<td>76</td>
</tr>
<tr>
<td>250/523</td>
<td>150</td>
<td>0.0147</td>
<td>39.78</td>
<td>7.50 \times 10^{-9}</td>
<td>0.000750</td>
<td></td>
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<tr>
<td>300/573</td>
<td>200</td>
<td>0.0069</td>
<td>85.92</td>
<td>1.63 \times 10^{-4}</td>
<td>0.0163</td>
<td>16</td>
</tr>
</tbody>
</table>

9.5 Why does bumping occur in a test tube, but not in a teakettle?

The test-tube is very smooth so \( (r_c)_{\text{test-tube}} \ll (r_c)_{\text{teakettle}} \). It follows that, since \( r_c = 2g/\mu_{\text{sat}}(T_{\text{nuc}} - T_{\text{sat}}) \) is small, \( \rho_{\text{sat}}(T_{\text{nuc}} - T_{\text{sat}}) \) is large. Thus \( T_{\text{nuc}} \) is also much higher in the test-tube than in the teakettle.

It is beyond our scope here, but the thermodynamic availability is a measure of the damage that a superheated liquid can do when it nucleates. We can show (Jour. Heat Transfer, Feb. 1981, Vol. 103, pp. 61-64) that the availability rises as \( (T_{\text{sup}} - T_{\text{sat}})^2 \). Thus \( \Delta a \) (and the possible damage) increase strongly with superheat.
9.6 Use van der Waals' equation to estimate how much superheat water can sustain at low pressure.

In reduced form, the van der Waals equation is:

\[ p_r = \frac{8Tr}{3(1-v_r)} - \frac{3}{v_r^2} \]

Set \( p_r = 0 \) and solve for \( T_{r_{max}} \):

\[ T_{r_{max}} = \frac{9}{8} \frac{1-v_r}{v_r^2} \]

But at the limiting point:

\[ \frac{\partial p_r}{\partial v_r} = 0 = -\frac{8}{3} \frac{1}{(1-v_r)^2} + \frac{6}{v_r^3} \]

Substitute \( T_{r_{max}} \):

\[ 0 = -\frac{8}{3} \frac{1}{(1-v_r)^2} + \frac{6}{v_r^3} \]

or:

\[ 0 = -\frac{1}{1-v_r} + \frac{2}{v_r} \quad \frac{v_r}{v_r} = \frac{2}{3} \]

Thus:

\[ T_{r_{max}} = \frac{9}{8} \left( \frac{2}{3} \right)^{\frac{1}{3}} = \frac{27}{32} \]

\[ T_{max_\text{H}_2\text{O}} = \frac{27}{32} T_c = \frac{27}{32} 647.2 = 546^\circ\text{R} \]

So, at 1 atm, the limiting superheat is \( \Delta T = (546 - 373) = 173^\circ\text{C} \)

(The measured extremes are just a little greater than this.)

9.7 Find \( c \) in \( n \sim \Delta T^c \) such that the result is consistent with Berenson's curves in Fig. 9.14 and Yamagata's equation:

\[ q \sim n^{1/3} \Delta T^{1.2} \]

From the log-log plots in Fig. 9.14 we measure the slopes in the nucleate boiling range. Call this slope, \( d \). The 5 values are 6, 5.7, 5.3, 4, 2.2. Then:

\[ q \sim \Delta T^d \sim \Delta T^{c/3} \Delta T^{1.2} \quad \text{or} \quad c = 3d - 1.2 \]

Thus the 5 values of \( c \) in \( n = \Delta T^c \) are:

\[ c = 16.8; \quad c = 15.9; \quad c = 14.7; \quad c = 10.8; \quad \text{and} \quad c = 5.4 \]
9.8 Suppose $C_{sf}$ for a given surface is reported as being 50% higher than is really is. How much error will this contribute to the calculated $q$?

From eqn. (9.4) we have: 
\[ q \sim \left( \frac{\Delta T}{C_{sf}} \right)^3 \]

Then 
\[ q_{\text{calculated}} \sim \frac{1}{1.53} \left[ \frac{\Delta T}{C_{sf \text{correct}}} \right]^3 \]

or 
\[ q_{\text{calculated}} = 0.296 q_{\text{correct}} \]

Thus the calculation is low by 70%.

9.9 Water at 100 atm boils on a nickel heater. \( \Delta T = 60^\circ \text{C} \).

Find $q$ and $h$.

Properties at $T_{sat} = 310^\circ \text{C}$:
\begin{align*}
\rho_f &= 650 \text{ kg/m}^3 \\
\rho_g &= 54.7 \text{ kg/m}^3 \\
\lambda_f &= 1325000 \text{ J/kg} \\
\lambda_g &= 5600 \text{ J/kg}^\circ \text{C} \\
\mu_f &= 0.0000075 \text{ kg/m} \cdot \text{s} \\
\mu_g &= 0.0117 \text{ kg/m} \cdot \text{s} \\
Pr &= 1.02 \\
C_{sf} &= 0.006
\end{align*}

Then, from eqn. (9.4)
\[ \left( \frac{C_f \Delta T}{h_g Pr^2} \right)^3 = \frac{C_{sf}}{\lambda_f \mu_g \sqrt{\rho_g/\rho_f}} q \]

So:
\[ q = \left( \frac{56000(6)}{1325000(1.02)} \right)^3 \frac{0.0000075(1.325)10^6}{(0.006)^3} \sqrt{\frac{0.0117}{9.8(0.353)}} \]

\[ q = 6.017 \times 10^6 \text{ W/m}^2 \]

And:
\[ h = \frac{q}{\Delta T} = \frac{6.017 \times 10^6}{6} = 1,003,000 \text{ W/m}^2 \cdot \text{K} \]

This is very high.

9.10 Compute $q_{\text{max}}$ for saturated water at 1 atm on a flat plate -- very large in extent -- at $g/g_e = 1/6$ and $10^{-4}$.

At earth-normal gravity:
\[ q_{\text{max}} = 1,260,000 \text{ W/m}^2 \] (Example 10.5)

Thus, at $g/g_e = 1/6$:
\[ q_{\text{max}} = 1,260,000 \sqrt{\frac{1}{6}} = 805,000 \text{ W/m}^2 \]

And at $g/g_e = 10^{-4}$:
\[ q_{\text{max}} = 1,260,000 \sqrt{10^{-4}} = 126,000 \text{ W/m}^2 \]

Since, in accordance with eqn. (9.11)
\[ q_{\text{max, flat plate}} \sim g^{1/4} \]
9.11 Water boils on a 0.001 m radius copper wire. Plot as much of the boiling curve as you can, for this case.

We go through the regimes of the boiling curve, one at a time, starting with natural convection.

\[
\bar{N}_D = 0.36 + \frac{0.518 R_a^{1/4}}{\left[1 + 0.552 R_a^{1/6}\right]^{4/9}} = 0.36 + \frac{0.518}{\left[1 + 0.552 \left(\frac{0.001}{0.292}\right)^{1/6}\right]^{4/9}} \Delta T^{1/4}
\]

\[
q = \bar{N}_D \frac{k}{D} \Delta T = 122 \Delta T + 844 \Delta T^{5/6}
\]

The nucleate boiling heat flux is given by eqn. (9.4)

\[
q = \frac{c_p \Delta T}{h_{fg} \gamma_{fg}} \sqrt{\frac{q_0 \rho}{\sigma}} = \left[\frac{2.18}{\left(236(0.0303) \cdot 1.24(0.003)\right)}\right]^3 \cdot 0.265(10) \cdot 2.26(10) \cdot \sqrt{\frac{9.893 k}{0.0589}} \Delta T^3
\]

\[
q = 124 \Delta T^3
\]

The peak heat flux at \( R' = R' \left(\frac{33.55 \Delta T}{0.0589}\right) = 0.399 \) is given by eqn. (9.20)

\[
q_{\text{max}} = \frac{1}{1.14} \left[ q_{\text{max}} \left(R'\Delta T\right)^{1/4}\right] = \frac{1.26}{1.14} (10)^{6} \frac{0.94}{0.399^{1/4}} = 1,307,000 \text{ W/m}^2
\]

The minimum heat flux for \( R' = 0.399 \) is

\[
q_{\text{min}} = 0.515 \left[\frac{18}{R'^2 (2R'^2 + 1)}\right]^{1/4} \left[0.097 h_{fg} \sqrt{\frac{\sigma (p_f - p_0)}{\gamma_{fg}} (1 + \frac{1}{1.14})^{1/4}}\right]
\]

\[
= 0.094 \left[\frac{18}{0.399^2 (2 \cdot 0.399 + 1)}\right]^{1/4} \left(0.597(2,257,000)\right)^{1/4} \sqrt{\frac{0.0589(0.88 \times 0.589)}{0.589}}
\]

\[
q_{\text{min}} = 23,800 \text{ W/m}^2
\]

In the film boiling regime \( h = h_{f,b} + \frac{3}{4} h_{\text{rad}} \). Thus:

\[
q = \frac{k \Delta T}{D} \left\{ (0.661 + 0.243) \frac{R}{D}^{1/4} \right\} \left[ \frac{(250^2)}{2.73 \cdot h_{fg} \Delta T^{3/4}} \right] + \frac{3}{4} \frac{k}{D} (T_w - T_{sat})
\]

where we assume \( e = 1 \) and in which:

\[
h_{fg} = h_{fg} (1 + 0.968 - \frac{0.163}{Pr_g}) = 2,257,000 \left(1 + \left[0.968 - \frac{0.163}{1.052}\right] \frac{2030 \Delta T}{2,257,000}\right)
\]

\[
= 2,257,000 \left(1 + 0.00075 \Delta T\right)
\]
9.11 (continued)

\[ q = \frac{0.0237}{0.002} \left(0.661 + \frac{0.243}{0.679} \right) 0.399^{1/4} \left[ \frac{958 \left(9.8 \times 2.257/10^6 + 10^3 \cdot 0.002 \cdot \Delta T^{3/4} \right)}{5.884 \times 10^{-5} \cdot 0.04} 
+ 0.75 \left(5.67 \cdot 10^8 \left( T_w^4 - 1.936 \left(10^4 \right)^4 \right) \right) \right] \] for film boiling

\[ q = 683 \left(1 + 0.00075 \Delta T\right)^{1/4} \Delta T^{3/4} + 0.925 (10)^8 \left( T_w^4 - 1.936 \left(10^4 \right)^4 \right) \] for film boiling with correction for radiation using \( G = 1 \)

Typical nucleate-transition behavior (not calculated)

Film boiling with natural convection

Typical film-transition boiling behavior (not calculated)

\( q_{\text{max}} = 1,307,000 \)

\( q_{\text{min}} = 29,800 \)

Film boiling with no radiation. Properties evaluated at 593 °K or 320 °C

\( \Delta T_{\text{incept}} \) occurs where \( 122 \Delta T + 844 \Delta T^{3/4} = 124 \Delta T^3 \)

Trial & error gives \( \Delta T_{\text{incept.}} = 3.2^\circ \text{C} \) or 5.8^\circ \text{F} .

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9.12 This problem is the same as 9.11 with the following exceptions: Since the heater is a sphere, we use eqn. (8.32) instead of eqn. (8.28) for natural convection. The peak heat flux is given by eqn. (9.22) instead of (9.20). Film boiling is still given by \( h = h_{fb} + 0.75h_{rad} \), and \( h_{rad} \) is still the same. But we no longer need include a curvature correction in calculating \( h_{fb} \), because a 0.03 m sphere is sufficiently large not to need it.) There is no reliable eqn. for \( q_{min} \) in this case.

9.13 Predict \( q_{max} \) for a small flat plate with only one jet on it.

\[
q_{max} = \frac{(A_{heater})_{actual}}{(A_{heater})_{ideal}} q_{max} \text{ for a flat plate}
\]

but \( (A_{heater})_{ideal} = \lambda d_{1}^2 \), therefore

\[
\frac{q_{max}}{q_{max}} = \frac{1.14}{\lambda d_{1}^2} A_{heater}
\]

9.14 Show how to locate points of maximum and minimum \( h \) during pool boiling.

Use Fig. 9.2

\[\frac{h}{\Delta T} \text{ is slope of lines which pass through the origin & touch the curve} \]

(Notice that this can only be done on linear coordinates. Fig. 9.2 is semi-logarithmic. Therefore we must locate points A & C by trial & error in Fig. 9.2) we get:

\[
h_{max} = \frac{960,000}{2} = 48,000 \text{ W/m}^2\text{°C} \quad h_{min} = \frac{170,000}{250} = 680 \text{ W/m}^2\text{°C}
\]

9.15 A 0.002 m diam. jet of saturated water flows normal to a 0.015 m diam. disc, at 1 m/s. How much energy can the disc dissipate?

\[
\frac{\rho^2}{\rho} = \frac{958.3}{0.597} = 1605 \quad \text{so eqn. (9.41) gives } A = 0.329.
\]

Then eqn. (9.40) gives:

\[
q_{max} = \frac{2.939 \rho \beta h_{u,jet} (0.002)^{1/3} (1000(1605))^{1/3}}{\sqrt{u_{j,D'/D}}} \]

\[
= 2.939(0.597)(2.257	imes10^4)(0.1133)^{1/3}(558.3(0.015)/0.0589)^{0.329} = 3.65 \times 10^7 \text{ W/m}^2
\]

So the maximum heat dissipation is \( Q_{max} = q_{max} \pi D^2 = 6.448 \text{ W} \)

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9.16 Saturated water at 1 atm. boils on a 0.005 m diam. rod of platinum. What is $T_{\text{rod}}$ at burnout?

$$q_{\text{max}} = (0.94/R^{1/3}) q_{\text{max}} \text{ (eqn. 9.20)}$$

$$R' = \sqrt{\frac{\rho_s - \rho_a}{\rho_a}} R = \frac{\frac{\pi}{3} (957.6)}{0.0585}$$

$\rho_a = 957.6$

$$R' = \frac{\pi}{24} \frac{\rho}{(2257000) \sqrt{0.0585(9.8)(957.6)}}$$

$$q_{\text{max}} = \frac{\rho}{24} \frac{(0.599)(2257000) \sqrt{0.0585(9.8)(957.6)}}{1,107,000 \text{ W/m}^2}$$

$$\text{so } q_{\text{max}} = (0.94/R^{1/3}) q_{\text{max}} = 1,041,000 \text{ W/m}^2$$

From eqn. (9.4)

$$T_{\text{rod, burnout}} = T_{\text{sat}} + \frac{C_s F r h_g}{c_p \mu} \left( \frac{\sigma}{g (p_2 - \rho)} \right)^{1/6} \frac{q_{\text{max}}}{1/3}$$

$$= 100 + \frac{0.013(1.7)(2.257)(10^3)}{42.9(0.2910^{-6} \times 957.2)^{1/3}} (0.0589)^{1/6} (1.04 \times 10^6) = 119.2^\circ C$$
9.19 Verify the form of eqn.(9.8) using dimensional analysis.

\[ u_g = fn(\delta, \rho_g, \lambda_H) \], 4 variables in m,s, and kg. Thus we look for 4-3, or 1 \( \Pi \)-group. Let's write that like a Weber number:

\[ \pi = \frac{\rho u_g^2 \lambda_H}{\sigma} = \text{const. or } u_g = C\sqrt{\frac{\rho}{g^3 \lambda_H}} \]

eqn.(9.8) is of this form with \( C = \sqrt{2\pi} \)

9.20 Compare the value of \( q_{\text{max}} \) implied by data for pool boiling from a 1 in. diam. sphere in Problem 5.6, with the appropriate prediction.

The measured value of \( q_{\text{max}} \) can be obtained using the expression derived in the solution of Problem 5.6.

\[ q_{\text{max}} = \bar{h} \Delta T_{\text{sat}} = 0.712 \frac{\text{Btu}}{\text{ft}^2 \cdot \circ\text{F}} \left( \frac{dT}{dt} \right)_{\circ\text{F}} \]

From the figure associated with Problem 5.6 we read

\[ \left. \frac{dT}{dt} \right|_{\text{max}} = 102 \frac{\circ\text{F}}{s} \text{ so } q_{\text{max}} = 72.62 \frac{\text{Btu}}{\text{ft}^2 \cdot \circ\text{F}} = 261,446 \frac{\text{Btu}}{\text{ft}^2 \cdot \text{hr}} \]
9.20 (continued)

Now for this sphere, \( R' = \frac{0.0254 \text{ m}/2}{\sqrt{0.0589 / 9.8(958.2-0.6)}} = 5.07 \text{ m} \)

Therefore we use eqn. (9.21):

\[
q_{\text{max}} = 0.84 \frac{q_{\text{max}_F}}{1.14} = 0.737 q_{\text{max}_F}
\]

where \( q_{\text{max}_F} \) is given in Example 9.5 as 1,260,000 \( \frac{W}{m^2} \)

So:

\[
q_{\text{max}} = 0.737(1,260,000)/3.154 = 294,426 \frac{\text{Btu}}{\text{ft}^2\text{-hr}}
\]

In this case the measurement is 11% below the prediction.
9.22 Verify equation (9.53) which gives \( \delta \) for a condensing film subject to a shear stress, \( \tau_\delta \).

We first integrate eqn. (8.50) twice and get:

\[
\frac{du}{dy} = \frac{\rho_f - \rho_g}{\rho_f} gy + C_1 \quad \text{and} \quad u = -\frac{\rho_f - \rho_g}{\rho_f} \frac{y^2}{2} + C_1y + C_2
\]

The first b.c., \( u(y=0) = 0 \) gives \( C_2 \) & the second \( \left. \frac{2u}{3y} \right|_{y=\delta} = \frac{\tau_\delta}{\mu} \)
gives \( C_1 = \frac{\tau_\delta}{\mu} + \frac{\rho_f - \rho_g}{\rho_f} g \delta. \)

Thus

\[
u = \frac{(\rho_f - \rho_g)g\delta^2}{2\mu} \left[ 2\frac{\chi}{\delta} - \left(\frac{\chi}{\delta}\right)^2 \right] + \frac{\tau_\delta}{\mu} \frac{Y}{Y}
\]

Then equation (8.53) gives

\[
m = \int_0^\delta \rho_f u dy = \frac{\rho_f - \rho_g}{3\mu} g \delta^3 + \frac{\tau_\delta}{2\mu} \delta
\]

and equation (8.54) becomes:

\[
\frac{k_\delta}{h_f} \frac{\Delta T}{T_\delta} = \frac{d^2}{dx} \left[ \frac{\rho_f - \rho_g}{\nu} g \delta^2 + \frac{\tau_\delta}{\nu} \delta \right] \frac{d\delta}{dx}
\]

which we integrate, subject to \( \delta(x=0) = 0 \):

\[
\frac{2k_\delta}{h_f} \frac{\Delta T}{T_\delta} dx = (\rho_f - \rho_g)g \delta^2 d\delta^2 + \tau_\delta \delta^2 d\delta^2
\]

or

\[
\frac{4k_\delta}{g(\rho_f - \rho_g)h_f^T \gamma_\delta} \frac{\Delta T}{T_\delta} \delta^4 + \frac{4}{3} \frac{\tau_\delta}{g(\rho_f - \rho_g)} \delta^3 = \text{eqn. (9.53)}
\]

Now if \( \tau_\delta = -\frac{3g(\rho_f - \rho_g)}{4} \delta \) equation (9.53) reduces to:

\[
\frac{4k_\delta}{g(\rho_f - \rho_g)h_f} \frac{\Delta T}{T_\delta} = 0
\]

Which means that only \( \Delta T = 0 \) will work (otherwise the film must grow to a larger value of \( \delta \) so \( \tau_\delta \) no longer equals \(-\frac{3g(\rho_f - \rho_g)}{4} \delta\).)

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9.23 A 0.07 m D.D. pipe is at 40°C. Saturated steam at 80°C blows across it. Plot $\bar{h}_{\text{cond.}}$ for $0 \leq \text{Re}_D \leq 10^6$.

We are given the following expression for flow over a cylinder, (where we evaluate $\mu_f$ and $k_f$ at 60°C, and the other properties at 80°C):

$$\bar{h} = 0.64 \frac{k_f}{D} \sqrt{\frac{\beta_D u_m D}{\mu_f}} \left[ 1 + \left( 1 + 1.69 \frac{0.251 \mu_f \alpha_e D}{u_m k_f (T_x - T_w)} \right)^{1/2} \right]$$

Note: $u_m = \text{Re}_D \frac{k_f}{D}$

where $h_f' = 2,308,000 \left( 1 + 0.483 + \frac{0.0731}{2} \right) = 2,369,000$

so:

$$\bar{h} = 0.64 \frac{0.451}{0.07} \sqrt{1 + \left( 1 + 1.69 \frac{9.8 (2,369,000)(0.0000355)}{\text{Re}_D^2 (3.61 \times 10^{-7})^{3/2} 0.0731} \right)} \text{Re}_D^{1/2}$$

$$= 5.952 \sqrt{1 + \left( 1 + \frac{1.486 \times 10^{12}}{\text{Re}_D^2} \right)^{1/2}} \text{Re}_D^{1/2}$$

When $\text{Re}_D \gg 0$, $\bar{h} \Rightarrow 6480 \text{W/m}^2 \cdot \text{°C}$, or eqn. (9.55)

reduces to:

$$\bar{h} = 0.7257 \frac{k_f}{D} \sqrt{\frac{q h_f' \rho_e D^3}{2 \alpha_k k_f \Delta T}}$$

which is virtually the same as equation (8.67) for static condensation.

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9.24 a) Suppose you have pits of roughly 0.002mm diameter in a metallic heater surface. At about what temperature might you expect water to boil on that surface, if the pressure is 20 atm.

b) Measurements have shown that water at atmospheric pressure can be superheated about 200°C above its normal boiling point. Roughly how large an embryonic bubble would be needed to trigger nucleation in water in such a state.

\[ \sigma = 235.8(1 - 0.7512) \cdot 1.256 \cdot (1 - 0.625(1 - 0.7512)) = 34.70 \text{ dyne/cm} = 34.7 \text{ mN/m} \]

then \[ \Delta p = \frac{2(\sigma)}{R} = \frac{2(34.7)}{0.000001} = 69400 \text{ N/m}^2 = 10064 \text{ psi} \]

\[ T_{\text{sat at (20(14.7) + 10.064)psia is 419.8°F}} \]

\[ \Delta T = 4.60°F \text{ or } 2.56°C \quad \text{much less } \Delta T \text{ is needed to drive boiling at elevated pressures} \]

b) \[ p_{\text{sat (300°C) = p_{sat (572°F) = 1246.6psia}}} \]

\[ R = \frac{2\sigma}{\Delta p} \quad \text{but what is } \sigma? \quad \text{Probably it should be evaluated at } 300°C \text{ or } T_r = \frac{300 + 273}{647.2} = 0.885 \]

\[ \sigma = 235.8(1 - 0.885) \cdot 1.256 \cdot (1 - 0.625(1.115)) = 15.57 \text{ dyne/cm} \]

\[ = 0.001067 \text{ lb/f}t \]

\[ R = \frac{2(0.001067)}{(1246.6 - 14.7)(144)} = 1.203 \times 10^{-8} \text{ ft} \]

\[ = 1.443 \times 10^{-7} \text{ ft} \]

\[ = 3.666 \times 10^{-6} \text{ mm} \]

\[ = 36.6 \text{ Å} \quad \text{And that is very small indeed.} \]
9.25 Obtain the dimensionless functional form of the pool boiling \( q_{\text{max}} \) equation, and the \( q_{\text{max}} \) equation for flow boiling on external surfaces, using dimensional analysis.

The pool boiling result is worked out fully in the solution of Problem 4.28. It takes the form:

\[
\frac{\pi q_{\text{max}}}{24 q_{\text{max}2}} = f(L')
\]

This called the "Kulateladze" No.

All solutions for \( q_{\text{max}} \) in Table 9.3 take this form.

For external flows we have:

\[
q_{\text{max}} = f_n \left( \frac{\rho_f}{\rho_g}, \frac{h_{fg}}{\rho_g L}, \frac{L_{\text{in}}}{u_{\infty}} \right)
\]

J/m² s kg/m³ kg/m² J/kg kg/s m m/s

There are 7 variables in \( J, m, k_y, \frac{L_{\text{in}}}{u_{\infty}} \) or \( 1-4 = 3 \) \( \Pi \)-groups:

\[
\Pi_1 = \frac{q_{\text{max}}}{\rho_g h_{fg} u_{\infty}} \quad \Pi_2 = \frac{\rho_f}{\rho_g} \quad \Pi_3 = \text{We}_L = \frac{\rho_f u_{\infty}^2 L}{\sigma}
\]

Thus:

\[
\frac{q_{\text{max}}}{\rho_g h_{fg} u_{\infty}} = f_n \left( \frac{\rho_f}{\rho_g}, \text{We}_L \right)
\]

We see that the flow boiling burnout expressions in the text take this form unless there is an additional characteristic length in the problem. (See the expression for \( q_{\text{max}} \) when a jet of diameter, \( d \), impinges on a disc of diameter, \( D \). This introduces an additional group \( d/D \).) (See also the Katto flow boiling burnout correlation form.)

9.26 A (magical?) additive to water increases \( \sigma \) tenfold at 1 atm. By what factor will it improve \( q_{\text{max}} \) during pool boiling on: (a) infinite flat plates and (b) small horizontal cylinders; and (c) when a jet impinges on a disc.

a) from eqn. (9.11)

\[
\frac{q_{\text{max}}(\sigma_{\text{high}})}{q_{\text{max}}(\sigma_{\text{low}})} = \left( \frac{\sigma_{\text{high}}}{\sigma_{\text{low}}} \right)^{1/4} = (10)^{1/4} = 1.78
\]

b) from eqn. (9.20)

\[
\frac{q_{\text{max}}(\sigma_h)}{q_{\text{max}}(\sigma_i)} = \left( \frac{\sigma_h}{\sigma_i} \right)^{1/4} \left( \frac{R'(\sigma_i)}{R'(\sigma_h)} \right)^{1/4} = (10)^{1/4} \left( \frac{\sigma_h}{\sigma_i} \right)^{1/4} = 10^{1/4} = 1.603 \approx 1.6
\]

and from eqn. (9.40)

\[
\frac{q_{\text{max}}(\sigma_h)}{q_{\text{max}}(\sigma_i)} = \left( \frac{\text{We}_0(\sigma_i)}{\text{We}_0(\sigma_h)} \right)^{1/4} = \left( \frac{\sigma_h}{\sigma_i} \right)^A
\]

\[A=0.329\]

Thus:

\[
\frac{q_{\text{max}}(\sigma_h)}{q_{\text{max}}(\sigma_i)} = 10^A = 10^{0.329} = 2.133
\]

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9.27 Steam at 1 atm. is blown at 26 m/s over a 1 cm OD cylinder at 90°C. What is \( h \)? Suggest a physical process within the cylinder that could sustain this temperature in this flow.

\[
h' = 2257,000 \left( 1 + \frac{0.683 + 0.228 \cdot 4219(10)}{2257,000} \right) = 2,291,908
\]

\[
\bar{N_u} = 0.664 \left( \frac{2.6(0.01)}{0.78(10)^{0.3}} \delta \left( 1 + 0.69 \frac{0.8(2.291)0.6(0.0002776)(0.01)}{2.6^2(0.6811)(100-30)} \right)^{1/2} \right) \}
\]

\[
= 858.4, \quad \text{so} \quad \bar{h} = \bar{N_u} \frac{k}{D} = 858.4 \frac{(0.6811)(0.01)}{0.01}
\]

This means that we need a powerfully effective heat removal process in the cylinder -- enough to carry \( q = 584,460 \text{ W/m}^2 \) away from the surface. Nucleate boiling to water at least 1 atm. could do it, especially at high velocity. The right liquid -- one that is very cold and moves at high velocity -- might be made to do it.

9.28 The water shown in Fig. 9.17 is at one atmosphere and the nichrome heater can be approximated as nickel. What is \( T_w - T_{sat} \)?

\[
q_{fC} = \frac{\Delta T k}{L} \bar{N_u} \text{ where we scale } L = 18 \text{ cm from the photo, } \text{ use eqn. (6.68) for } \bar{N_u}.
\]

\[
q_{fC} = \frac{\Delta T (0.6811)}{0.18} \frac{0.664 \left( \frac{1.18(0.52)}{0.29(10)^{0.3}} \right)^{1/2}}{1.72^{1/3}} = 1712 \Delta T
\]

And from eqn. (9.4):

\[
q_{B} = \frac{\Delta T^3}{C_{sp} \rho_{sp} \theta_{sp}} \frac{0.8 \delta}{h_{sp} \rho_{sp} \theta_{sp}} \frac{\sigma}{\pi^2} = \frac{\Delta T^3}{0.0043^3} \frac{951.2(0.29)}{2.25710^2} \frac{4219^3}{2.589(0.6811)}
\]

\[
q_{B} = 1485 \Delta T^3
\]

Then, noting that \( q \) is high we use the limiting form of eqn. (9.37), namely \( q = \sqrt{T_{B} q_{fC}} \):

\[
480,000 = \sqrt{1712(1485) \Delta T^2}, \quad \Delta T = 17.55 \text{ C}
\]

This is quite low. It gives \( h = 27,700 \text{ W/m}^2 \cdot \text{C} \). The process is very efficient.

9.29 For film boiling on horizontal cylinders, eqn. (9.6a) is modified with Fig. 9.3d to:

\[
\frac{d}{d} = \frac{2 \pi^{1/3} \left( \frac{1}{\rho_{sp} \rho_{d}} + \frac{2}{\text{diam.}} \right)^{1/2}}{2}
\]

for saturated acetone, compare this \( \lambda_d \) and the flat plate value,

\[
2.2 \frac{g_{sp} \rho_{d}}{g_{sp} \rho_{d} + g_{d}} \Rightarrow 0.0006(6) \text{ m}
\]

\[
T_{sat, acetone} = 56 \text{ C}
\]

\[
\text{so} \quad g = 0.20 \text{ kg/s}^2
\]

\[
\lambda_d = \frac{2 \pi^{1/3}}{\sqrt{0.3(1/689) + \frac{2}{0.020} \cdot 0.0006(6)^2}} = 0.0047 \text{ m/m}
\]

\[
= 0.477 \text{ cm}
\]
9.29 (continued)

In Fig. 9.3d we find 4 wavelengths in 2.92 cm so

\[ \lambda_{\text{exp}1} = 0.73 \text{ cm} \]

deviation from theory

\[ \frac{0.73 - 0.917}{0.73} = 35\% \]

\[ \lambda_{\text{plan}} = \frac{2\pi(3)}{\sqrt{9.8(194) \frac{0.020}{0.018}}} = 0.018 \text{ cm} = 1.8 \text{ cm} \]

which is 147\% above the exp1. value & 277\% above the data.

9.30 Water at 47°C flows through a 13 cm diameter thin-walled tube at 8 m/s. Saturated water vapor, at one atmosphere, flows across the tube at 50 m/s. Evaluate \( T_{\text{tube}} \), \( U \), and \( q \).

Guess \( T_{\text{tube}} = 67°C \) for property evaluation. Then \( T_{H_2O} = 57°C \)

\( T_{\text{film}} = 83.5°C \)

\( \mu = 0.493 \times 10^{-5}, \rho = 0.6917, \rho = 3.14, \mu_w = 0.00056 \)

in the pipe \( \mu = 0.493 \times 10^{-5}, \rho = 0.6917, k = 0.6943 \)

(This guess is already a second iteration. We don't present the first one here.)

Now we use eqn. (7.41)

\[ Re_p = \frac{8(0.13)}{0.493(10)^6} = 2.11(10)^6 \]

\( f/B = 1/8[1.82 \log_{Re_p} - 1.69]^2 = 0.001283 \)

\[ \frac{1}{0.77(2.11 \times 10^6)^{1.07 + 12.7(0.03582)(3.14^{2/3})(0.00056)^{0.25}}} = 5774 \]

\[ \overline{h}_{H_2O \text{ in tube}} = 5774(0.6917)/0.13 = 28,768 \text{ W/m}^2\text{-°C} \]

\( f \) for condensation outside use eqn. (9.56) -- use of eqn. (9.55) would be more accurate; but only by about 1% at \( f \) this very high \( Re_p \) (see below)

\( \frac{1}{0.13(50)} = 1.879(10)^7 \)

\[ Nu_{D} = 0.64 \sqrt{2(1.879)^10^7} = 3,923, \overline{T}_{h} = 3923 \frac{0.6743}{0.13} = 20,398 \frac{W}{\text{m}^2\text{-°C}} \]

Then:

\[ U = \frac{1}{28,768 + \frac{1}{20,348}} = 11,918 \text{ W/m}^2\text{-°C} \]

\( f = UAT = 11,918(100 - 47) = 631,663 \text{ W/m}^2 \)

\( f = \overline{h}_{\text{film}} (100 - T_{\text{tube}}); \overline{T}_{\text{tube}} = 100 - \frac{631,663}{20,398} = 69°C \)

This temperature is within 20°C. Further iteration is not needed.
9.31 A 1 cm diameter, thin-walled tube carries liquid metal through saturated water at one atmosphere. The throughflow of metal is increased until burnout occurs. At that point the metal temperature is 250°C and inside the tube is 9600 W/m²·°C.

What is the wall temperature at burnout?

\[
R' = R/\sqrt{\frac{q_f}{j_c p_g}} = 0.005/\sqrt{0.0589/9.8(958.2-0.6)} = 1.996, \text{ so we use eqn. (9.15) for burnout.}
\]

\[
q_{\max} = 0.9 q_{\max\text{ plate}} = 0.9(1,260,000) = 1,134,000 \text{ W/m}^2
\]

See Example 9.5

Then:

\[
1,134,000 = 9600(250 - T_w) \quad \text{so} \quad T_w = 132°C
\]

9.32 At about what velocity of liquid metal flow does burnout occur in Problem 9.31 if the metal is mercury?

The Nusselt number \( q_{\max} \) is \( \frac{q_{\max}}{\nu/\lambda} = 15.69 \) so reading from Fig. 7.9,

\[
Re_d = 2200 = \frac{u_\infty D}{\nu} = \frac{u_\infty (0.01)}{5.94 \times 10^{-6}} \quad \text{so} \quad u_\infty = 1.2 \text{ m/s}
\]

9.33 Explain, in physical terms, why equations (9.23) and (9.25) instead of differing by a factor of two, are almost equal. How do these equations change when \( H' \) is large?

In both cases, burnout occurs when enough vapor is generated to cause Helmholtz instability to occur -- it does not matter whether from one side or two. Thus, when \( H' \) is large, \( q_{\max} \) is equal to 0.9 \( q_{\max 2} \) -- the same value in both cases.

Indeed, if we have the same vapor volume at \( q_{\max} \) in both cases, and if both \( q_{\max} \) values are the same, then \( H' \) must be twice as large in the insulated case. That is why the constant, 1.4, in eqn. (9.24) is exactly \( 2^{1/4} \) times the constant, 1.18, in eqn. (9.23). i.e.:

\[
\frac{1.18}{H'^{1/4}} = \frac{1.4}{(2H')^{1/4}}
\]
### Problem 9.37

<table>
<thead>
<tr>
<th>( P_{\text{vap}} ) (Pa)</th>
<th>( T_{\text{sat}} ) (K)</th>
<th>( T_{\text{sat}} ) (°C)</th>
</tr>
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<tr>
<td>1000</td>
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<tr>
<td>10000</td>
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<td>45.81</td>
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<tr>
<td>100000</td>
<td>372.76</td>
<td>99.61</td>
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<table>
<thead>
<tr>
<th>( t ) (°C)</th>
<th>( \Delta T ) (K)</th>
<th>( q ) (kW/m(^2))</th>
<th>( h ) (kW/m(^2)K)</th>
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<td>25.1</td>
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<td></td>
<td>8</td>
<td>279.8</td>
<td>35.0</td>
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| 45.81        | 1               | 113.0           | 113.0           |
|              | 2               | 238.8           | 119.4           |
|              | 3               | 377.3           | 125.8           |
|              | 4               | 528.7           | 132.2           |
|              | 5               | 692.9           | 138.6           |
|              | 6               | 869.8           | 145.0           |
|              | 7               | 1059.6          | 151.4           |
|              | 8               | 1262.1          | 157.8           |

| 99.61        | 1               | 210.3           | 210.3           |
|              | 2               | 444.5           | 222.2           |
|              | 3               | 702.5           | 234.2           |
|              | 4               | 984.2           | 246.1           |
|              | 5               | 1289.8          | 258.0           |
|              | 6               | 1619.2          | 269.9           |
|              | 7               | 1972.4          | 281.8           |
|              | 8               | 2349.4          | 293.7           |
### Problem 9.38

#### Surface at 100 C

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<th>Delta T</th>
<th>P_0</th>
<th>Delta P</th>
<th>rho_0</th>
<th>factor</th>
<th>mdot (kg/m^2s)</th>
<th>q (MW/m^2)</th>
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</thead>
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#### T_0 373.15
#### p_0 101420
#### R 461.404
#### coef 1.6678
#### sigma 0.31
#### factor1 3.0329914
#### hfg 2246000 treat as constant

#### Surface at 40 C

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<th>Delta P</th>
<th>rho_0</th>
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<th>mdot (kg/m^2s)</th>
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#### 40 C

#### T_0 313.15
#### p_0 7384.9
#### R 461.403996
#### coef 1.6678
#### sigma 0.31
#### factor1 3.0329914
#### hfg 2306000 treat as constant

---

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10.1 What will be the apparent values of $\varepsilon_{\lambda, 0.2}$ and $\varepsilon_{\lambda, 0.65}$ for the sun as viewed from the earth's surface.

From Fig. 10.2 we scale

$$
\varepsilon_{\lambda, 0.2} = \frac{\varepsilon}{\varepsilon_0} \bigg|_{\lambda = 0.2} = 0
$$

$$
\varepsilon_{\lambda, 0.65} = \frac{\varepsilon}{\varepsilon_0} \bigg|_{\lambda = 0.65} = 0.77
$$

These are low. They show energy has been removed by the earth's atmosphere. (The sun itself is virtually black.)

10.2 Plot $\varepsilon_{\lambda, b}$ vs. $T$ for $T = 300^\circ K$ and $10,000^\circ K$. What portion of the total energy is radiated in the visible range.

**equation (1.30):**

$$
\varepsilon_{\lambda, b} = \frac{2\pi (6.6256 \times 10^{-34}) (2.998 \times 10^8)^2 \frac{J \cdot m^2}{s}}{\lambda^5 \left[ e^{6.6256(2.998)10^{-26}/1.3805(10^8)^{23}\lambda} - 1 \right] m^5}
$$

$$
= \frac{3.7417 \times 10^{-25}}{\lambda^5 \left[ e^{0.04343/\lambda} - 1 \right]} \frac{kW}{m^2 \cdot \mu m}
$$

where $\lambda$ is in $\mu m$ and $T$ is in $^\circ K$.

**Visible range contains virtually no visible light at $300^\circ K$.**

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10.3 A 0.0006 m diam. wire ($\epsilon = 0.85$) at 950°C is on the center of a 0.07 m diam. thin metal tube ($\epsilon = 0.09$). The tube is horizontal in air at 25°C. Find $T_{\text{tube}}$.

First guess $h_{\text{conv.}} = 6 \text{ W/m}^2\text{-K}$. Then from eqn. (10.14)

$$Q = \sigma A (T_1^4 - T_2^4) = \frac{\sigma}{1 - \epsilon} \pi D (T_1^4 - T_2^4) = \frac{\frac{1}{\epsilon} + \frac{1}{D_1}}{D_1} (T_1^4 - T_2^4) = \frac{5.67 \times 10^8 (2.87 \times 10^12 - T_2^4)}{0.95 + 0.0006 \frac{1}{0.09}}$$

$$Q = 8.46 \times 10^{11} (2.237 \times 10^12 - T_2^4) = \bar{h} A \Delta T = 6 \pi (0.01) (T_2 - 298)$$

Trial and error gives: $T_2 = 437^\circ \text{K} = 167^\circ \text{C}$

Then, evaluating all properties but $\beta$ at $\frac{167+25}{2} = 96^\circ \text{C} = 369^\circ \text{K}$

$$R_a = \frac{\frac{9 \beta \Delta T \rho}{2 \sigma}}{2 \sigma} = \frac{9.8 \times 10^9 (167-25) \times 0.01^3}{2.266 (3.215) \times 10^{-9}} = 2.20 \times 10^6$$

so:

$$N_{\text{D}} = 0.36 + \frac{0.518 (2.20 \times 10^6)^{1/4}}{1 + \left(\frac{0.518}{0.707}\right)^{1/9}} = 15.1$$

and

$$h = 15.1 \times 0.0397 = 0.68 \frac{\text{W}}{\text{m}^2\text{-C}}$$

using 6.68 instead of 6 in the heat balance equation above, we get $T_2 = 422^\circ \text{K} = 149^\circ \text{C}$. That gives $h = 6.46 \frac{\text{W}}{\text{m}^2\text{-C}}$. Then $T_2$ will drop in proportion:

$$Q = 6.68 (149-25) = 6.46 (T_2 - 25)$$

$$T_2 = T_{\text{Shield}} = \frac{153^\circ \text{C}}{1.24 \text{ ft}}$$

$$h_{\text{rad.}} = \frac{T_w - T_\infty}{A_{\text{shiel}} \Delta T} = 1.24$$, $h_{\text{conv.}}$ should be less than this.

$$h_{\text{rad.}} = \frac{1}{\epsilon w} \frac{T_w^4 - T_\infty^4}{A_{\text{shiel}} \Delta T} = 1.0$$, this is about $(1/6)^{th}$ $h_{\text{conv.}}$.

Thus the present assumptions are not bad, but a refined calculation would account for convection inside and radiation outside.
10.4 A 1 ft² shallow pan with adiabatic sides is filled to the brim with water at 32°F. It radiates to the night sky whose temperature is 360°F while a 50°F breeze blows over it at 1.5 ft/s. Will the water freeze or warm up?

\[
T_{\text{sky}} = 360°F, \quad T = 50°F, \quad T_{\text{water}} = 32°F
\]

\[\therefore C_{H_2O} = 0.96\]

- Find \(q_{\text{conv}}\) using \[N_U = \frac{0.664 \Pr^{1/3} \Re_{\text{L}}^{1/2}}{0.309 \Pr} \] (Evaluate properties at \(50 + 32 = 82°F\)).

\[\begin{align*}
W & = 0.02495 \frac{W}{m^2} \frac{W}{C} \\
& \cdot 0.664 \cdot (0.717)^{1/3} \left( \frac{1.5 \cdot (0.3048 \cdot 0.3048)}{1.371 \times 10^{-5}} \right)^{1/2} \approx 4.802 \frac{W}{m^2}\end{align*}\]

\[q_{\text{conv}} = 4.802 \times \left[ \frac{(50 - 32)}{1.8} \right] = 48.02 \text{ W/m}^2\]

- Find \(q_{\text{rad}} = \frac{L}{\varepsilon \left( T_{\text{water}}^4 - T_{\text{sky}}^4 \right)} \)

\[\therefore q_{\text{rad}} = 0.96 \times 10^{-6} \times (173^4 - 300^4) = 215 \frac{W}{m^2}\]

Thus about four times as much heat radiates away as flows into the water by convection. It is, in fact, possible to freeze water in the desert in this way, on warmish nights.

10.5 Find the temperature, \(T_n\), of a thermometer in 10°C air and 27°C walls if it and the room are black.

Let's treat the thermometer bulb as a vertical wall, 0.01 m in height; and evaluate properties at 291.5 K & \(\beta\) at 283 K.

Then, using the simple Squire-Eckert equation (8.27)

\[\begin{align*}
\tilde{h} &= \frac{0.0255}{0.01} \cdot 0.678 \left[ \frac{0.713}{0.952 + 0.713} \right]^{1/4} \left[ \frac{9.8 \left( \frac{1}{283} \right) \left( T - 10 \right) \left( 0.01 \right)}{1.49 \left( 2.092 \right)^{10^{-10}}} \right]^{3/4} \\
q &= 4.54 \left( T - 10 \right)^{5/4} = 4.54 \left( (\text{T}^9 - 283) \right)^{5/4}
\end{align*}\]

And:

\[q_{\text{rad}} = -\sigma \left( T_{T}^4 - 300^4 \right) = -5.67 \times 10^{-8} \left( T_{T}^4 - 8.1 \times 10^9 \right)\]

Setting these equations equal to one another and solving them simultaneously for \(T\), we get

\[T = 292.6 \text{ K} = 19.6°C\]

Notice that we should have evaluated properties at (19.6 + 10)/2 = 14.8°C = 287.8 K. That's only 3°C off the mark so we let the calculation stand.
10.6 What will $T_t$ be in Problem 11.5 if $\varepsilon_t = 0.94$ and $\varepsilon_w = 0.92$ to 0.58.

\[ q_{rad} \text{ is now } = \frac{-1}{\frac{1}{T_t} + \frac{1}{T_w}} \frac{1}{1 - \varepsilon_t} \sigma (T_t^4 - 300^4) = 5.33 \times 10^{-8} (T_t^4 - 300^4) \]

neglect

The trial and error calculation now gives: $T_t = 19.4^\circ C$

The black body assumption was very good in this case.

10.7 Two thin aluminum sheets, one polished ($\varepsilon = 0.05$) and the other painted black ($\varepsilon = 0.9$) are placed horizontally outdoors in $10^\circ C$ air. $h \approx 5W/m^2 - ^\circ C$ on both the top and bottom. The top is irradiated with $750W/m^2$ and it re-radiates to a sky which is at $170^\circ K$. The earth beneath each plate is black at $10^\circ C$. Find the temperature of each plate. (Assume the sky to be black).

In either case we can write:

\[ q_{sun} + \varepsilon_{plate} \sigma (T_{sky}^4 - T_{pl}^4) + \varepsilon_{pl} \sigma (T_{earth}^4 - T_{pl}^4) \]

\[ \approx 750 \alpha_{pl} \]

\[ \approx 750 \varepsilon_{pl} \]

\[ = 2h(T_{pl} - T_{air}) \]

\[ = \frac{37.5 + 20.55 + 2830}{2888} = 5.67 \times 10^{-9} T_{pl1}^4 + 10T_{pl1} \]

\[ = \frac{675 + 270 + 2830}{3875} = 1.0206 \times 10^{-7} T_{pl2}^4 + 10T_{pl2} \]

Trial and error using a pocket calculator gives

\[ T_{pl1} = 285^\circ K = 12^\circ C \]

\[ T_{pl2} = 302.3^\circ K = 29.3^\circ C \]

Notice that the highly reflecting surface is, as we would anticipate, almost unaffected by solar radiation. For this reason, unpainted aluminum rooting is sometimes used in sunny climates.
10.8 Find the tip temperature of the sample holder shown. Assume 

\[ F_{\text{sample holder-sky}} = 1 \]

and take the holder to be a finite fin.

\[ h_r = \frac{\frac{4}{T_h - T_x}}{\sum \frac{4}{T_h - T_x}} \]

where \( T_h \approx 0 \, ^\circ C \)

\[ = \frac{5.67 \times 10^{-8} (0.09)(273^4 - 30^4)}{(273-30)} = 0.1166 \, \text{W/m}^2\text{°C} \]

Now for the fin: \( mL = \sqrt{\frac{0.1166 (\pi \times 0.05)}{204 (\pi \times 0.005^2)}} \) \( (0.16) = 0.0765 \)

Then \( \frac{T_{\text{tip}} - T_{\text{root}}}{T_{\text{root}} - T_{\text{oo}}} = \frac{2}{e^{0.0765} + e^{-0.076}} = 0.9971 \)

so \( T_{\text{tip}} = 0.9971 (273 - 30) + 30 = 272.3 \, ^\circ C \)

or \( T_{\text{tip}} = -0.7 \, ^\circ C \)

(Note that with so little temperature drop, it is justifiable to base \( h_r \) on a constant fin temperature.)

10.9 Find the percentages of leaving the bottom of the box that reach sides 1, 2, 3, 4, and the top.

These percentages are equal to \( F_{\text{bottom-1}}, F_{b-2}, F_{b-3}, F_{b-4}, \) and \( F_{b-top} \).

From Fig. 10.9, \( a/c = \frac{2}{0.8} = 2.5 \), \( b/c = \frac{3}{0.8} = 3.75 \), \( F_b = 0.53 \)

From Fig. 10.9, \( h/l = \frac{a}{3} = 0.267 \), \( w/l = \frac{2}{3} = 0.667 \), \( F_{b-2} = F_{b-4} = 0.14 \)

From Fig. 10.9, \( h/l = \frac{a}{2} = 0.4 \), \( w/l = \frac{2}{3} = 1.5 \), \( F_{b-1}, F_{b-3} = 0.095 \)

Check the result \( \sum F_{b-n} = 0.53 + 2(0.14) + 2(0.095) = 1.00 \)
**Problem 10.10**  Consider Fig. (10.11). Find \( F_{1-(2+4)} \) and \( F_{(2+4)-1} \).

\[ \text{Solution} \]

First note that \( F_{1-(24)} = F_{1-4} \). We use shape factor algebra to break this down.

\[ F_{4-(123)} = F_{4-1} + F_{4-2} + F_{4-3} \]
\[ = 2F_{4-1} + F_{4-2} \]

\[ \frac{A_{(123)}}{A_4} F_{(123)-4} = \frac{2A_1}{A_4} F_{1-4} + \frac{A_2}{A_4} F_{2-4} \]

Divide through by \( 2A_1/A_4 \) and rearrange:

\[ F_{1-4} = \frac{A_{(123)}}{2A_1} F_{(123)-4} - \frac{A_2}{2A_1} F_{2-4} \]

\( F_{2-4} \) is the subject of Problem 10.11, and the answer is \( F_{2-4} = 0.255 \). The other shape factor may be found using Fig. 10.9, letting surface 4 be the \( h \) surface:

\[ F_{(123)-4} = 0.23 \quad \text{with} \quad \left\{ \begin{array}{l} h/l = 0.5/1.2 = 0.42 \\ w/l = 0.6/1.2 = 0.5 \end{array}\right. \]

Hence

\[ F_{1-4} = \frac{(1.2)(0.6)}{2(0.4)(0.6)} (0.23) - \frac{1}{2} (0.255) = 0.218 \]

Then,

\[ F_{(24)-1} = F_{(4)-1} = \frac{A_1}{A_4} F_{1-4} = \frac{(0.4)(0.6)}{(1.2)(0.5)} (0.218) = 0.087 \]
10.11 Find $F_{2-4}$ for the situation shown.

To solve this problem, we make use of Fig. 10.10. But notice that the upright and horizontal surfaces are inverted here.

$$F_{1-4} = \frac{F_{1-4,1} + F_{2-4,2} + F_{3-4} + F_{4-1}}{2} = \frac{F_{1-4,1} + F_{2-4,2}}{2} = 2F_{2-4,1}$$

but from Fig. 10.10(2), $h/l = \frac{0.5}{0.4} = 1.25$; $w/l = \frac{0.6}{0.4} = 1.5$ \[ F_{2-4} = 0.165 \]

and: $h/l = \frac{0.5}{0.8} = 0.625$, $w/l = \frac{0.6}{0.8} = 0.75$ \[ F_{1-4,2} = 0.21 \]

Thus $F_{2-4} = 0.21 - 0.165 = 0.045$

Then $F_{2-4} = F_{2-4,1} + F_{2-4,2} = F_{2-4} + F_2 + F_{2-4} = 0.045 + 0.165 + 0.045 = 0.255$
10.12 Find $F_{1-2}$ for the configuration shown.

\[
\text{Therefore:} \quad A_{14} \frac{F_{14-32}}{A_{4}} = A_{1} \left( \frac{F_{1-3}}{A_{4}} + \frac{F_{1-2}}{F_{4-2}} + \frac{F_{4-3}}{F_{4-3}} \right)
\]

So:

\[
2F_{1-3} + 2F_{1-2} = 2F_{14-32}
\]

From Fig. 10.9 we then read (1):

\[
\begin{align*}
\frac{a}{c} &= \frac{1}{0.7} = 1.43 \quad \text{so} \quad F_{1-3} = 0.3 \\
\frac{b}{c} &= \frac{1}{0.7} = 1.43 \\
\frac{a}{c} &= \frac{2}{0.7} = 2.86 \quad \text{so} \quad F_{14-32} = 0.38 \\
\frac{b}{c} &= \frac{1}{0.7} = 1.43
\end{align*}
\]

It follows that:

\[
F_{1-2} = \frac{2F_{14-32} - 2F_{1-3}}{2} = 0.08
\]

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10.13 Compute the net heat transfer between the black cylinders shown.

First find \( F_{1-2} \). From Table 10.2 we read:

we read:

\[
F_{1-2} = \frac{1}{\pi} \left[ \sqrt{\left(1 + \frac{0.03}{0.04}\right)^2 - 1} + \sin^{-1} \frac{1}{1 + \frac{0.03}{0.04}} - 1 - \frac{0.03}{0.04} \right]
\]

\[
F_{1-2} = 0.0937
\]

so:

\[
Q = (\pi D)(0.0937)(T_1^4 - T_2^4) = \pi (0.04)(0.0937)(5.67(8.73^4 - 2.93^4)) = 38.3 \text{ W m of length}
\]

10.14 Develop the string method for evaluating \( F_{1-2} \) between two-dimensional surfaces.

Noting that areas are proportional to the distances shown we write view factors for the two triangles, \( L_1-a-c \), and \( L_1-b-d \), using case 4 in Table 10.2.

Thus:

\[
F_{1-c} = \frac{L_1 + c - a}{2L_1} \quad \text{and} \quad F_{1-d} = \frac{L_1 + d - b}{2L_1}
\]

Then, since \( F_{1-2} = 1 - F_{1-c} - F_{1-d} \),

\[
F_{1-2} = \frac{2L_1}{2L_1} - \frac{L_1 + c - a + L_1 + d - b - (a + b) - (c + d)}{2L_1} = \frac{(a + b) - (c + d)}{2L_1}
\]

So it would be possible to obtain \( F_{1-2} \) by comparing the difference between the lengths of the crossed strings, \((a + b)\), and the edge strings, \((c + d)\), with \(2L_1\). Hotte and Sarofim [10.15] show that this will also work if \( L_1 \) and \( L_2 \) are curved in complicated ways.

(Note: If the student is not clever in attacking this problem, he can easily embark on some pretty complicated, albeit correct, strategies.)
10.15 Find $F_{1-5}$ for the configuration shown:

We shall write:

$$F_{1-5} = F_{1-4_r} - F_{1-4_l}$$

Then:

$$A_1 F_{1-4_r} = A_{12} F_{12-4} - A_{1} F_{1-4_l} - A_{2} F_{2-4} - A_{23} F_{23-4}$$

$$= \frac{A_{12} + A_{23}}{A_1} F_{1-4_r} - \frac{A_1}{A_1} F_{1-4_l} - \frac{A_{23}}{A_1} F_{23-4}$$

$$= A_1 F_{1-4_r} - \frac{A_{23}}{A_1} F_{23-4} = A_1 F_{1-4_r} - \frac{A_{23}}{A_1} F_{23-4}$$

so

$$F_{1-4_r} = \frac{3}{2} F_{12-4} - F_{23-4} - \frac{1}{2} F_{2-4}$$

Likewise

$$A_1 F_{1-4_l} = A_{12} F_{12-4} - A_{1} F_{1-4_l} - A_{2} F_{2-4} - A_{23} F_{23-4}$$

$$= \frac{A_{12} + A_{23}}{A_1} F_{1-4_l} - \frac{A_1}{A_1} F_{1-4_l} - \frac{A_{23}}{A_1} F_{23-4}$$

$$= A_1 F_{1-4_l} - \frac{A_{23}}{A_1} F_{23-4} = A_1 F_{1-4_l} - \frac{A_{23}}{A_1} F_{23-4}$$

so

$$F_{1-4_l} = \frac{3}{4} F_{12-4} - \frac{1}{2} F_{1-4_l} - \frac{1}{4} F_{2-4}$$

Finally:

$$F_{1-5} = F_{1-4_l} - F_{1-4_r} = \frac{3}{2} F_{12-4} - F_{23-4} - \frac{3}{4} F_{12-4} + \frac{1}{4} F_{2-4}$$

Now from Fig. 10.9:

$$\frac{a}{b} = \frac{0.8}{1.2} = 0.67, \quad \frac{w}{l} = \frac{0.6}{1.2} = 0.5$$

$$F_{123-4} = 0.265$$

Thus:

$$F_{1-5} = \frac{3}{2} (0.265) - 0.23 - \frac{3}{4} (0.218) + \frac{1}{4} (0.125) = 0.035$$

This result could easily suffer 10 or 20% error from accumulative inaccuracy of graph reading. Notice, too, that without recognizing some tricks in manipulating $F$'s, one could have a hard time solving this one.
10.16 Find $F_{1-2,3,4}$ for the configuration shown.

$$F_{1-2,3,4} = F_{1-2,3,4}^\text{with no blockage by the cy} + F_{1-5}$$

However: (equation (10.12))

$$F_{1-2,3,4} = F_{1-2} + F_{1-3} + F_{1-4} = 1$$

To get $F_{1-5}$ we use eqn. (5) in Table 10.2, with

$$r = 0.05m, \ b = 0.125m, \ c = 0.125m$$

and $a = -0.125$ (notice the important inclusion of a minus sign in front of $a$.)

$$F_{1-5} = \frac{0.05}{0.125 - (-0.125)} \left[ \tan^{-1} \frac{0.125}{0.125} - \tan^{-1} \frac{-0.125}{0.125} \right] = 0.3142$$

Then:

$$F_{1-2,3,4} = 1 - F_{1-5} = 0.6858$$

(Some students will use Table 11.2 to calculate the un-blocked values of $F_{1-2} = F_{1-4} = 0.293$ and $F_{1-3} = 0.4142$ and only then discover that $0.293 + 0.293 + 0.414 = 1$.)

10.17 All sides of the box, except 1 and 2 are insulated. Find $Q_{1-2}$.

From eqn. (11.31):

$$Q_{1-2} = \frac{e_b - e_{b_2}}{1 - \frac{e_1}{e_1 A_1} + \frac{1}{A F_{1-3}} + \frac{1}{A F_{1-2}}} + \frac{1-e_2}{e_2 A_2}$$

but $e_1 = 1$ so $\frac{1-e_1}{e_1 A_1} = 0$ and from Fig. 10.9, case 2 we read
10.17 (continued)

for \(a/c = b/c = 1\), \(F_{1-2} = 0.2\). It follows that \(F_{1-3} = 0.8\) and, by symmetry, \(F_{2-3} = 0.8\). Then, since \(A_1 = A_2 = 1\) and \(A_3 = 4\), the equation gives:

\[
Q_{1-2} = \frac{1}{1 - 0.2} + \frac{1}{0.2(1)} + \frac{1}{1(0.8)} + \frac{1}{1(0.2)} = \frac{5.67 \times 10^5}{(773^4 - 573^4)} = 2494 \text{ W}
\]

**Note:** One could, alternatively, write the three nodal equations:

\[\begin{align*}
\text{Node 1: } & e_{b1}^A = \frac{5.67 \times 10^5 (773^4) - B_z}{1(0.2)} + \frac{5.67 \times 10^5 (773^4) - B_3}{1(0.2)} \\
\text{Node 2: } & \frac{5.67 \times 10^5 (573^4) - B_z}{1 - 0.2} = \frac{B_2 - 5.67 \times 10^5 (773^4)}{1(0.2)} + \frac{B_2 - B_3}{1(0.2)} \\
\text{Node 3: } & B_2 \text{ and } B_3 \text{ are already specified. This equation is redundant.}
\end{align*}\]

Solve for \(B_2 = 16.088\) Then \(Q = \frac{B_1 - B_2}{1(0.2)} + \frac{B_2 - B_3}{1(0.2)} = 2494 \text{ W}\)

10.18 Find \(Q_{1-2}\) and \(T_{\text{ins.-walls}}\) for Problem 10.17 is \(\epsilon_{\text{ins. wall}} = 0.6\), and if it is \(1.0\).

Note that, since node 3 is at an insulated wall, there is no heat flow across the thermal resistance, \((1 - \epsilon_3)/A_3\). Thus \(e_{b3}^A = B_3\) and \(\epsilon_3\) is irrelevant to the determination of either \(Q_{1-2}\) or \(T_{\text{ins.-wall}}\). With reference to the solution of Problem 11.17, we can immediately write, for \(\epsilon_{\text{ins.-wall}}\) equal to either 1.0 or 0.6:

\[
Q_{1-2} = 2494 \text{ W}
\]

\[
T_{\text{ins.-wall}} = \frac{\sqrt{\epsilon_{b3}^A}}{1 - \epsilon_3} = \frac{\sqrt{B_3}}{1 - \epsilon_3} = \frac{\sqrt{18.166}}{1 - \epsilon_3} = 752^\circ K = 479^\circ C
\]
10.19 Find $F_{1-3}$ within the insulated cylinder shown.

First find $F_{1-2}$:

$$R_1 = \frac{2.5}{10} = R_2 = 0.25, \quad X = 1 + \frac{1.05}{0.25} = 18$$
so $F_{1-2} = \frac{1}{2} \left[ 18 - \sqrt{18 - 4} \right] = 0.05573$

Furthermore: $A_1 = \frac{\pi}{4} (0.05)^2 = 0.00196 \text{m}^2$, $A_2 = 0(0.05)(0.01) = 0.00078 \text{m}^2$

$$B_1 = e_{b_1} = 5.67 \times 10^8 \text{ergs} \cdot \text{cm}^{-2} \cdot \text{sec}^{-1} \cdot \text{mole}^{-1} \cdot \text{K}^{-1}$$

$$b_1 = 3.15 \times 10^{-3} \text{K}$$

Nodal balances:

1: $O = \frac{1097.5 - B_2}{9155} + \frac{1097.5 - B_3}{540.3}$
2: $\frac{9155 - B_2}{2452} = \frac{B_3 - 1097.5}{9155} + \frac{B_2 - B_3}{540.3}$
3: $0 = \frac{B_3 - 1097.5}{540.3} + \frac{B_2 - B_3}{540.3}$

So we use the second two of these three equations:

$$B_2 = 0.0583 + B_2/2 \quad 37.06 - 0.117 B_2 - 0.05302 B_2 + 64.77 - B_2 + 548.8$$

so $B_2 = 961.45$, $B_2 = e_{b_2} = 1029.5$

and $\frac{Q}{\text{cylinder}} = \sqrt{e_{b_2}/\sigma} = \frac{367.1 \text{K}}{94 \text{K}}$

and:

$$Q_{\text{net}} = \frac{B_1 - B_2}{9155} + \frac{B_3 - B_2}{540.3} = 0.1408 \text{W}$$

10.20 Rework Example 10.3 if $\epsilon_{\text{shield}} = 0.34$. (Refer to the text for the sketch and numbers.)

$$e_{b_4} = e_{b_4} = 0.388 \left(\frac{16}{14} \times 0.34 \right) = 158$$

$$e_{b_1} = 5.67 \times 10^8 \text{ergs} \cdot \text{cm}^{-2} \cdot \text{sec}^{-1} \cdot \text{mole}^{-1} \cdot \text{K}^{-1}$$

$$b_1 = 3.15 \times 10^{-3} \text{K}$$

Since neither $h$ or $i$ is adiabatic, we can only write a nodal energy balance at node $s$:

$$297$$
10.20 (continued)

\[ O = \frac{B_s - 1098}{20} + \frac{B_s - 418}{11.02} + \frac{B_s - 266,928}{158} \]

or

\[ O = 0.14707B_s - 17,822.25; \quad B_s = 12,118 \]

Then:

\[ Q_{h-s} = \frac{B_h - B_s}{A_{h,F_{h-s}}} = \frac{266,928 - 12,118}{158} = 1613 \text{ W} \]

\[ Q_{h-i} = \frac{B_h - B_i}{A_{h,F_{h-i}}} = \frac{266,928 - 418}{66.3} = 402 \text{ W} \]

\[ Q_{s-i} = \frac{B_s - B_i}{A_{s,F_{s-i}}} = \frac{12,118 - 418}{11.02} = 1062 \text{ W} \]

Thus the net cooling of the shield must be \( Q_{h-s} - Q_{s-i} = 551 \text{ W} \)
10.23 Calculate the heat transfer between the same shown and its base, by radiation. \((\epsilon_s = 0.4 \text{ and } \epsilon_{cu} = 0.15)\)

\[
A_1 = 2\pi R^2, \quad A_2 = \pi R^2, \quad A_2/A_1 = 1/2
\]

\[
Q = \frac{\epsilon_1 A_1}{1 - \epsilon_1} (\sigma T_1^4 - B_1)
\]

so we need \(B_1\). To get \(B_1\), write:

\[
B_1 = \epsilon_1 \sigma T_1^4 + (1 - \epsilon_1) [B_1 F_{1,1} + B_2 F_{1,2}]
\]

\[
B_2 = \epsilon_2 \sigma T_2^4 + (1 - \epsilon_2) [B_2 F_{2,2} + B_1 F_{2,1}]
\]

so:

\[
B_1 [1 - (1 - \epsilon_1) F_{1,1}] + B_2 [-F_{1,2} (1 - \epsilon_1)] = \epsilon_1 \sigma T_1^4
\]

\[
B_1 [-F_{2,2} (1 - \epsilon_2)] + B_2 [1 - (1 - \epsilon_2) F_{2,1}] = \epsilon_2 \sigma T_2^4
\]

\[
0.7B_1 - 0.3B_2 = 0.4\sigma T_1^4
\]

\[-0.85B_1 + 1.1B_2 = 0.15\sigma T_2^4
\]

Therefore:

\[
B_1 = \frac{\left| \begin{array}{cc}
0.4\sigma T_1^4 & -0.3 \\
0.15\sigma T_2^4 & 1
\end{array} \right|}{0.7 - 0.3(0.85)} = \frac{0.4\sigma T_1^4 - 0.045\sigma T_2^4}{0.495}
\]

so:

\[
Q = 2\pi (0.1) \frac{0.4}{1 - 0.4} 5.67 \cdot 10^{8} (573)^4 \left( 1 - 0.4 \right) - 0.045 \frac{(373)^4}{0.495} \left( \frac{0.995}{0.495} - 373 \right)
\]

\[
Q = 21.24 \text{ W}
\]
10.24 A hemispherical indentation in a smooth wrought iron plate has a 0.008 m radius. How much heat radiates from the 40°C dent to the -20°C surroundings?

\[
A_1F_{-2} = A_2F_{-1} \quad \text{and} \quad F_{-2} = F_{-\infty} \quad \text{and} \quad F_{-\infty} = \frac{A_2}{A_1}
\]

\[
\varepsilon_{b_1} = \frac{1 - \varepsilon_i}{\varepsilon_i A_1} \quad B_1 = \frac{1}{A_1 F_{-\infty}} \quad \varepsilon_{b_2}
\]

So:

\[
Q = \frac{\varepsilon_{b_1} - \varepsilon_{b_2}}{1 - \varepsilon_i} + \frac{1}{\varepsilon_i A_1 + A_1 F_{-\infty}} = \frac{1}{(1 - \varepsilon_i) A_1 + \varepsilon_i} \frac{A_2}{A_1} \frac{\varepsilon_i A_2}{A_1} \frac{(T_1 - T_{-\infty})}{\text{heat flow without a dent}}
\]

\[
= \frac{1}{(1 - 0.35) \frac{\tau (0.009)^2}{2 \pi (0.008)^2} + 0.35} \times \left[ \frac{1.31^3 - 2.5^3}{1.31} \right]
\]

\[
Q = 0.0325 \text{W}
\]

10.25 A conical hole in a block of metal, for which \( \varepsilon = 0.5 \), is 5 cm in diameter at the surface and 5 cm deep. By what factor will the radiation from the area of the hole be changed by the presence of the hole?

(This following solution breaks down if the cone is very deep and slender since the the apex receives little and we cannot use the network analogy.)

From the solution to problem 10.24 we find that

\[
\frac{Q_{\text{with hole}}}{Q_{\text{no hole}}} = \frac{1}{(1 - \varepsilon_i) A_1 + \varepsilon_i} \frac{A_2}{A_1} = \frac{1}{0.5 \pi (2.5)^2} + 0.5 = 1.382
\]

where the area of the cone is the product of its average circumference, 2.5\( \pi \), and its slant height, 5/\( \cos(\tan^{-1} 2.5/5) \) = 43.9 cm².
10.26 A single-pane window in a large room is 4 ft wide and 6 ft high. The room is kept at 70°F but the pane is at 67°F owing to heat loss to the colder outdoor air. Find: a) the heat transfer by radiation to the window; b) the heat transfer by natural convection to the window; and c) the fraction of heat transferred to the window by radiation.

a) \[
\frac{Q}{W_{\text{window to room}}} = \frac{1}{\epsilon_{\text{window}}} + \frac{A_{\text{window}}}{A_{\text{room}}} \left( \frac{1}{\epsilon_{\text{room}}} - 1 \right) = \epsilon_{\text{window}} \approx 0.94
\]

\[
Q_{\text{rad window to room}} = \epsilon_{\text{window}} A_{\text{window}} (T_{\text{window}}^4 - T_{\text{room}}^4)
\]

\[
= 0.94(5.67 \times 10^{-8})(4 \times 6)(0.3048)^2 [292.59^4 - 294.26^4]
\]

\[
Q_{\text{rad room to window}} = -Q_{\text{rad window to room}} = 20.06 \text{ W} = 68.43 \text{ Btu/hr}
\]

b) Use eqn. (8.13a) with \[
R_{L} = \frac{9.8 (1/294.26)(1.67)(6 \times 0.3048)^3}{(1.508 \times 10^{-5})(2.117 \times 10^{-5})} = 1.06 \times 10^9
\]

and \[
Pr = 0.713:
\]

\[
\overline{Nu}_L = 0.68 + 0.67 \overline{R}_{L}^{4/3} \left[ 1 + \left( \frac{0.492}{Pr} \right)^{9/4} \right]^{-4/9} = 93.64
\]

So \[
\overline{h} = \overline{Nu}_L k / ℓ = 93.64 (0.02562) / 6 (0.3048) = 1.31 \text{ W/m}^2\text{°C}
\]

(two is a very low \(\overline{h}\).)

Then: \[
Q_{\text{conv.}} = \overline{h} A (ΔT) = 1.31 (4 \times 9)(0.3048)^2 (1.67) = 4.88 \text{ W} = 16.64 \text{ Btu/hr}
\]

c) \[
\% \text{ of heat transfer by radiation} = \frac{20.06}{20.06 + 4.88} = 0.804 = 80.4 \%
\]
10.27 Suppose the window-pane temperature is unknown in Problem 10.26. The outdoor air is at $40^\circ F$ and $h = 62 \text{ W/m}^2 \cdot \text{K}$ on the outside. It is night and the effective $T_{\text{sky}} = 15^\circ \text{C}$. Assume $F_{\text{window-sky}} = 0.5$ and the other surroundings are at $40^\circ \text{C}$. Evaluate $T_{\text{window}}$ and draw the analogous electric circuit evaluating the thermal resistances. (The window is opaque to infra-red radiation but it offers little resistance to conduction so $T_{\text{window}}$ is approximately uniform.)

\[
(Q_{\text{rad.}} + Q_{\text{nat'l conv.}})_{\text{indoors}} = (Q_{\text{conv.}} + Q_{\text{rad. sky}} + Q_{\text{rad. other}})_{\text{outdoors}}
\]

\[
e_{\text{glass}} A_w (T_i^4 - T_w^4) + h_i A_w (T_i - T_w) = T_{\text{indoor}} A_w (T_i - T_o) + e_g F_{w-s} \alpha_A w (T_w^4 - T_s^4)
\]

\[
+ e_g F_{w-o.s} \alpha_A w (T_w^4 + T_s^4)
\]

Let's cut through a potentially terrible lot of trial and error computation by noting that natural convection is not going to be very important on the inside. (We take our cue in this from the solution of Problem 10.26. Therefore we'll guess that $h_i$ is $3 \text{ W/m}^2 \cdot \text{K}$ and correct this assumption later if we must. Then divide that equation above by $e_g \alpha A_w$ and get (in S.I. units):

\[
294.14^4 - T_w^4 + \frac{3(10)^6}{0.54(5.6)} (294.14 - T_w) = \frac{62(10)^6}{0.54(5.6)} (T_w - 273.15) + \frac{1}{2}(271.64^4 - 263.7^4)
\]

or

\[
1.496(10)^9 - T_w^4 + 339.5(10)^9 - 122(10)^9 T_w = T_w^4 - 5.387(10)^9
\]

so:

\[
T_w^4 + 0.61(10)^9 T_w = 176.2(10)^9
\]

solving by trial and error we get: $T_w = 279.0^\circ \text{K}$

$= 42.53^\circ \text{F}$

Now check $F_{\text{nat'l conv.}}$ using eqn. (8.13a). (Details are given in solution to Problem 10.26.)

\[
Ra_L = \frac{1.066 \times 10^9}{1.67} \frac{294.14 - 279.0}{T_w}
\]

when $T_w = 1.67^\circ \text{C}$

so:

\[
Nu_L = 0.68 + 0.67 Ra_L^{1/4} \left[1 + \left(\frac{0.492}{0.5625}\right)^{1/4}\right] = 162.3
\]

and $\bar{h} = 162.3 \frac{0.0256}{6(0.309)} = 2.27 \text{ W/m}^2 \cdot \text{K}$

Going back through the trial and error solution based on this value of $\bar{h}$ we get $42.3^\circ \text{F}$ -- almost no change.

So $T_w = 42.35^\circ \text{F}$
10.27 (continued)

Next calculate resistances: \( R_{rad} = \frac{1}{h_{rad}A_w} = \frac{\Delta T}{Q_{rad}} = \frac{15.2}{170.7} = 0.0893 \degree C/W \)

\[ R_{rad_{sky}} = \frac{30.5}{12.71} = 0.920; \quad R_{rad_{0.3}} = \frac{16.64}{7.17} = 2.32; \quad R_{convo} = \frac{1}{h_{convo}} = 0.0723, \]

the hitherto neglected \( R_{pane} = \frac{\text{thickness}}{kA_w} = \frac{0.001725}{1}; \quad R_{convo} = 0.1975. \)

Heat is basically short-circuited to the window by radiation.

Heat is short-circuited to the outdoors by convection.

So convection is irrelevant inside. Radiation is irrelevant outside. And conduction through the glass causes no \( \Delta T. \)

10.28 An effective low-temperature insulation is made by evacuating the space between metal sheets. Calculate \( q \) between 150\( ^{\circ} \)K and 100\( ^{\circ} \)K for: (a) two sheets of highly polished al., (b) three sheets of highly polished al., and (b) three sheets of rolled sheet steel.

In all cases, \( \bar{T}_{1-2} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{A_1(1-\varepsilon_1)}} \text{ or } \frac{1}{\frac{1}{\varepsilon_1} + 1(\frac{1}{\varepsilon_2} - 1)} = \frac{1}{\varepsilon_1 - 1} \)

Case a.) \( \bar{T}_{1-2} = \frac{1}{0.041 - 1} = 0.02041 \) (where we use \( \varepsilon_{ai} \) for a higher temp.)

\( q = \bar{T}_{1-2} \sigma (T_1^4 - T_2^4) = \frac{0.02041(5.67)10^8(150^4 - 100^4)}{1157(10)^4} = 0.470 \text{ W/m}^2 \)

b.) \( q = 1.157(10)^{-9} [150^4 - T_{middle}^4] = 1.157(10)^{-9} [T_{middle}^4 - 100^4] \)

\( T_{middle} = \sqrt{[(150^4 + 100^4)/2]}^4 = 131.95 \text{ K} \)

\( q = 1.157(10)^{-9} [150^4 - 131.95^4] = 0.235 \text{ W/m}^2 \)

C.) \( \bar{T}_{1-2} = \frac{1}{0.4925 - 1} = 0.4925, \quad q = 0.4925 \sigma [131.95^4 - 100^4] = 5.67 \text{ W/m}^2 \)
10.29 Three parallel black walls, 1m wide, form an equilateral triangle.
One wall is held at 400°K, one at 300°K, and the third is insulated.
Find Q W/m and the temperature of the third wall.

\[ T_1 = 400°K, \quad T_2 = 300°K, \quad T_3 = ? \]

By symmetry, we see:
\[ F_{1-2} = F_{1-3} = F_{2-1} = F_{2-3} = \frac{1}{2} \]
\[ e_{b_1} = 0.400 = 14.52 \text{ W/m}^2 \]
\[ e_{b_2} = 0.300 = 45.93 \text{ W/m}^2 \]

Using equation (10.35) we get:
\[ Q = \frac{1452 - 45.93}{\left(\frac{1}{2} + \frac{1}{2}\right)} \approx 74.4 \text{ W/m} \]

And to get \( e_{b_3} \) we write for the node #3:
\[ O = \frac{e_{b_2} - e_{b_3}}{2} + \frac{e_{b_2} - e_{b_3}}{2} \]
\[ e_{b_3} = (e_{b_2} + e_{b_3})/2 \]
\[ 955.7 = 0.744 \]
\[ 0 = \frac{1}{2} T_3^4 \]
\[ T_3 = 360.3°K \]

10.30 Two 1cm diameter rods run parallel with centers 4 cm apart. One is
at 1500°K and black. The other is unheated and \( \varepsilon = 0.66 \). They are
both encircled by a cylindrical black radiation shield at 400°K.

Evaluate Q W/m and the temperature of the unheated rod.

From Table 10.2, \( x = 1 + \frac{3}{1} = 4 \)
\[ F_{1-3} = \frac{1}{4} \left[ 15 + 5 \cdot \frac{1}{4} - 4 \right] = 0.0400 \]
\[ = F_{3-1} \]
\[ F_{1-2} = 1 - F_{1-3} = 0.960 = F_{3-2} \]
\[ A_1 = A_3 = 0.031416 \text{ m}^2/m \]

Using equation (10.35) we get:
\[ Q_{1-2} = \frac{287.094 - 1452}{1 - \frac{1}{796 + 33.16} + \frac{1}{33.16}} = \frac{8957}{80} \text{ W/m} \]

Now at node #1, \( Q = -8957 = \frac{B_2 - B_1}{33.16} + \frac{B_3 - B_1}{796} \)
\[ 0 = 0.031416(8957) = 281.4 \text{ W/m} \]
\[ T_3 = 690°K \]

(We note that \( \varepsilon \) does not enter the problem.)
10.31 A small diameter heater is centered in a large cylindrical shield. Discuss the relative importance of the emittance of the shield during specular and diffuse radiation.

In this case $A_{\text{inside}}/A_{\text{outside}}$ is very small so (see eqn. (10.30)):

$$
\frac{G_{1-2}}{A_{\text{diffuse}}} = \frac{1}{\varepsilon_i + 0} = \varepsilon_i, \quad \text{and} \quad \frac{G_{1-2}}{A_{\text{specular}}} = \frac{1}{\varepsilon_i + \varepsilon_0 - 1}
$$

In the pure diffuse limit, $\varepsilon_2$ is irrelevant, while in the pure specular limit, $\varepsilon_1$ and $\varepsilon_2$ are equally important.

10.32 Two, 1m wide, commercial aluminum sheets are joined at a 120° angle along one edge. The back (or 240°-angle) side is insulated. The plates are both held at 120°C. The 20°C surroundings are distant.

What is the net radiant heat transfer from the left hand plate to the right hand side and to the surroundings?

The business about heat transfer from left to right is a red-herring. We see at once that symmetry requires this to be zero. We may therefore treat the plates as a common heater, calculate $Q$ from both, and then divide it by two.

\[ A_{\text{1/2F_{1-2}} - 5} = 2A_{\text{F_{1-5}}} = 2(1 - F_{1 - 2}) = 1.732 \]

\[ \varepsilon_1 = \varepsilon_0 = 0.05 \]

Then eqn. (11.24) gives:

\[ Q = \frac{1}{2} \left[ \frac{1}{0.05} \left( \frac{1}{0.05} + \frac{1}{1.732} + \frac{1}{\infty} \right) \right] \]

Then:

\[ Q = 82.96 \text{ W/m} \]

10.33 Two parallel discs of diameter equal to 0.5m are separated by an infinite parallel plate, midway between them, with a 0.2m diameter hole in it. What is the view factor between the two discs, if they are 0.5m apart.

\[ F_{1-2} = F_{\text{hole}} = \frac{1}{2} \left( X - \sqrt{X^2 - 4 \left( R_{\text{hole}} / R_i \right)^2} \right) \]

Table 10.3

No. 3

\[ X = 1 + \frac{\left( R_{\text{h1}} + \varepsilon \right)^2}{\left( R_1 / h \right)^2} = 1 + \frac{\left( 0.2 / 0.5 \right)^2}{(0.5 / 0.5)^2} = 2.6 \]

\[ F_{1-\text{hole}} = \frac{1}{2} \left( 2.6 - \sqrt{2.6^2 - 4 \left( R_{\text{hole}} / R_i \right)^2} \right) \]

\[ = 0.0435 \]
10.34 An evacuated spherical cavity, 0.3 m in diameter in a zero-gravity environment, is kept at 300°C. Saturated steam at 1 atmosphere is then placed in the cavity. a) What is the initial flux of radiant heat transfer to the steam? b) Determine how long it will take for $q_{\text{cond}}$ to become less than $q_{\text{rad}}$. (Correct for the rising steam temperature if it is necessary to do so.)

In this case: $p_l = 0.3$ so at 373°C, $\varepsilon_g = \int \varepsilon_g = 0.275(1.21) = 0.333$

$\frac{573}{373} = 0.46$ at 573°C, $\alpha_g = \varepsilon_g \left( \frac{373}{573} \right)^{0.46} = 0.32$

Then $q_{\text{net}} = q_{\text{w-g}} - q_{\text{g-w}} = 0.333 \sigma (373)^4 - 0.324 \sigma (373)^4 = 1680 \frac{W}{m^2}$

and from eqn. (5.54) $q_{\text{cond}} = \frac{k_{\text{steam}} d T}{\sqrt{\pi \alpha_{\text{steam}} t}} = \frac{0.0237(200)}{\sqrt{\pi \times 2.032 \times 10^{-5} t}}$

Thus $q_{\text{cond}}$ will equal 1680 after $t = 0.353$ sec

Should we have accounted for temperature rise? It would appear not, but let's check:

Heat capacity of steam = $\rho c_p V_0 = 0.597(2030) \frac{4\pi}{3} (0.15)^3 = 17.13 \frac{W}{^\circ C}$

$Q_{\text{rad}} = 1680(4\pi(0.15)^2) 0.353 = 167.7 \frac{W}{^\circ C}$

$\int Q_{\text{cond}} dt = \frac{\alpha_g(t) r t}{1680} = 2(167.7) \frac{W}{^\circ C}$

So $\Delta T = \frac{Q_{\text{total}}}{\text{Heat cap}} = \frac{3(167.7)}{17.13} = 29.4 ^\circ C$

That's a lot more $\Delta T$ than one might first expect, but it's still a small number. We can probably ignore it.

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10.35 Verify cases 1, 2, and 3 in Table 10.2 using the "string method" described in Problem 10.14.

\[ F_{1-2} = \frac{(a+b)-(c+d)}{2L_1} \]

\[ c = d = h \]
\[ a = b = \sqrt{h^2 + \omega^2} \]

\[ F_{1-2} = \frac{(a+b)-(c+d)}{2\omega} = \sqrt{1 + \left(\frac{h}{\omega}\right)^2} - \frac{h}{\omega} \]

2) \[ a = b = \omega \]
\[ c = \omega \]
\[ d = 2\omega \sin \frac{\alpha}{2} \]

\[ F_{1-2} = 1 - \sin \alpha/2 \]

3) \[ F = \frac{(h+\omega)-(0+\sqrt{h^2 + \omega^2})}{2\omega} = \frac{1}{2} \left[ 1 + \frac{h}{\omega} - \sqrt{\left(\frac{h}{\omega}\right)^2 + 1} \right] \]

10.36 Two long parallel heaters consist of 120° segments of 10 cm diameter parallel cylinders, whose centers are 20 cm apart. The segments are those nearest each other, symmetrically placed on the line connecting their centers. Find \( F_{1-2} \) using the "string method" described in Problem 10.14.

\[ F_{1-2} = \frac{(a+b)-(c+d)}{2L_1} \]

(see general sketch in solution above.)

\[ a = 20 \sin 60° = b \]
\[ c = (20 \sin 60°) \sin 60° = d \]
\[ L_1 = \frac{1}{3} (10 \pi) \]

So:
\[ F_{1-2} = \frac{40 \sin 60° - 40 \sin 60° \cdot \sin 60°}{20\pi/3} = \frac{6}{\pi} \sin 60° (1 - \sin 60°) \]

\[ F_{1-2} \approx 0.2216 \]

308
10.37 Two long parallel strips of rolled steel sheet lie along sides of an imaginary 1 m equilateral triangular cylinder. One piece is 1 m wide and kept at 20°C. The other is (1/2) m wide, centered in an adjacent leg, and kept at 400°C. The surroundings are distant and they are insulated. Find Q. (You will need a shape factor. It can be found using the method described in Problem 10.14.)

\[
m = \frac{d \sin 60^\circ}{4} = \frac{\sqrt{a^2 - (1 - \frac{d}{4} \cos 60^\circ)^2}}{4} = \frac{0.2165}{0.866} = 0.24964
\]

So \( \alpha = 0.9014 \)

\[
\frac{C}{c} = 0.9014 \quad \text{by symmetry}
\]

Then:

\[
F_{1-2} = \frac{(a+b)-(c+d)}{2h} = \frac{0.9014 + 0.75 - 0.9014 - 0.25}{2(0.5)} = \frac{0.5}{2(0.5)} = 0.5
\]

Using \( \varepsilon_1 = 0.60 = \varepsilon_2 \), calling the surroundings, 3:

\[
\frac{1 - 0.60}{0.60/2} = 1.03 \quad \frac{1}{A_{1F_{1-2}}} = 4 \quad \frac{1 - 0.60}{1(0.60)} = 0.515
\]

\[
\frac{1}{A_{1F_{1-2}}} = A_1(1-F_{1-2}) = 4
\]

\[
\frac{1}{A_{2F_{2-3}}} = \frac{q}{3}
\]

\[
A_1F_{1-2} = A_2F_{2-1}
\]

but \( F_{2-1} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \)

so \( F_{2-3} = 1 - F_{2-1} = \frac{3}{4} \)

Then eqn. (10.35) gives:

\[
Q = \frac{\sigma (c_{73}^4 - 293^4)}{1.03 + \frac{1}{4 + \frac{q}{3} + \frac{1}{4}}} + 0.515 = 2927 \text{ W}
\]
10.38 Find the shape factor from the hot to the cold strip in Problem 11.37 using, not the string method, but Table 10.2. If your instructor asks you to do so, complete Problem 10.37 when you have \( F_{1-2} \).

(see details of dimensions in Problem 10.37 solution.)

\[ A_{1, 1'} \frac{F_{1-2}}{2} = A_{1} F_{1-2} + A_{1'} F_{1-2} \]

But from Table 10.2, #4:

\[ F_{1, 1'} - 2 = \frac{3/4 + 1 - 0.9014}{2 (\frac{3}{4})} = 0.5657 \]

\[ F_{1-2} = \frac{1/4 + 1 - 0.9014}{2 (\frac{1}{4})} = 0.6972 \]

so:

\[ \frac{3}{4} (0.5657) = \frac{1}{2} F_{1-2} + \frac{1}{4} (0.6972) \]

so: \( F_{1-2} = 0.5 \)

(From this point forward, details of solution are given in Prob 10.37.)

11.39 Prove that, as the figure becomes very long, the view factor for the 2nd case in Table 10.3 reduces to that given for the 3rd case in Table 10.2.

In this case \( H \to 0 \), \( W \to 0 \), \( H/W \to h/w \), so the equation reduces to:

\[ F_{1-2} = \frac{1}{1+\left(\frac{h}{w}\tan^{\prime} \omega \tan^{\prime} \theta - \sqrt{\left(\frac{h}{w}\right)^2 + 1} \tan^{\prime} \omega \tan^{\prime} \theta + \frac{1}{4} (h + 0 + 0 + 0 + 0)\right)} \]

\[ = \frac{1}{1+\left(\frac{h}{w} \frac{\pi}{2} - \sqrt{\left(\frac{h}{w}\right)^2 + 1} \frac{\pi}{2} + 0\right)} = \frac{1}{2} \left(1 + \frac{h}{w} - \sqrt{1 + \left(\frac{h}{w}\right)^2}\right) \]

This is given in Table 10.2.

10.40 Show that \( F_{1-2} \) for the first case in Table 10.3 reduces to the expected result when plates 1 and 2 are extended to infinity.

\[ X \text{ and } Y \to \infty \text{ so} \]

\[ F_{1-2} \to \frac{2}{\pi} \left[ \frac{\tan^{-1} X}{X} + \frac{\tan^{-1} Y}{Y} \right] \]

\[ \frac{\tan^{-1} X}{X} \to 0 \text{ but } \tan^{-1} a + \tan^{-1} b = \frac{\pi}{2} \]

so: \( F_{1-2} = 1 \) which is correct for two infinite facing plates.

310
10.42 In problem 2.26 you were asked to neglect radiation in showing that 
q was equal to 8227 W/m² as the result of conduction alone. Discuss 
the validity of the assumption, quantitatively.

In this case we have: \( q_{\text{rad}} = \frac{15}{\varepsilon} \left( \left[ 10000 + 273 \right]^4 - \left[ 200 + 273 \right]^4 \right) \)

where \( \varepsilon \) = the emittance of both plates. When \( q_{\text{cond}} = q_{\text{rad}} = 8227, \ \varepsilon = 0.1066 \). This is a reasonable value for polished metal, but it doesn't give a negligible value of \( q_{\text{rad}} \).

If \( q_{\text{rad}} = \text{only } 10\% \) \( q_{\text{cond}} = 822.7 \), then \( \varepsilon = 0.0112 \). This would be hard to achieve. It would require, for example, a very highly polished silver.

10.42 A 100°C sphere with \( \varepsilon = 0.86 \) is centered within a second sphere at 
300°C with \( \varepsilon = 0.47 \). The outer diameter is 0.3 m and the inner

diameter is 0.1 m. What is the radiant heat flux?

\[
\begin{align*}
\frac{1-0.47}{\pi(0.3)^2} \cdot 0.47 & = \frac{1}{A_1F_{12}} \\
& = 3.988 \\
\frac{1-0.86}{\pi(0.3)^2} \cdot 0.86 & = \frac{1}{\pi(0.3)^2} \left( \frac{0.05}{0.15} \right) \\
& = 31.83
\end{align*}
\]

(We get \( F_{1-2} \) from Table 10.3, case 4.)

Then, using eqn. (10.35), we get:

\[
Q = \frac{\sigma(573)^4 - \sigma(373)^4}{3.988 + 31.83 + 5.182} = 122.3 \text{ W}
\]

\[
q = \frac{Q}{A} = \frac{122.3}{\pi(0.3)^2} = 432.6 \frac{\text{W}}{\text{m}^2}
\]

311
Problem 10.52: The fraction of blackbody radiation between wavelengths of 0 and \( \lambda \) is

\[
f = \frac{1}{\sigma T^4} \int_0^\lambda e_{\lambda,b} \, d\lambda
\]  

(11)
a) Work Problem 10.51.
b) Show that

\[
f(\lambda T) = \frac{15}{\pi^4} \int_{c_2/\lambda T}^\infty \frac{t^3}{e^t - 1} \, dt
\]  

(12)

where \( c_2 \) is the second radiation constant, \( hc/k_B \), equal to 1438.8 \( \mu m \cdot K \).
c) Use the software of your choice to plot \( f(\lambda T) \) and check that your results match Table 10.7.

Solution. Following the solution to Problem 10.51:

\[
f = \frac{1}{\sigma T^4} \int_0^\lambda e_{\lambda,b} \, d\lambda
\]  

(13)

\[
= \frac{1}{\sigma T^4} \int_0^\lambda \frac{2\pi h c_0^2}{\lambda^5 \left[ \exp(h c_0/k_B \lambda) - 1 \right]} \, d\lambda
\]  

(14)

\[
= \frac{1}{\sigma T^4} \int_{c_0/\lambda}^\infty \frac{2\pi h \nu^3}{\nu^5 \left[ \exp(h \nu/k_B T) - 1 \right]} \, d\nu
\]  

(15)

\[
= \frac{1}{\sigma T^4} \frac{2\pi k_B^4 T^4}{h^3 c_0^2} \int_{c_0/\lambda T}^\infty \frac{t^3}{e^t - 1} \, dt
\]  

(16)

\[
= \frac{15}{\pi^4} \int_{c_2/\lambda T}^\infty \frac{x^2}{e^x - 1} \, dx
\]  

(17)

\[
= \frac{15}{\pi^4} \int_0^\infty \frac{x^3}{e^x - 1} \, dx - \frac{15}{\pi^4} \int_0^{c_2/\lambda T} \frac{x^3}{e^x - 1} \, dx
\]  

(18)

\[
= 1 - \frac{15}{\pi^4} \int_0^{c_2/\lambda T} \frac{x^3}{e^x - 1} \, dx
\]  

(19)

The numerical integration can be done in various ways, depending on the software available. (On a sophisticated level, the last integral can be written in terms of the Debye function which is available in the Gnu Scientific Library.) This equation is plotted in Fig. 1.

Problem 10.53: Read Problem 10.52. Then find the central range of wavelengths that includes 80% of the energy emitted by blackbodies at room temperature (300 K) and at the solar temperature (5777 K).

Solution. From Table 10.7, \( f = 0.10 \) at \( \lambda T = 2195 \) \( \mu m \cdot K \) and \( f = 0.90 \) at \( \lambda T = 9376 \) \( \mu m \cdot K \). Dividing by the absolute temperatures gives:

<table>
<thead>
<tr>
<th>( T ) [K]</th>
<th>( \lambda_{0.1} ) [( \mu m )]</th>
<th>( \lambda_{0.9} ) [( \mu m )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>7.317</td>
<td>31.25</td>
</tr>
<tr>
<td>5777</td>
<td>0.380</td>
<td>1.62</td>
</tr>
</tbody>
</table>

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**Problem 10.54**: Read Problem 10.52. A crystalline silicon solar cell can convert photons to conducting electrons if the photons have a wavelength less than $\lambda_{\text{band}} = 1.11$ μm, the bandgap wavelength. Longer wavelengths do not produce an electric current, but simply get absorbed and heat the silicon. For a solar cell at 320 K, make a rough estimate of the fraction of solar radiation on wavelengths below the bandgap? Why is this important?

**Solution.** The relevant temperature is that of the sun, 5777 K, not that of the solar cell. We approximate the sun as a blackbody at 5777 K, ignoring atmospheric absorption bands.

$$\lambda_{\text{band}} T = (1.11)(5777) \, \text{μm} \cdot \text{K} = 6412 \, \text{μm} \cdot \text{K}$$

Referring to Table 10.7, a bit less than 80% of solar energy is on these shorter wavelengths (with a more exact table, 77%). This is significant because the solar cell can convert less than 80% of the solar energy to electricity; additional considerations lower the theoretical efficiency still further, to less than 50%.
**Problem 10.55** Two stainless steel blocks have surface roughness of about 10 µm and \( \varepsilon \approx 0.5 \). They are brought into contact, and their interface is near 300 K. Ignore the points of direct contact and make a rough estimate of the conductance across the air-filled gaps, approximating them as two flat plates. How important is thermal radiation? Compare your result with Table 2.1 and comment on the relative importance of the direct contact that we ignored.

**Solution** The gaps are very thin, so little circulation will occur in the air. Heat transfer through the air will be by conduction. Radiation and conduction act in parallel across the gap. The temperature difference across the gap will likely be small, so we may use a radiation thermal resistance. The conductance is the reciprocal of the thermal resistance, per unit area, so \( h_{\text{gap}} = h_{\text{cond}} + h_{\text{rad}} \).

Letting the gap width be \( \delta = 10 \) µm and taking \( k_{\text{air}} = 0.0264 \) W/m·K, we can estimate

\[
h_{\text{cond}} \approx \frac{k}{\delta} = \frac{0.0264}{10 \times 10^{-6}} = 2,640 \text{ W/m}^2\text{K}
\]

With eqns. (2.29) and (10.25):

\[
\mathcal{F}_{1-2} = \left( \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 \right)^{-1} = \left( \frac{2}{0.5} - 1 \right)^{-1} = \frac{1}{3}
\]

\[
h_{\text{rad}} = 4\sigma T_{1}^3 \mathcal{F}_{1-2} = 4(5.67 \times 10^{-8})(300)^3 (0.3333) = 2.041 \text{ W/m}^2\text{K}
\]

Then

\[
h_{\text{gap}} = h_{\text{cond}} + h_{\text{rad}} = 2640 + 2.041 = 2,642 \text{ W/m}^2\text{K}
\]

This conductance is on the lower end of the range of given in Table 2.1. Conduction through contacting points will add significantly to the heat transfer, although it will be highly multidimensional and not easily calculated. Thermal radiation, however, is negligible.
11.1 Derive equations 11.9
\[ m_i = \frac{\rho_i}{\rho} = \frac{M_i C_i}{M C} = \frac{\chi_i M_i}{\sum_k \chi_k M_k} \quad \text{by eqns. (11.2, 4, 6, 8)} \]
\[ \chi_i = \frac{C_i}{C} = \frac{\rho_i}{\rho} \frac{M}{M_i} = \frac{(m_i/M_i)}{\sum (m_k/M_k)} \]

11.2 A 1000 liter cylinder at 300 K contains a gaseous mixture comprised of 0.10 kg/mole of NH₃, 0.04 kg/mole of CO₂, and 0.06 kg/mole of He. 

a.) Find the mass fraction each species and the pressure in the cylinder. 

b.) If the cylinder is heated to 600 K, what are the new mole fractions, mass fractions, and molar concentrations. 

c.) The cylinder is now compressed isothermally to a volume of 600 liters. What are the molar concentrations, mass fractions, and partial densities? 

d.) If 0.40 kg of gaseous N₂ is injected into the cylinder while the temperature remains at 600 K, find the mole fractions, mass fractions, and molar concentrations.

(a) By eqn. (11.6): \( \chi_{NH₃} = 0.1/0.2 = 0.5 \), \( \chi_{CO₂} = 0.04/0.2 = 0.2 \), \( \chi_{He} = 0.06/0.2 = 0.3 \). By eqn (11.8): \( M = (0.5)(17.03) + (0.2)(44.01) + (0.3)(4.003) = 18.52 \) kg/kgmole. Thus, from eqn. (11.9):

\[ m_{NH₃} = (0.5)(17.03)/18.52 = 0.46 \]
\[ m_{CO₂} = (0.2)(44.01)/18.52 = 0.475 \]
\[ m_{He} = (0.3)(4.003)/18.52 = 0.0648 \]

The pressure is: \( P = C R₀ T = (0.2 \text{ kgmole} / \text{m}^3)(8314.3 \text{ J/kgmoleK}) \cdot (300 \text{ K}) = 4.99 \times 10^5 \text{ Pa} = 4.92 \text{ atm} \)

(b) The mass fractions are unchanged. The total pressure, like the temperature, is doubled: \( P = 2 \times 4.92 = 9.84 \) atm.

By eqn. (11.17):
\[ P_{NH₃} = (0.5)(4.92) = 2.46 \text{ atm} \]
\[ P_{CO₂} = (0.2)(4.92) = 0.984 \text{ atm} \]
\[ P_{He} = (0.3)(4.92) = 1.476 \text{ atm} \]

(c) The mass fractions are still unchanged. The molar concentrations are:
\[ c_{NH₃} = (0.1 \text{ kgmole} / 0.6 \text{ m}^3) = 0.167 \text{ kgmole} / \text{m}^3 \]
\[ c_{CO₂} = (0.04 \text{ kgmole} / 0.6 \text{ m}^3) = 0.0667 \text{ kgmole} / \text{m}^3 \]
\[ c_{He} = (0.06 \text{ kgmole} / 0.6 \text{ m}^3) = 0.100 \text{ kgmole} / \text{m}^3 \]
By eqn. (11.4), the partial densities are:

\[ \rho_{\text{NH}_3} = (0.167)(17.03) = 2.84 \text{ kg/m}^3 \]
\[ \rho_{\text{CO}_2} = (0.0667)(44.01) = 2.94 \text{ kg/m}^3 \]
\[ \rho_{\text{He}} = (0.100)(4.003) = 0.400 \text{ kg/m}^3 \]

(d) We have added \((0.4 \text{ kg})/(28.01 \text{ kg/kg mole}) = 0.0143 \text{ kg/mole} \) \(\text{N}_2\).

\[ \chi_{\text{NH}_3} = 0.1 / 0.2143 = 0.467 \]
\[ \chi_{\text{CO}_2} = 0.04 / 0.2143 = 0.187 \]
\[ \chi_{\text{He}} = 0.06 / 0.2143 = 0.280 \]
\[ \chi_{\text{N}_2} = 0.0143 / 0.2143 = 0.0667 \]

The molecular weight of the mixture is:

\[ M = (0.467)(17.03) + (0.187)(44.01) + (0.280)(4.003) + (0.0667)(28.01) = 19.17 \text{ kg/kg mole} \]

Thus,

\[ M_{\text{NH}_3} = (0.467)(17.03)/(19.17) = 0.415 \]
\[ M_{\text{CO}_2} = (0.187)(44.01)/(19.17) = 0.429 \]
\[ M_{\text{He}} = (0.280)(4.003)/(19.17) = 0.0585 \]
\[ M_{\text{N}_2} = (0.0667)(28.01)/(19.17) = 0.0975 \]

The molar concentrations of part (c) are unchanged.

\[ C_{\text{N}_2} = 0.0143 \text{ kg/mole}/0.6 \text{ m}^3 = 0.0238 \text{ kg/mole/m}^3 \]

11.3 Planetary atmospheres show significant variations of temperature and pressure in the vertical direction. Observations suggest that the atmosphere of Jupiter has the following composition at the tropopause level:

- Number density (molecules/m³) of \( \text{H}_2 = 5.7 \times 10^{21} \)
- \( \text{He} = 7.2 \times 10^{20} \)
- \( \text{CH}_4 = 6.5 \times 10^{18} \)
- \( \text{NH}_3 = 1.3 \times 10^{18} \)

Find the mole fraction and partial density of each species at this level if \( p = 0.1 \text{ atm} \) and \( T = 113^\circ \text{K} \). Estimate the number densities at the level where \( p = 10 \text{ atm} \) and \( T = 400^\circ \text{K} \), deeper within the Jovian troposphere. (Surface pressures on Jupiter may actually be as high as \( 10^5 \) atm.)

The number density \( N = \sum_{i} N_i = 6.428 \times 10^{21} \) molecules/m³. The molar concentration is:

\[ C = N/N_A = (6.428 \times 10^{21})/(6.0225 \times 10^{23}) \]

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= 1.067 \times 10^{-5} \text{ kg/m}^3, \text{ by eqn (12.10). The partial molar concentrations are: } C_{H_2} = (6.7 \times 10^{21})/(6.0225 \times 10^{23}) = 9.46 \times 10^{-6} \text{ kg/m}^3, C_{He} = (7.2 \times 10^{20})/(6.0225 \times 10^{23}) = 1.19 \times 10^{-6} \text{ kg/m}^3, C_{CH_4} = 1.079 \times 10^{-6} \text{ kg/m}^3, C_{NH_3} = 2.159 \times 10^{-9} \text{ kg/m}^3. \text{ From eqn (12.6), we find: } \\
\chi_{H_2} = 9.464 \times 10^{6}/1.067 \times 10^{5} = 0.887 \\
\chi_{He} = 1.196 \times 10^{6}/1.067 \times 10^{5} = 0.112 \\
\chi_{CH_4} = 1.079 \times 10^{-6}/1.067 \times 10^{5} = 0.001 \\
\chi_{NH_3} = 2.159 \times 10^{-6}/1.067 \times 10^{5} = 0.002 \\

\text{From eqn (12.4), the partial densities are } \rho_i = C_i M_i: \\
\rho_{H_2} = (9.46 \times 2.04)\times 10^{-6} = 1.9 \times 10^{-5} \text{ kg/m}^3 \\
\rho_{He} = (1.19 \times 4.03) \times 10^{-6} = 4.8 \times 10^{-6} \text{ kg/m}^3 \\
\rho_{CH_4} = (1.08 \times 16.04) \times 10^{-6} = 1.7 \times 10^{-5} \text{ kg/m}^3 \\
\rho_{NH_3} = (2.16 \times 17.03) \times 10^{-6} = 3.7 \times 10^{-6} \text{ kg/m}^3 \\

\text{To estimate the number densities at lower levels, assume that relative amounts of each species don't change, i.e. the mole fractions are the same. We have: } N_i = C_i N_A = \frac{x_i N_A}{C} \\

\text{From eqn (12.15): } C = \rho / R^* T = 10(101325)/(8314.3)(400) = 0.305 \text{ kg/m}^3 \\
\text{So: } N_{H_2} = 1.8 \times 10^{27} \text{ molecules/m}^3, N_{CH_4} = 2.0 \times 10^{24} \text{ molecules/m}^3, N_{He} = 2.2 \times 10^{26} \text{ molecules/m}^3, N_{NH_3} = 3.9 \times 10^{23} \text{ molecules/m}^3.

11.4 \text{ Using the definitions of the fluxes, velocities, and concentrations, derive equation (11.29) from equation (11.26) for binary diffusion} \\
\text{From eqns (12.21, 2.7, 31, 38): } \mathbf{J}_1^k = -\rho D_{ij} \nabla x_i = \rho_i (\mathbf{\nabla} \mathbf{v}_1 - \mathbf{\nabla} \mathbf{v}) \\
\mathbf{J}_1^k = -c D_{ij} \nabla x_i = c_i (\mathbf{\nabla} \mathbf{v}_1 - \mathbf{\nabla} \mathbf{v}). \text{ With (12.18, 24), (12.3, 7):} \\
\mathbf{\nabla} \mathbf{v}_1 - \mathbf{\nabla} \mathbf{v} = \mathbf{\nabla} \mathbf{v}_1 - (x_1 \mathbf{\nabla} \mathbf{v}_1 + x_2 \mathbf{\nabla} \mathbf{v}_2) = \chi_2 (\mathbf{\nabla} \mathbf{v}_1 - \mathbf{\nabla} \mathbf{v}_2) \\
\mathbf{\nabla} \mathbf{v}_1 - \mathbf{\nabla} \mathbf{v} = \mathbf{\nabla} \mathbf{v}_1 - (m_1 \mathbf{\nabla} \mathbf{v}_1 + m_2 \mathbf{\nabla} \mathbf{v}_2) = m_2 (\mathbf{\nabla} \mathbf{v}_1 - \mathbf{\nabla} \mathbf{v}_2)
Thus: \[ \vec{J}^* = c_1 x_z (\vec{\jmath}_1 - \vec{\jmath}_2) = c_1(m_2 M/M_z) (\vec{\jmath}_1 - \vec{\jmath}_2) = \frac{M}{M_1 M_z} \vec{J}_1 \]

where we use the equation before (11.25). It also gives us

\[ \nabla m_1 = \nabla \left( \frac{x_1 m_1}{M_1} \right) = \frac{M_1}{M} \nabla x_1 - \frac{x_1 m_1}{M^2} \left\{ m_1 \nabla x_1 + m_z \nabla x_z \right\} \]

\[ = \frac{M_1}{M} \left\{ 1 - \frac{x_1}{m_1} M_1 + \frac{x_1}{m_1} M_z \right\} \nabla x_1 = \frac{M_2 M_1}{M^2} \nabla x_1 \]

(The relation \( x_1 + x_z = 1 \) and eqn (11.8) were used here, also)

Hence, \[ \frac{\vec{J}}{M_{1, M_z}} (- \rho D_{12} \nabla m_1) = -c D_{12} \left( \frac{m_2^2}{m_{1, M_z}} - \frac{M_2 M_1}{M^2} \nabla x_1 \right) \]

\[ \vec{J}^* = -c D_{12} \nabla x_1 \]

11.5 Show that \( D_{12} = D_{21} \) in a binary mixture.

From eqns. (11.25, 26): \[ \frac{\vec{J}}{M_{1, M_2}} + \frac{\vec{J}}{M_{2, M_2}} = 0 \Rightarrow D_{12} \nabla m_1 = -D_{21} \nabla m_2 \]

But: \( \nabla m_1 = \nabla (1 - m_z) = -\nabla m_z \). Thus: \( D_{12} \nabla m_1 = -D_{21} \nabla m_2 \)

\( = -D_{21} (-\nabla m_1) = D_{21} \nabla m_1 \). Therefore: \[ D_{12} = D_{21} \]

11.6 Fill in the details involved in obtaining equation (11.32) from equation 11.31.

The Taylor expansions are: \[ \frac{N_A}{N} \bigg|_{x_0 \pm a} = \frac{N_A}{N} \bigg|_{x_0} + \frac{d}{dx} \left( \frac{N_A}{N} \right) \bigg|_{x_0} (\pm a) \]

+ \( O(a^2) \), so that we obtain

\[ \left\{ \left( \frac{N_A}{N} \right) \bigg|_{x_0 - a} - \left( \frac{N_A}{N} \right) \bigg|_{x_0 + a} \right\} \approx -2a \frac{d}{dx} \left( \frac{N_A}{N} \right) \bigg|_{x_0} \]

to an error \( O(a^3) \) (not \( a^2 \)). Thus, the flux through the plane \( x = x_0 \) is: \[ \vec{j}_a = -2 \eta a (\nu \vec{\epsilon} \vec{\ell}) \frac{M_A}{N_A} \frac{d}{dx} \left( \frac{N_A}{N} \right) \]

Now:

\[ \frac{d}{dx} \left( \frac{N_A}{N} \right) = \frac{d}{dx} \left( \frac{\rho a M A_N}{N A} \right) = \frac{d}{dx} \left( \frac{M_A M}{M_A} \right) = \frac{M}{M_A} \frac{dM_A}{dx} + \]

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\[
\frac{M_A}{M_A} \frac{d}{dx} M = \frac{M_A}{M_A} \frac{dM_A}{dx} + \frac{M_A}{M_A} \frac{d}{dx} \left( x_A M_A + x_A' M_A' \right) \\
= (M_A - M_A') \frac{d}{dx} x_A = 0
\]

because we assume identical molecules. Eqn. (11.34) follows.

11.7 Batteries commonly contain an aqueous solution of sulfuric acid with lead plates as electrodes. Current is generated by the reaction of the electrolyte with the electrode material. At the negative electrode, the reaction is

\[
Pb(s) + SO_4^{2-} + PbSO_4 + 2e^-
\]

where the (s) denotes a solid phase component and the charge of an electron is \(1.609 \times 10^{19}\) Coulombs. If the current density at such an electrode is \(i = 5\) milliamperes/cm\(^2\), what is the mole flux of \(SO_4^{2-}\) to the electrode? (1 Amp = 1 Coulomb/s.) What is the mass flux of \(SO_4^{2-}\)? At what mass rate is \(PbSO_4\) produced? If the electrolyte is to remain electrically neutral, at what rate does \(H^+\) flow toward the electrode? Hydrogen does not react at the negative electrode.

The current is:

\[
i = \text{(charge per ion)(no. ions per kg mole)(flux of ions at electrode surface, kg mole/m}^2\text{.s)}
\]

From which

\[
(5 \times 10^{-3} A) / (10^{-4} m^2) = i = 2(1.609 \times 10^{-19} C)(N_A)(N_{SO_4^{2-}})
\]

Solving, we have:

\[
N_{SO_4^{2-}} = \frac{(5 \times 10^1)}{2(1.609 \times 10^{19}) (6.0225 \times 10^{23})} = 2.58 \times 10^{-7} \text{ kg mole/m}^2\text{.s}
\]

The mass flux is:

\[
n_{SO_4^{2-}} = M_{SO_4^{2-}} \cdot N_{SO_4^{2-}} = (96.06)(2.58 \times 10^{-7})
\]

One mole of \(PbSO_4\) is formed for every mole of \(SO_4^{2-}\) reaching the electrode. Thus:

\[
m_{PbSO_4} = \frac{M_{PbSO_4} \cdot N_{SO_4^{2-}}}{(303.3)(2.58 \times 10^{-7})}
\]

In steady state, the hydrogen ions will be effectively stationary near the negative electrode. \(H^+\) doesn't react and \(SO_4^{2-}\) diffuses past it. Thus:

\[
N_{H^+, s} = 0
\]
11.8 The salt concentration in the ocean increases with increasing depth, $z$. A model for the concentration distribution in the upper ocean is $S = 33.25 + 0.75 \tanh(0.026z - 3.7)$ where $S$ is the salinity in grams of salt per kg of ocean water and $z$ is the distance below the surface in m. a.) Plot the mass fraction of salt as a function of $z$. (the region of rapid transition of $m_{salt}(z)$ is called the halocline.) b.) Ignoring the effects of oceanic motion, compute $j_{salt}(z)$. Use a value of $D_{salt}$, water $= 1.5 \times 10^{-5}$ cm$^2$/s. Indicate the position of maximum diffusion on your plot of the salt concentration. c.) The upper region of the ocean is well-mixed by wind-driven waves and turbulence while the lower region and halocline tend to be calmer. Using $j_{salt}(z)$ from b.), make a simple estimate of the amount of salt carried upward in one week in a 5 km$^2$ horizontal area of the sea.

![Mass fraction graph](image)

(a) Salinity, $S = 1000 \times m_{salt}$

(b) Under this assumption, just differentiate $m_{salt}(z)$ and use Fick’s Law, i.e. assume pure diffusion.

$$j_{salt}(z) = -\rho D_{12} \frac{d}{dz} m_{salt}$$

$$= 7.5 \times 10^{-9}(0.026) \text{sech}^2(0.026z - 3.7)(-\rho D_{12})$$

The density of salt-water may be approximated as having a uniform value:

$$\rho = \rho + \rho_{H_2O} = \rho_{H_2O}(1 + \frac{m_{salt}}{1 + m_{salt}}) \approx \rho_{H_2O}(1 + m_{salt})$$

We make the additional approximation that the partial density, $\rho_{H_2O}$, is the same as the density of pure water $\approx 999 \text{ kg/m}^3$. So:

$$\rho \approx 999(1 + 0.03325) = 1032 \text{ kg/m}^3.$$

Then:

$$j_{salt}(z) = -3.02 \times 10^{-6} \frac{kg}{m \cdot s} \text{ sech}^2(0.026z - 3.7)$$

The flux is vertically upward. The point of maximum $j_{salt} is the maximum slope point of $m_{salt}$ (at A):

$$z_A = 0.026 \times 142 \text{ m.}$$

(c) The flat form of the salt concentration profile in the upper region may be attributed to turbulent mixing. (The fresher
water near the surface originates from rainfall and nearby land masses.) We may reason that salt diffuses slowly from lower water into the fresher water above at a rate given approximately by \( j_{\text{salt max}} \): transport is primarily diffusive at this level and the upward flux shouldn't exceed this value. Then:

\[
\text{mass flow upward per 5 sq km per week} = j_{\text{max}} \, \text{sq km/1 wk} = (3.02 \times 10^{-11} \, \text{kg/m}^2\text{s})(1000 \, \text{m})^2(3600 \times 24.7) \times 5 = 91.3 \, \text{kg}
\]

This flux complicates oceanographic modeling.

11.9 Butane reacts with hydrogen on the surface of a nickel catalyst to form methane and propane. This heterogeneous reaction, referred to as "hydrogenolysis," is

\[
\text{C}_4\text{H}_{10} + \text{H}_2 \xrightarrow{\text{Ni}} \text{C}_3\text{H}_8 + \text{CH}_4
\]

The molar rate of consumption of \( \text{C}_4\text{H}_{10} \) per unit area in the reaction is \( \dot{R}_{\text{C}_4\text{H}_{10}} = A \left( e^{-\frac{AE}{RT}} \right) p_{\text{C}_4\text{H}_{10}} p_{\text{H}_2} \), where

\[ A = 6.3 \times 10^{10} \, \text{kg mole/m}^2\text{s} \text{ and } AE = 1.9 \times 10^8 \, J/\text{kg mole}. \]

(a) If \( p_{\text{C}_4\text{H}_{10}} = 0.2 \, \text{atm}, p_{\text{H}_2} = 0.17 \, \text{atm}, \) and \( p_{\text{H}_2} = 0.3 \, \text{atm} \) at a nickel surface with conditions of 440°C and 0.87 atm total pressure, what is the total rate of consumption of butane? (b) What are the mole fluxes of butane and hydrogen to the surface? What are the mass fluxes of propane and ethane away from the surface? (c) What is \( \dot{m} \)? What are \( v, v^m, \) and \( v_{\text{CH}_4} \)? (d) What is the diffusional mole flux of butane? What is the diffusional mass flux of propane? What is the flux of \( \text{Ni} \)?

(a) \( \dot{R}_{\text{C}_4\text{H}_{10}} = (6.3 \times 10^{10}) \exp \left\{ \frac{-1.9 \times 10^8}{(8.314.3)(440+273.15)} \right\} \times (0.2)(0.3)^{-2.4} \)

\[ = 2.746 \times 10^{-3} \, \text{kg mole/m}^2\text{s} \]

(b) From stoichiometry: \( \dot{R}_{\text{C}_4\text{H}_{10}} = -N_{\text{C}_4\text{H}_{10}} - N_{\text{H}_2} \), \( N_{\text{C}_3\text{H}_8} = N_{\text{CH}_4} = \dot{R}_{\text{C}_4\text{H}_{10}} \). Signs reflect fluxes of \( \text{C}_4\text{H}_{10} \), \( \text{H}_2 \) toward the surface and \( \text{C}_3\text{H}_8 \), \( \text{CH}_4 \) away. Because \( n_i = M_i N_i \) and \( M_{\text{C}_3\text{H}_8} = 44.09 \, \text{kg/kg mole}, M_{\text{CH}_4} = 16.04 \, \text{kg/kg mole}, \) we have

\[
N_{\text{C}_3\text{H}_8} = (44.09)(2.746 \times 10^{-3}) = 0.1211 \, \text{kg/m}^2\text{s}
\]

\[
N_{\text{CH}_4} = (16.04)(2.746 \times 10^{-3}) = 0.04405 \, \text{kg/m}^2\text{s}
\]
(c) There is no net flux through the surface: \( \vec{V} = 0 \). There is no net mole flux through the surface, either: \( \vec{V}^m = \nabla C = 0 \).

\[
V_{C_{4}H_{10}} = N_{C_{4}H_{10}} / \rho_{C_{4}H_{10}} = N_{C_{4}H_{10}} / C_{C_{4}H_{10}} ; C_{C_{4}H_{10}} = V_{C_{4}H_{10}} / (R^T T) = (0.1225)(0.2) / (831.43)(175.315) = 3.23 \times 10^3 \text{ kg mole/m}^3. V_{C_{4}H_{10}} = \frac{0.5964 \times 10^3}{13.25 \times 10^3} = -0.4585 \text{ m/s toward the surface.}
\]

\[
\int \left. V_{C_{4}H_{10}} \right|_{0}^{s} = \frac{V_{C_{4}H_{10}}}{s} = -2.746 \times 10^3 \text{ kg mole/m}^2 \cdot \text{s}.
\]

\[
\int \frac{d}{dC_{3}H_{8}} \left( V_{C_{3}H_{8}} - \int \frac{d}{dC_{3}H_{8}} \right) = N_{C_{3}H_{8}} = 0.1211 \text{ kg/m}^2 \cdot \text{s}.
\]

Nickel does not have a net reaction. All fluxes of Ni are zero.

11.10 Consider two chambers held at temperatures \( T_1 \) and \( T_2 \), respectively, and joined by a small insulated tube. The chambers are filled with a binary gas mixture, with the tube open, and allowed to come to steady state. If the Soret effect is taken into account, what is the concentration difference between the two chambers? Assume an effective mean value of the thermal diffusion ratio is known.

In steady state, there is no mass flow between the chambers. Assuming the tube is too small to allow convection (recirculation)

Eqn. (11.36) gives:

\[
\frac{d}{dx} f_1 = 0 = \frac{d}{dx} \left[ \frac{M_1 M_2}{M_2} k_T \frac{d}{dx} \ln T \right]
\]

so

\[
\frac{d}{dx} M_1 = - \frac{M_1 M_2}{M_2} k_T \frac{d}{dx} \ln T \Rightarrow \int_{x_1}^{x_2} k_T \frac{d}{dx} \ln T \, dx = \frac{M_1 M_2}{M_2} k_T \ln \left( \frac{T_2}{T_1} \right)
\]

for a mean value \( k_T \). The difference in concentration between the chambers is:

\[
M_1(x_2) - M_1(x_1) = \frac{M_1 M_2}{M_2} k_T \ln \left( \frac{T_2}{T_1} \right)
\]
11.11 Compute $D_{12}$ for oxygen gas diffusing through nitrogen gas at $p = 1 \text{ atm}$ using equations (11.113) and (11.116) for $T = 200^\circ \text{K}$, $500^\circ \text{K}$, and $1000^\circ \text{K}$. Observe that (11.113) shows large deviations from equation (11.116) even for such similar molecules.

\[
\sigma_{\text{O}_2-\text{N}_2} = \frac{1}{2} (3.467 + 3.790) = 3.633 \text{ Å}
\]
\[
\frac{e}{k} = 106.7 \text{ kcal/°K}
\]
\[
T = 32.00 \text{ kg/mole}
\]

For eqn (11.113), use
\[
d = \sigma_{\text{O}_2-\text{N}_2} \text{ and } M = 30.0;
\]
convert $p$ to $N/m^2$ and $d$ to $m$, rather than $Å$. At $200^\circ \text{K}$, we have:

\[
D_{\text{AA}} = \frac{(1.38 \times 10^{-23}/\pi)^{3/2}}{(3.633 \times 10^{-10})^2} \left( \frac{6.0225 \times 10^{26}}{30.0} \right)^{3/2} \left( \frac{200}{1001 \times 2.5} \right)^{3/2} = 8.724 \times 10^{-6} \text{ m}^2/\text{s}
\]

\[
D_{\text{AB}} = \frac{(1.8583 \times 10^{-7})(200)^{3/2}}{\left( \frac{1}{1} \right)(3.633)^2(1.027)} \left( \frac{1}{28.01} + \frac{1}{32.00} \right)^{3/2} = 1.003 \times 10^{-5} \text{ m}^2/\text{s}
\]

<table>
<thead>
<tr>
<th>$T$</th>
<th>$D_{\text{AA}}$</th>
<th>$D_{\text{AB}}$</th>
<th>(D_{\text{O}_2-\text{N}_2}(T)) in $\text{m}^2/\text{s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>$8.724 \times 10^{-6}$</td>
<td>$1.003 \times 10^{-5}$</td>
<td>0.87</td>
</tr>
<tr>
<td>500</td>
<td>$3.446 \times 10^{-5}$</td>
<td>$4.763 \times 10^{-5}$</td>
<td>0.69</td>
</tr>
<tr>
<td>1000</td>
<td>$9.754 \times 10^{-5}$</td>
<td>$1.575 \times 10^{-4}$</td>
<td>0.62</td>
</tr>
</tbody>
</table>

11.12 a.) Compute the binary diffusivity of each of the noble gases when they are individually mixed with nitrogen gas as 1 atm and 300°K. Plot the results as a function of the molecular weight of the noble gas. What do you conclude? b.) Consider the addition of a small amount of helium ($x_{\text{He}} = 0.04$) to a mixture of nitrogen ($x_{\text{N}_2} = 0.48$) and argon ($x_{\text{Ar}} = 0.48$). Compute $D_{\text{He},\text{m}}$ and compare it with $D_{\text{Ar},\text{m}}$. Note that the higher concentration of argon does not improve its ability to diffuse through the mixture.

<table>
<thead>
<tr>
<th>$\sigma$ (Å)</th>
<th>$e/k$ (°K)</th>
<th>$M$ (kg/mole)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ar</td>
<td>He</td>
<td>Kr</td>
</tr>
</tbody>
</table>
| 3.542 | 2.551 | 3.655 | 2.820 | 4.047 | 3.799
| 93.3 | 10.22 | 118.9 | 32.8 | 231.0 | 71.4
| 39.95 | 4.003 | 83.80 | 20.18 | 131.1 | 28.01
Compute $D_{AB}$ \((\mathcal{E}/k)_A\) for each gas in collision with $N_2$ from eqns. on page 676; read $\Omega_d$ from Table 11.3 and compute $D_{AB}$:

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{A} & \text{He} & \text{Kr} & \text{Ne} & \text{Xe} \\
\hline
\mathcal{E}/k_A & 3.670 & 3.175 & 3.727 & 3.309 & 3.923 \\
\hline
\text{He} & 27.01 & 113.0 & 40.39 & 128.4 & 2.336 \\
\hline
\Omega_d & 0.7337 & 0.9819 & 0.8074 & 1.021 & 1.959 & 6.978 & 1.546 & 3.187 & 1.279 \\
\hline
\end{array}
\]

Lighter gases diffuse more easily.

b) From eqn. (11.116), $D_{He,Ar} = 7.310 \times 10^{-5} \text{ m}^2/\text{s}$.

From eqn. (11.118):

\[
D_{He,Ar} = \left[ \frac{0.48}{7.310} + \frac{0.48}{6.978} \right] \times 10^{-5} \text{ m}^2/\text{s} = 7.436 \times 10^{-5} \text{ m}^2/\text{s}
\]

Since helium is a trace gas, eqn. (11.119) gives: $D_{Ar,He} = D_{Ar,N_2} = 1.959 \times 10^{-5} \text{ m}^2/\text{s}$

The heavier Ar still diffuses more slowly than He.

11.13 a.) One particular correlation shows that gas phase diffusion coefficients vary as $T^{1.81}$ and $p^{-1}$. If an experimental value of $D_{12}$ is known at $T_1$ and $p_1$, develop an equation to predict $D_{12}$ at $T_2$ and $p_2$. b.) The diffusivity of water vapor (1) in air (2) was measured to be $2.39 \times 10^{-5}$ m$^2$/s at 80°C and 1 atm. Provide a formula for $D_{12}(T,p)$.

(a) $D_{12} \propto p^{-1} \cdot T^{1.81}$ so $D_{12}(p,T) = D_0 p^{-1} T^{1.81}$ for a constant $D_0$. Thus, $D_{12}(p,T)/D_{12}(p_{ref},T_{ref}) = (p/p_{ref})^{-1} (T/T_{ref})^{1.81}$.

or:

\[
D_{12}(p,T) = D_{12}(p_{ref},T_{ref}) \left( \frac{p_{ref}}{p} \right) \left( \frac{T}{T_{ref}} \right)^{1.81}
\]

(b) $D_{12}(p,T) = (2.39 \times 10^{-5} \text{ m}^2/\text{s}) \left( \frac{4 \text{ atm}}{p} \right) \left( \frac{T}{281 \text{ K}} \right)^{1.81}$
11.14 Kinetic arguments lead to the Stefan-Maxwell equation for a
dilute-gas mixture:

\[
\nabla x_i = \sum_{j=1}^{n} \frac{c_i c_j}{c^2 D_{ij}} \left( \frac{j_j^*}{c_j} - \frac{j_i^*}{c_i} \right)
\]

a.) Derive equation (11.118) from this, making the appropriate
assumptions. b.) Show that if \( \sigma_{ij} \) has the same value for
each pair of species, then \( \sigma_{im} = \sigma_{ij} \).

(a) \( \vec{j}_j^* \approx 0 \) for \( j \neq i \) so:

\[
\nabla x_i = \sum_{j=1}^{n} \frac{c_i c_j}{c^2 D_{ij}} \left( \frac{j_j^*}{c_j} - \frac{j_i^*}{c_i} \right) = -\frac{j_i^*}{c_i} \sum_{j=1}^{n} \frac{c_i}{c_j}
\]

or, with Fick's Law:

\[
\nabla x_i = (\nabla \text{Dim} \nabla x_i) \left( \sum_{j=1}^{n} \frac{c_j}{c^2 D_{ij}} \right) \cdot \text{Dim} = \left\{ \sum_{j=1}^{n} \frac{x_j}{D_{ij}} \right\}^{-1}
\]

(b) In this case, \( D_{ij} \) is independent of \( i \) and \( j \), so:

\[
\nabla x_i = \frac{1}{c^2 D_{ij}} \left( c_i \sum_{j=1}^{n} \frac{j_j^*}{c_j} - \frac{j_i^*}{c_i} \sum_{j=1}^{n} c_j \right) = -\frac{j_i^*}{c D_{ij}}
\]

With Fick's Law, \( j_i^* = -c \text{Dim} \nabla x_i \) and \( D_{ij} = \text{Dim} \).

11.15 Compute the diffusivity of methane in air using a.) equation
(11.118) and b.) Blanc's Law. For part b.) ignore argon; use
\( x_{\text{methane}} = 0.05, T = 420^\circ \text{F}, \) and \( p = 10 \) psia.

a) \( \sigma_{\text{CH}_4-\text{air}} = 3.735 \times 10^{-7}, (\varepsilon/k)_{\text{CH}_4-\text{air}} = 108.1 \) K from eqns. (11.114/5).

\[ kT/\varepsilon = 4.523 \] so, from Table 12.2, \( \Omega = 0.8601 \). The pressure
\( p = (10/14.696) = 0.6805 \) atm. Eqn (12.43) gives:

\[
D_{\text{CH}_4-\text{air}} = \left( \frac{1.8563 \times 10^{-7}}{(4.869)^{3/2}} \right) \left( \frac{0.6805}{0.8601} \right)^{3/2} \sqrt{\frac{4}{28.96} + \frac{4}{16.04}}
\]

\[ = 7.658 \times 10^{-5} \text{ m}^2/\text{s} \]

b) We need \( D_{N_2-\text{CH}_4}, D_{O_2-\text{CH}_4}, \chi_{O_2}, \) and \( \chi_{N_2} \) for eqn. (11.118).

\( O_2-\text{CH}_4 \): \( \sigma = 3.613 \times 10^{-7}, (\varepsilon/k) = 125.9 \) K, \( \Omega = 0.8097 \cdot D = 7.476 \times 10^5 \text{ m}^2/\text{s} \)

\( N_2-\text{CH}_4 \): \( \sigma = 3.778 \times 10^{-7}, (\varepsilon/k) = 103.0 \) K, \( \Omega = 0.8572 \cdot D = 7.608 \times 10^5 \text{ m}^2/\text{s} \)

\( \chi_{\text{air}} = 0.95 \), so that: \( \chi_{N_2} = 0.70/0.95 = 0.74, \chi_{O_2} = 0.21/0.95 = 0.22 \)

Thus, eqn. (11.118) yields:
11.16 Diffusion of solutes in liquids is driven by the chemical potential, \( \mu \). Work is required to move a mole of solute A from a region of low chemical potential to a region of high chemical potential, i.e.

\[
dW = d\mu_A = \frac{d\mu_A}{dx}
\]

under isothermal, isobaric conditions. For an ideal (very dilute) solute, \( \mu_A \) is given by

\[
\mu_A = \mu_0 + RT \ln(c_A)
\]

where \( \mu_0 \) is a constant. Using an elementary principle of mechanics, derive the Nernst-Einstein equation. Note that the solution must be assumed to be very dilute.

From mechanics, if \( F_m \) is the force resisting the motion, \( dW = -F_m \, dx \). Thus, \( F_m = -\frac{dW}{dx} \) is the force per mole resulting from the potential gradient. \( F_m \) is opposed by the drag on the solute. \( dF_A \) is drag per molecule, \( F_m = -F_A \cdot N_A \), \( N_A \), Avagadro. Thus, \( F_A = -(R^0T/C_A \cdot N_A) \frac{dc_A}{dx} \). From Sections 11.263 \( J_A = -c \, D_A \frac{d(c_A)}{dx} = c_A (V_A - \nu^*) \). Since the solution is very dilute: (i) \( c = c_B \) constant, (ii) \( \nu^* = x_A \cdot V_A + x_B \cdot V_B = V_B = V \), the mass average velocity. Collecting these results, we have

\[
(V_A - \nu^*) = (V_A - V) = \frac{c \, D_{AB} \frac{d(c_A)}{dx}}{c_A} = -D_{AB} \frac{dc_A}{dx} = \frac{D_{AB} \cdot F_A \cdot N_A}{R^0T}
\]

Steady average velocity of solute relative to solvent = \( V_A \) in (12.44)

Thus,

\[
D_{AB} = k_B T \left( \frac{V_A - V}{F_A} \right)
\]

11.17 A dilute aqueous solution at 300°C contains potassium ions, \( K^+ \). If the velocity of aqueous \( K^+ \) ions is \( 6.61 \times 10^{-4} \text{ cm}^2/\text{s} \), estimate the effective radius of \( K^+ \) ions in an aqueous solution. Criticize this estimate. The charge of an electron is \( -1.609 \times 10^{19} \text{ C} \) and \( V = 5 \text{ V} \).

The force on a \( K^+ \) ion in a 1 V/cm electric field is \( F = qE = (1.609 \times 10^{19} \text{ C})(1 \text{V/cm}) = 1.609 \times 10^{17} \text{ N} \) so that the mobility of a \( K^+ \) ion is
11.18 a.) Obtain the following diffusion coefficients: (i) for
dilute CCl₄ diffusing through liquid methanol at 340°K, (ii)
for dilute benzene diffusing through water at 290°K,
(iii) for dilute ethyl alcohol diffusing through water at
350°K, and (iv) for dilute acetone diffusing through methanol
at 370°K. b.) Estimate the effective radius of a methanol
molecule in a dilute aqueous solution.

a) Use eqn (11.124) and App. A. (i) μₘethanol = 3.3×10⁻⁴ kg/m·s

\[ D_{CCl₄-meth} = \frac{(340)}{(3.3 \times 10^{-4})} \left(4.4 \times 10^{-15}\right) \left(\frac{0.042}{0.102}\right) \left(\frac{35.53}{29.93}\right)^{1/2} \]

= 4.3×10⁻⁹ m²/s

(ii) μ₇H₂O = 1.123×10⁻³ kg/m·s. Eqn (11.124) gives:

\[ D_{benzene-H₂O} = 9.94×10⁻¹⁰ m²/s \]

(iii) μ₇H₂O = 3.65×10⁻⁴ kg/m·s. Eqn (11.124) gives:

\[ D_{ethyl-H₂O} = 3.49×10⁻⁹ m²/s \]

(iv) μ₇meth = 2.4×10⁻⁴ kg/m·s. Using eqn (11.124) we have:

\[ D_{acet.-meth} = \frac{(370)}{(2.4 \times 10^{-4})} \left(4.4 \times 10^{-15}\right) \left(\frac{0.042}{0.074}\right) \left(\frac{35.53}{28.90}\right)^{1/2} \]

= 6.8×10⁻⁹ m²/s

b) Use eqns (11.123b) and (11.124). From (11.124), Dμ/T =

4.11×10⁻¹⁵ kg/m·s². Solving (11.123b) for Rₗmeth we have:

\[ Rₗ_{meth} = \frac{k_B}{4\pi} \left(\frac{D_{meth-H₂O}μ_{H₂O}}{T}\right)^{-1} = \frac{1.3805×10^{-²³}}{4\pi (4.11 \times 10^{-¹⁵})} = 2.67×10^{-¹⁰} m \]

= 2.67 Å
11.20 a.) Show that \( k = (5/2)\mu c_V \) for a monatomic gas. b.) Obtain Eucken’s formula for the Prandtl number of a dilute gas

\[
Pr = 4\gamma/(9\gamma - 5)
\]

c.) Recall that for an ideal gas \( \gamma = (D + 2)/D \) where \( D \) is the number of modes of energy storage of its molecules. Obtain an expression for \( Pr \) as a function of \( D \) and describe what it means. d.) Use Eucken’s formula to compute \( Pr \) for gaseous Ar, N\(_2\), and H\(_2\)O. Compare the result to data in Appendix A over the range of temperatures. Explain the results obtained for steam as opposed to Ar and N\(_2\). (Note that for each mode of vibration there are two modes of energy storage, but that vibration is normally inactive until \( T \) is very high.)

(a) The simplest approach is to divide eqn. (11.125) into (11.126)

\[
k = \left( \frac{0.083143}{2.6843} \right) \left( \frac{\mu}{M} \right) = \left( 3.118 \times 10^4 \right) \text{ J/kg K} \left( \frac{\mu}{M} \right)
\]

For a monatomic gas

\[
c_V = \frac{3}{2}(kR/M) \quad \text{and} \quad \frac{5}{2}(kR) = 3.118 \times 10^4 \text{ J/kg K}
\]

Therefore,

\[
k = \frac{5}{2} \mu c_V
\]

(b) The Eucken correction (11.127) gives

\[
k = \left( \frac{9\gamma - 5}{4\gamma} \right) \mu c_p
\]

and \( Pr = (\mu c_p/k) \) so:

\[
Pr = \left( \frac{4\gamma}{9\gamma - 5} \right)
\]

(c) Substituting the expression stated into Eucken’s formula for \( Pr \), we have:

\[
Pr = \frac{4(D+2)}{9(D+2)-5D} = \frac{2(D+2)}{2D+9}
\]

\( Pr \) tends to unity as \( D \) becomes large, as for complex molecules and at high temperature when vibrational modes become more important.

(d) The value of \( \gamma \) must be specified. For monatomic Ar, \( D = 3 \) (3 translational modes) so \( \gamma = 5/3 \) and \( Pr = 2/3 \). For diatomic N\(_2\) we add two rotational modes, \( D = 5 \) and \( \gamma = 7/5 \) so \( Pr = 28/85 = 14/19 \). For triatomic H\(_2\)O, we have an additional rotational mode, so \( D \) is at least 6. and
\[ Pr = 16/21. \] \( H_2O \) may also be more susceptible to vibrational modes.

<table>
<thead>
<tr>
<th></th>
<th>Ar</th>
<th>( N_2 )</th>
<th>( H_2O )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eucken ( Pr ):</td>
<td>0.666</td>
<td>0.757</td>
<td>0.762</td>
</tr>
<tr>
<td>Data:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100°K</td>
<td>1.000</td>
<td>100°K</td>
<td>0.767</td>
</tr>
<tr>
<td>311 K</td>
<td>0.676</td>
<td>300</td>
<td>0.715</td>
</tr>
<tr>
<td>533</td>
<td>0.664</td>
<td>500</td>
<td>0.696</td>
</tr>
<tr>
<td>811</td>
<td>0.660</td>
<td>800</td>
<td>0.713</td>
</tr>
<tr>
<td>1089</td>
<td>0.663</td>
<td>1100</td>
<td>0.723</td>
</tr>
<tr>
<td>1500</td>
<td>0.671</td>
<td>1500</td>
<td>0.704</td>
</tr>
</tbody>
</table>

The agreement for Ar and \( N_2 \) is within about 5% over the whole range. The agreement for \( H_2O \) is poor. The underprediction may result from active vibrational modes.

11.21 A student is studying the combustion of a premixed gaseous fuel with the molar composition: 10.3 percent methane, 15.4 percent ethane, and 74.3 percent oxygen. She passes 0.006 ft\(^3\)/s of the mixture (at 70°F and 18 psia) through a smooth 3/8 inch ID tube, 47 inches long. a) What is the pressure drop? b) The student’s advisor recommends pre-heating the fuel mixture, using a nichrome strip heater wrapped around the last 5 inches of the duct. If the heater produces 0.8 W/inch, what is the wall temperature at the outlet of the duct? Let \( c_p, CH_4 = 2280 \text{ J/kg} \cdot \text{K}, \gamma_{CH_4} = 1.3, c_p, C_2H_6 = 1730 \text{ J/kg} \cdot \text{K}, \gamma_{C_2H_6} = 1.2 \), and approximate the properties at the inlet conditions.

First determine the properties of the gas mixture at (70°F, 18 psia) = (294.4 K, 1.225 atm). Use (11.128,129) for the mixture. From Appendix A, \( \mu_{OL} = 2.032 \times 10^{-5} \text{ kg/m s}, \lambda_{OL} = 0.02629 \text{ W/m K}. \)

For \( CH_4, C_2H_6 \) use (11.125,127):

<table>
<thead>
<tr>
<th>( \sigma (\text{J}) )</th>
<th>( C_p (\text{K}) )</th>
<th>( M (\text{kg mol}) )</th>
<th>( kT/\nu )</th>
<th>( \rho_{\text{avg}} )</th>
<th>( \mu (\text{kg m s}^{-1}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( CH_4 )</td>
<td>3.758</td>
<td>148.6</td>
<td>16.04</td>
<td>1.981</td>
<td>1.179</td>
</tr>
<tr>
<td>( C_2H_6 )</td>
<td>4.443</td>
<td>215.7</td>
<td>30.07</td>
<td>1.365</td>
<td>1.368</td>
</tr>
</tbody>
</table>

and from the given data: \( k_{CH_4} = 0.03237 \text{ W/m K}, k_{C_2H_6} = 0.01944 \text{ W/m K} \) by (11.127). To compute \( \phi_{1j} \), let \( O_2 = 1, CH_4 = 2, C_2H_6 = 3 \). The results are: \( \phi_{12} = 0.9378, \phi_{13} = 1.484, \phi_{23} = 1.476, \phi_{21} = 1.015, \phi_{31} = 0.7226, \phi_{32} = 0.6645 \)
Representing the sums in the denominators of (11.128-9) as
\[ \Xi_i \equiv \frac{3}{j=1} \kappa_i \phi_{2j}, \] we have: \[ \Xi_1 = 1.068, \Xi_2 = 1.084, \Xi_3 = 0.7593. \]
Thus,
\[ \mu_\infty = \left\{ \frac{(0.743)(2.082)}{1.068} + \frac{(0.103)(1.102)}{1.084} + \frac{(0.154)(0.930)}{0.7593} \right\} \times 10^5 \]
\[ = 1.71 \times 10^{-5} \text{ kg/m s} \]
\[ k_\infty = \frac{0.0253}{\text{ W/m K}} \]

(a) Use eqn. (3.25): \[ \Delta p = (L/D)\left( \frac{\rho u^2}{2} \right) \phi, \] \[ u_{av} = \frac{m}{\rho A} \] where \[ \rho = 0.006 \text{ ft}^3/\text{s} = 1.70 \times 10^{-4} \text{ m}^3/\text{s} \] and \[ A = \frac{\pi}{4} \left( \frac{3}{8} \right)^2 \text{ m}^2 = 7.13 \times 10^{-5} \text{ m}^2, \] so that \[ u_{av} = 2.38 \text{ m/s}. \] The density of the mixture is, by (12.8) and the ideal gas law: \[ \Delta M = 30.06 \text{ kg/kg mol}, \] \[ \rho = (101.325)(1.125)(30.00)(8314.3)(294.4) = 1.524 \text{ kg/m}^3. \] The Reynolds number is: \[ Re_\infty = (1.524)(2.38)(3/8)(0.0254)/(1.71 \times 10^{-5}) \]
\[ = 2020, \text{ barely laminar}. \] From Fig. 7.6, \[ \phi = 64/Re_\infty = 0.0317 \]
so:
\[ \Delta p = \left( \frac{4\pi}{3\phi} \right) \left( \frac{1.524}{2}(2.38)^2 \right) (0.0317) = 1.72 \text{ N/m}^2 = 2.49 \times 10^3 \text{ psi} \]
The pressure drop is entirely negliqible.

(b) At the end of the tube we should have fully developed laminar flow: \[ Nu = 4.368, \text{ eqn. (7.21)}. \] From Ex. 7.1 and eqn. (7.21)
\[ T_w(L) = 294.4^\circ K + \frac{4 q_w c_p}{\kappa_\infty L} + \frac{11}{48} \frac{q_w D}{k}, \quad L = 5". \]
Now, \[ \kappa / k = k_{cp} / \kappa_p = 1/\phi. \] From App. A, \[ C_{p_\infty} = 919.8 \text{ J/kg K} \] and \[ C_{pm} = \Xi \kappa_i C_{p_i} = \frac{1}{M} \Xi \kappa_i C_{p_i} M_i = 1119 \text{ J/kg K}. \] Thus:
\[ T_w(\text{exit}) = 294.4^\circ K + q_w \left( \frac{11}{48} \frac{3/8}{(0.0254)^2} + \frac{1}{1119} \right)^2 \]
\[ = 294.4^\circ K + 0.09942 q_w \]
The wall heat flux is \[ q_w = (0.8 \text{ W/in})/\pi D = (0.8)/\pi (3/8)(0.0254)^2 \]
\[ = 1053 \text{ W/m}^2 \] so that \[ T_w(\text{exit}) = 399.1 K \]
11.22 a.) Work Problem 6.36. b.) A fluid is said to be incompressible if the density of a fluid particle does not change as it moves about in the flow (i.e., if $\frac{DP}{Dt} = 0$). Show that an incompressible flow satisfies $\nabla \cdot \vec{u} = 0$. c.) How does the condition of incompressibility differ from that of constant density? Describe a flow that is incompressible but which does not have "constant density."

(b) From Problem (6.36):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = \left( \frac{\partial \rho}{\partial t} + \vec{u} \cdot \nabla \rho \right) + \rho \nabla \cdot \vec{u} = 0$$

or

$$\frac{\partial \rho}{\partial t} = -\rho \nabla \cdot \vec{u} \quad \text{(For an incompressible fluid, we clearly have $\nabla \cdot \vec{u} = 0$.)}$$

(c) "Constant density" is generally taken to mean that $\rho$ is spatially uniform and constant in time. Thus, all fluid particles would have the same density. In an incompressible flow each particle may have a different, but unchanging, density. Thus, the flow may have spatial gradients of density, i.e. it may be 

stratified. The most significant example of an incompressible, stratified fluid is the ocean, which is colder and saltier at greater depths and thus shows increasing density as one moves downward from the surface.

11.23 Carefully derive equation (11.43). Note that $\rho$ is not assumed constant in (11.43).

Eqns. (11.39) and (11.41) give:

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot \rho_i \vec{u} - \nabla \cdot \frac{\rho_i}{\partial \rho_i \partial t} \vec{u} + \vec{r}_i \quad \text{(* 2nd term in incompressible form)}$$

Introduce Fick's Law and assume $\rho \vec{D}_{im}$ spatially uniform:

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot \rho_i \vec{u} = -\nabla \cdot (\rho \vec{D}_{im} \nabla m_i) + \vec{r}_i = +\rho \vec{D}_{im} \nabla^2 m_i + \vec{r}_i$$

Now: $\frac{DP}{Dt} = \rho \vec{D}_{ni} \frac{m_i}{Dt} + \frac{m_i}{Dt} \frac{DP}{Dt} = \rho \vec{D}_{mi}$, for incompressible flow
Thus:
\[
\frac{Dm_i}{Dt} = D_{m_i} \nabla^2 m_i + \dot{r}_i / \rho_i \quad (11.43)
\]

To get (12.66) let \( p_i \), \( D_{m_i} \) separately be independent of \((x, y, z)\):
\[-\nabla \cdot \bar{F}_i = +D_{m_i} \nabla (p \nabla m_i) = D_{m_i} \nabla^2 p_i \]

Substitute into (*) above to get:
\[
\frac{Dp_i}{Dt} = D_{m_i} \nabla^2 p_i + \dot{r}_i
\]

11.24 Derive the equation of species conservation on a molar basis, using \( c_i \) rather than \( \rho_i \). Also obtain an equation in \( c_i \) alone, analogous to equation (11.43), but without introducing incompressibility. What assumptions must be made to obtain the latter result?

\[
\frac{d}{dt} \int_{R} c_i \, dR = - \int_{S} \bar{N}_i \cdot d\vec{S} + \int_{R} \bar{R}_i \, dR \quad (\bar{R}_i = \text{molar rate of creation of species } i, \text{ kg mole/m}^3 \cdot \text{s})
\]

\[
= - \int_{S} \bar{c}_i \bar{\nabla} \cdot d\vec{S} - \int_{S} \bar{J}_i \cdot d\vec{S} + \int_{R} \bar{R}_i \, dR
\]

using (11.24) and (11.22). In the first line, the first term is the rate of storage, the second is the rate of inflow, and the third the rate of creation of species \( i \) in the region \( R \).

By Gauss’ theorem and the usual arguments, this leads to:
\[
\frac{\partial c_i}{\partial t} + \nabla \cdot (c_i \bar{\nabla} \bar{\nabla} \bar{J}_i \cdot d\vec{S}) = -\nabla \cdot \bar{J}_i + \bar{R}_i \quad \text{molar analog of (11.38)}.
\]

Now use Fick’s Law, \( \bar{J}_i = -cD_{m} \nabla \bar{r}_i \) and assume both \( c \) and \( D_{m} \) are independent of \((x, y, z)\). The boxed eqn becomes:
\[
\frac{\partial c_i}{\partial t} + \nabla \cdot (c_i \bar{\nabla} \bar{\nabla} \bar{J}_i \cdot d\vec{S}) = D_{m_i} \nabla^2 c_i + \bar{R}_i \quad \text{closest analog of (11.38)}
\]

The first two terms are not \( D_{m} \). The eqn may be recast to give \( \partial \bar{c}_i / \partial t \), but the result is rather awkward.
11.25 Find the following concentrations:  
(a) The mole fraction of air in solution with water at 50°C and 1 atm, exposed to an atmosphere at the same conditions. \( H = 4.88 \times 10^4 \) atm.  
(b) The mole fraction of ammonia in air above an aqueous solution with \( x_{NH_3} = 0.05 \) at 0.9 atm. and 40°C, and \( H = 1522 \) mm Hg.  
(c) The mole fraction of \( SO_2 \) in an aqueous solution at 150°C and 1 atm., if \( \rho_{SO_2} = 28.0 \) mm Hg and \( H = 1.42 \times 10^4 \) mm Hg.  
(d) The partial pressure of ethylene over an aqueous solution at 25°C and 1 atm. with \( x_{C_2H_4} = 1.75 \times 10^{-5} \) and \( H = 11.4 \times 10^3 \) atm.

(a) Under these conditions, the air is essentially dry and \( \rho_{air} \approx 1 \) atm. Thus, \( X_{air} = (1 \text{ atm})/(4.88 \times 10^4 \text{ atm}) = 2.05 \times 10^{-5} \) by Henry's Law, (11.45).

(b) From Henry's Law, \( \nu_{NH_3} = (1522 \text{ mm Hg})(0.05) = 76.1 \text{ mm Hg} \).  

(c) Again use Henry's Law: \( X_{SO_2} = \rho_{SO_2}/H = 28.0/1.42 \times 10^4 \)  

(d) \( \rho_{C_2H_4} = (11.4 \times 10^3 \text{ atm})(1.75 \times 10^{-5}) = 0.200 \)

11.26 Use a steam table to estimate:  
(a) The mass fraction of water vapor in air over water at 1 atm and 20°C, 50°C, 70°C, and 90°C.  
(b) The partial pressure of water over a 3 percent by weight aqueous solution of HCl at 50°C.  
(c) The boiling point at 1 atm of salt water with a mass fraction \( m_{NaCl} = 0.18 \).

(a) Using Raoult's law and the reasoning of example 11.3,  
\[ P_{H_2O} = P_{sat,H_2O}(T) \]  

<table>
<thead>
<tr>
<th>( T (°C) )</th>
<th>( P_{sat} (kPa) )</th>
<th>( X_{H_2O} )</th>
<th>( M_6 ) (kg/mole)</th>
<th>( M_{H_2O} ) (kg/mole)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>2.379</td>
<td>0.02308</td>
<td>28.71</td>
<td>0.01449</td>
</tr>
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<td>50</td>
<td>12.344</td>
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<td>27.63</td>
<td>0.07951</td>
</tr>
<tr>
<td>70</td>
<td>31.19</td>
<td>0.3076</td>
<td>25.59</td>
<td>0.2147</td>
</tr>
<tr>
<td>90</td>
<td>70.14</td>
<td>0.6922</td>
<td>21.39</td>
<td>0.5832</td>
</tr>
</tbody>
</table>

(b) \( M_{H_2O} = 36.46 \) kg/mole. By (11.9), with \( \mu_{H_2O} = 0.05 \), \( M_{soln} = 18.30 \).  

By (11.9), \( \nu_{NaCl,soln} = 0.0151 \). Raoult's law then yields:
\[
P_{\text{H}_2\text{O}, s} = P_{\text{sat}, \text{H}_2\text{O}} (1 - 0.0151) = (12.249 \text{ kPa})(0.9849) = 12.16 \text{ kPa}
\]

(c) \(M_{\text{NaCl}} = 58.44 \text{ kg/kg mole}\). The molecular weight of the solute is 20.58 kg/kg mole, and \(x_{\text{NaCl, soln}} = 0.06340\). The solution's boiling point is the temperature at which \(P_{\text{H}_2\text{O}, s} = 1\) atm:

\[
101225 \text{ Pa} = P_{\text{sat}, \text{H}_2\text{O}}(T_{\text{b.p.}})(1 - 0.06340)
\]

or

\[
P_{\text{sat}, \text{H}_2\text{O}}(T_{\text{b.p.}}) = 108.184 \text{ Pa}.
\]

From a steam table, we find

\[
T_{\text{b.p.}} = 101.8^\circ \text{C}
\]
11.28 a.) Write eqn. (11.43) and the b.c.'s in terms of a nondimensional mass fraction, $\psi$, analogous to the dimensionless temperature in eqn. (7.42). b.) For $\nu = D_{i,m}$, relate $\psi$ to the Blasius function, $f$, for flow over a flat plate. c.) Note the similar roles of Pr and Sc in the two boundary layer transport processes. Infer the mass concentration analog of eqn. (6.55) and sketch the concentration and momentum b.l. profiles for $Sc \ll 1$, $Sc = 1$, and $Sc \gg 1$.

(a) The appropriate nondimensionalization is: $\psi = \frac{M_i - M_{i,s}}{m_{i,e} - M_{i,s}}$

The analog to (6.42) is then:

$$u \frac{\partial \psi}{\partial x} + v \frac{\partial \psi}{\partial y} = D_{i,m} \frac{\partial^2 \psi}{\partial y^2}; \quad \psi(y=0) = 0, \quad \psi(y \to \infty) = 1,$$

$$\frac{\partial \psi}{\partial y} \bigg|_{y=\infty} = 0$$

(b) By eqns (6.21),(6.41) when $\nu = D_{i,m}$, the distributions of concentration and velocity are identical: $\psi = f'$

(c) The similarity of heat transfer and low-rate mass transfer shows that we need only replace $S_t$ by $S_c$ and $\alpha$ by $D_{i,m}$. Therefore: $\frac{S_c}{\delta} = \frac{S_{i,m}}{\delta}$ for $0.01 \leq Sc \leq 50$ and low rates.

![Diagram showing concentration and momentum profiles for different Sc values.](image-url)
11.29 When Sc is large, momentum diffuses more easily than mass, and the concentration b.l. thickness, $\delta_c$, is much less than the momentum b.l. thickness, $\delta$. On a flat plate, the small part of the velocity profile within the concentration b.l. is approximately $u/U_e = 3\nu/(2\delta)$. Compute $Nu_{\delta_{m,\infty}}$ based on this velocity profile, assuming a constant wall concentration. (Hint: Use the mass transfer analogs of eqn. (6.47) and (6.50) and note that $q_{\delta_{m,\infty}} / \rho C_p$ becomes $j_{i,s}/\rho$.)

The analog of (6.47) is: $\frac{d}{dx} \int_0^{\delta_c} u (m_i - m_{i,e}) dy = \frac{j_{i,s}}{\rho}$

and the analog of (6.47) is: $\frac{m_i - m_{i,e}}{m_{i,s} - m_{i,e}} = 1 - \frac{\frac{3}{2} \frac{\nu}{\delta_c}}{\frac{1}{2} \left( \frac{\delta_c}{\delta} \right)^3}$

Now, $\frac{j_{i,s}}{\rho} = -D_{i,m} \frac{d}{dy} (m_i - m_{i,e}) \bigg|_{y=0} = (m_{i,s} - m_{i,e}) \frac{3}{2} \frac{1}{\delta_c}$

And:

$$\frac{d}{dx} \int_0^{\delta_c} u (m_i - m_{i,e}) dy = U_e (m_{i,s} - m_{i,e}) \int_0^{\delta_c} \frac{d}{dy} \left( \frac{3}{2} \frac{\nu}{\delta_c} \right) \left( 1 - \frac{\frac{3}{2} \frac{\nu}{\delta_c}}{\frac{1}{2} \left( \frac{\delta_c}{\delta} \right)^3} \right) dy$$

$$= U_e (m_{i,s} - m_{i,e}) \frac{d}{dx} \left( \frac{3}{2} \frac{\nu}{\delta_c} \right) = U_e (m_{i,s} - m_{i,e}) \frac{3}{2} \left( \frac{\delta_c}{\delta} \right) \frac{d\delta_c}{dx}$$

since $\delta_c/\delta = f_u (Sc \text{ only})$ by analogy to eqn. (6.46).

Combining the above equations, we obtain a d.e. for $\delta_c(x)$:

$$D_{i,m} \frac{3}{2} \frac{1}{\delta_c} = U_e \frac{3}{20} \left( \frac{\delta_c}{\delta} \right) \frac{d\delta_c}{dx} \quad \text{where } \frac{\delta_c}{\delta} \neq f_u (x)$$

$$\frac{10 D_{i,m}}{U_e} \left( \frac{\delta_c}{\delta} \right) = \frac{d}{dx} \left( \frac{\delta_c^2}{2} \right) \quad \text{integrate with } \delta_c = 0 \text{ at } x = 0$$

$$\delta_c = \sqrt{\frac{20 D_{i,m} / U_e (\delta_c / \delta) \cdot x}{\delta_c}}$$

Next introduce eqn. (6.2) for $\delta$ and rearrange to find

$$\left( \frac{\delta_c}{\delta} \right)^2 = \sqrt{\frac{20 D_{i,m} / U_e \cdot Re_{\delta} / (4.64 \cdot x)}{\delta_c}} \quad \text{or } \quad \left( \frac{\delta_c}{\delta} \right) = \frac{1}{1.025 (\delta_c)^{1.5}}$$

The mass transfer coefficient is:

$$g_{j,m,i}^* = \frac{j_{i,s}}{(m_i, s - m_{i,e})} = \rho D_{i,m} \cdot \frac{3}{2} \frac{1}{\delta_c}$$

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and the Nusselt number for mass transfer is:

\[ \text{Nu}_{m,x} = \frac{q^*_{m,i} \cdot x}{\mu D_{m}} = \frac{3}{2} \frac{V_x}{D_{m}} \left[ \frac{D_{m}}{\nu} \right] \frac{\delta_c}{\delta} \]

\[ = \frac{3}{2 \nu D_{m}} \text{Re}_{x}^{\frac{1}{2}} \left( \frac{\delta_c}{(1.025)^{\frac{1}{2}}} \right) \quad \text{Nu}_{m,x} = 0.331 \left( \frac{\text{Re}_{x}^{\frac{1}{2}} \delta_c}{\text{Sc}} \right) \]

This may be compared to problem (6.21) and eqn (6.59).

11.30 Consider a 1-dimensional, binary gaseous diffusion process in which species 1 and 2 diffuse in opposite directions along the z-axis at equal molar rates. This process is known as equimolar counter diffusion. 

a.) What are the relations between \( N_1, N_2, J_1^*, \) and \( J_2^*? \)
b.) If steady state prevails and conditions are isothermal and isobaric, what is the concentration of species 1 as a function of \( z? \)
c.) Write the mole flux in terms of the difference in partial pressure of species 1 between locations \( z_1 \) and \( z_2 \).

(a) By assumption, \( N_1 = -N_2. \) Therefore \( N = N_1 + N_2 = 0 \) and we have by (11.24)

\[ N_1 = -N_2 = J_1^* = -J_2^* \]

(b) \( N_1 = \text{constant in steady state}, \) by eqn (11.49) with \( \dot{N}_1 = M_1 N_1. \)

With Fick's law: \( N_1 = -cD_{12} \frac{\partial \chi_1}{\partial z} = \text{constant.} \) Integrating this we obtain:

\[ \chi_1(z) = -\frac{N_1}{cD_{12}} z + \chi_1(z=0) \]

since \( cD_{12} \) is constant under isothermal, isobaric conditions.

(c) Use the above result: \( \chi_1(z_1) - \chi_1(z_2) = \frac{N_1}{cD_{12}} (z_2 - z_1) \)

\[ N_1 = \frac{cD_{12}}{(z_2 - z_1)} (\chi_1(z_1) - \chi_1(z_2)) \]

Now recall that \( \chi_1 = p_1/p \) and \( c/v = 1/R^0T. \) There obtains

\[ N_1 = \frac{(p_1 - p_2)}{R^0T(z_2 - z_1)} \]
11.31 Consider steady mass diffusion from a small sphere. When convection is negligible, the mass flux in the radial direction is $J_{i, r} = j_{i, r} = -\rho \frac{\partial \ln m_i}{\partial r} \frac{dm_i}{dr}$. For the case in which the concentration is $m_i = m_i,s$ far from the sphere and $m_i = m_i,s$ at its surface, use a mass balance to obtain the surface mass flux in terms of the overall concentration difference (assuming that $\rho \frac{\partial \ln m_i}{\partial r}$ is constant). Then apply the definitions (11.77) and (11.58) to show that $Nu_{m,D} = 2$ for this situation.

In steady state, the mass flow through any spherical shell of radius $r$ about the sphere is the same. Thus, $4\pi r^2 \cdot J_{i, r} = \text{constant with respect to } r = C_i$

Use the given information to obtain a differential eqn:

$C_i = -\rho \frac{\partial \ln m_i}{\partial r} \cdot 4\pi r^2 \cdot \frac{dm_i}{dr}$

Integrate:

$\int_{m_{i,s}}^{m_{i,0}} dm_i = -\frac{C_i}{\rho \frac{\partial \ln m_i}{\partial r}} \int_{r_s}^\infty \frac{dr}{r^2}$

where $R_s$ is the radius of the sphere

Rearrange to find:

$J_{i, r} = m_{i,s} = \frac{C_i}{4\pi R_s^2} (m_{i,s} - m_{i,\infty})$

Using (11.77) and (11.58), with $\frac{\partial \ln m_i}{\partial r} = \rho \frac{\partial \ln m_i}{\partial r}$ for this low-rate process:

$Nu_{m,D} = \frac{2R_s}{\rho \frac{\partial \ln m_i}{\partial r} \left( m_{i,s} - m_{i,\infty} \right)} = 2$

11.32 An experimental Stefan tube is 6 cm in diameter and 30 cm from the liquid surface to the top. It is held at 100°C and 8.0 x 10^4 Pa. Pure argon flows over the top and liquid CCl₄ is at the bottom. The pool level is maintained while 0.69 ml of CCl₄ evaporates during a period of 8 hrs. What is the diffusivity of carbon tetrachloride in argon measured under these conditions? The specific gravity of liquid CCl₄ is 1.59 and its vapor pressure is $\log_{10} p_v = 8.004 - 1771/ T$, where $p_v$ is expressed in mm hg and T in °K.

Use equation (11.71) after calculating the mole flux. First calculate the number of moles evaporated:
\[ S_{ad} = (1.59) \delta_{ad} = (1.59)(994.4) = 1589 \text{ kg/m}^3 \]

\[ C_{ce} = \frac{\rho}{M} = (1589)/(153.8) = 10.33 \text{ kg mole/m}^3 \]

The number of moles evaporated is \( 0.69 \times 10^{-6} \text{ m}^3/(10.33 \text{ kg mole/m}^3) = 7.13 \times 10^{-6} \text{ kg mole}. \) The mole flux is thus:

\[ N_s = \frac{\text{kg mole evaporated}}{\text{rad}^2 \text{m}^4 \text{time}} = \frac{(7.13 \times 10^{-6})}{(0.06)^2(0)(3600)} = 8.76 \times 10^{-8} \text{ kg mole/m}^2 \text{s} \]

The mole concentration of gas in the tube is \( C = \frac{\rho V}{RT} = \frac{(8.0 \times 10^4)}{(8314.3)(283.15)} = 0.0340 \text{ kg mole/m}^3. \) Using the given eqn, the vapor pressure of \( CCl_4 \) is \( P_v = 56.15 \text{ mm Hg} = 7.49 \times 10^3 \text{ Pa}; \) the mole concentration of \( CCl_4 \) at the liquid surface is \( x_{CCl_4,5} = P_v/\rho = 0.0936. \) Now apply (11.72) as:

\[ D_{12} = \frac{(N_s \cdot L)}{C} \left( \ln \left(1 + \frac{x_{1,5} - x_{1,5}}{x_{1,5} - 1}\right) \right)^{-1} = \frac{(0.84 \times 10^{-6})(0.3)}{\ln \left(1 + \frac{0.0936}{0.0936 - 1}\right)(0.034)} \]

\[ D_{12} = 7.87 \times 10^{-6} \text{ m}^2/\text{s} \]

11.33 Repeat the analysis given in Section 12.6 on the basis of mass fluxes, assuming that \( \rho D_{im} \) is constant and neglecting any buoyancy driven convection. Obtain the analog of eqn. (11.71).

From eqn (11.49), we find: \( n_1 = n_{1,5} \) and \( n_2 = n_{2,5}. \) From (11.20)

\[ n_{1,5} = n_1 n - \rho D_{12} \frac{d n_1}{dy}. \]

However, \( n \) is constant (at 2%), \( n_{1,5} = n_{1,5} + n_{2,5}, \) so we may integrate this d.e. as before, assuming \( (\rho D_{12}) \) is a constant:

\[ \rho D_{12} \frac{d n_1}{dy} = n_1 n_5 - n_{1,5} \Rightarrow \left( \frac{n_5}{\rho D_{12}} \right) = \ln \left( n_{1,5} n_1 - n_{1,5} \right) + \text{const}. \]

The b.c. at \( y = 0 \) is \( n_1 = n_{1,5} \) from which the constant is \( -\ln(n_{1,5} n_{1,5} - n_{1,5}). \) The boundary condition at \( y = L \) is \( n_1 = n_{1,5} \) and the result is:

\[ N_s = \left( \frac{\rho D_{12}}{L} \right) \ln \left(1 + \frac{n_{1,5} - n_{1,5}}{n_{1,5} - n_{1,5}}\right) \]

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11.34 In sections 11.4 & 11.7, it was assumed at points that \( c_{D_{12}} \) or \( \rho D_{12} \) was independent of position. a.) If the mixture composition varies in space this assumption may be poor. Using eqn. (11.116) and the definitions from Section 1.2, examine the composition-dependence of these two groups. For what type of mixture is \( \rho D_{12} \) most sensitive to composition? What does this indicate about molar versus mass-based analysis? b.) How do each of these groups depend on pressure and temperature? Is the analysis of Section 11.7 really limited to isobaric conditions? c.) Do the Prandtl and Schmidt numbers depend on composition, temperature, or pressure?

(a) From eqn (11.116), \( D_{12} \propto \frac{T^{1/2}}{\rho D_0} \) (molecular parameters).

The molecular parameters depend only upon which two species are involved and not on the relative amounts of each in the mixture. Now, \( c = \rho / R_0 T \) and \( p = \rho M / R_0 T \).

Thus, \( c D_{12} \propto \left( \frac{T^{1/2}}{\rho D_0} \right) \) (molecular parameters).

\( \rho D_{12} \propto \left( \frac{T^{1/2}}{\rho D_0} \right) \cdot M \) (molecular parameters).

We see that \( c D_{12} \) is independent of spatial variations of the mixture composition. \( \rho D_{12} \) is not because of its dependence on \( M \):

\[
M = x_1 M_1 + x_2 M_2 = M_1 \left\{ 1 + x_2 \left( \frac{M_2}{M_1} - 1 \right) \right\}
\]

\( \rho D_{12} \) is most sensitive to variations of composition (\( x_2 \)) when \( M_2 \) and \( M_1 \) differ greatly. When \( M_2 \approx M_1 \), the composition variation is small. For mixtures with greatly differing molecular weights, molar analysis assuming constant \( c D_{12} \) will yield better results than mass-based analysis assuming constant \( \rho D_{12} \).

(b) Both groups are independent of \( \rho \) and depend on \( T \) as \( (T^{1/2} / \rho D_0) \). The isobaric assumption in is unnecessary.
(c) $Pr = \mu cp/k$. Make reference to eqns. (11.125-129). The Prandtl number is independent of pressure and depends on mixture composition. The NT factors in $\mu, k$ will cancel out of the mixture rules, but the complicated dependence of $Pr$ on the temperature dependent $\mu/k, S_k$ may introduce a mild dependence of $Pr$ on $T$ in mixtures. $Sc = \mu/kD_{12}$. The Schmidt number is independent of $p$, depends on composition, and has some dependence on $T$.

11.35 A Stefan tube contains liquid bromine at 320$^\circ$K and 1.2 atm. Carbon dioxide flows over the top and is also bubbled up through the liquid at the rate of 40 ml/hr. If the distance from the liquid surface to the top is 16 cm and the diameter is 3 cm, what is the evaporation rate of Br$_2$? ($p_{sat, Br} = 0.680$ bar at 320$^\circ$K.).

From eqn (12.43), compute $D_{12} = 6.229 \times 10^{-6}$ m$^2$/s. From Raoult's law, assuming a negligible solubility of CO$_2$ in Br$_2$ liquid, and (11.15) find $X_{1,3} = n_{sat, Br_2}/P = 0.680/(1.01325)(1.2) = 0.559$. The flux of CO$_2$ through the liquid surface is:

\[
N_{CO_2,5} = \text{(volume flow rate)} \times c/(area \ of \ tube) \\
= (40 \times 10^{-6} \text{ m}^2/\text{3600} \text{ s}) (0.04570 \text{ kg mole/m}^3)/(\pi/4 (0.03)^2) \\
= 7.18 \times 10^{-7} \text{ kg mole/m}^2 \cdot \text{s}
\]
Now apply (11.73) and solve iteratively:

\[ N_{B_{R_{2}}, s} + (7.10 \times 10^{-7}) = \frac{(0.04570)(6.229 \times 10^6)}{(0.16)} \ln \left[ 1 + \frac{-0.559}{0.559 - \frac{N_{B_{R_{2}}, s}}{N_{B_{R_{2}}, s} + 7.18 \times 10^{-3}}} \right] \]

Guess \( N_{B_{R_{2}}, s} \), substitute into the ln term and solve for implied \( N_{B_{R_{2}}, s} \) on L.H.S.:

\[
\begin{array}{c|c}
1.5 \times 10^{-6} & 2.40 \times 10^{-6} \\
2.40 \times 10^{-6} & 1.59 \times 10^{-6} \\
1.59 \times 10^{-6} & 2.25 \times 10^{-6} \\
2.25 \times 10^{-6} & 1.66 \times 10^{-6} \\
1.95 \times 10^{-6} & 1.86 \times 10^{-6} \\
1.90 \times 10^{-6} & 1.90 \times 10^{-6} \\
\end{array}
\]

\[ N_{B_{R_{2}}, s} = 1.90 \times 10^{-6} \text{ kg mole/m}^2\text{s} \]

11.36 Show that \( g_{m,1} = g_{m,2} \) and \( B_{m,1} = B_{m,2} \) in a binary mixture.

From (11.77), (11.25), and (11.3), we have:

\[ g_{m,1} = \frac{i_{1,s}}{(m_{1,s} - m_{1,e})} = \frac{i_{2,s}}{(1-m_{2,s} - 1+m_{2,e})} = \frac{i_{2,s}}{(m_{2,s} - m_{2,e})} = g_{m,2} \]

From (12.91), (12.20), and (12.3), we have:

\[ B_{m,1} = \frac{m_{1,e} - m_{1,s}}{m_{1,s} - n_{1,s}/\mu''} = \frac{- (m_{2,e} - m_{2,s})}{1 - m_{2,s} - (\mu'' - n_{2,s})/\mu''} \]

\[ = \frac{m_{2,e} - m_{2,s}}{m_{2,s} - n_{2,s}/\mu''} = B_{m,2} \]
11.37 Demonstrate that stagnant film models of the momentum and thermal boundary layers reproduce the proper dependence of \(C_f,x \) and \(Nux \) on \(Re_x \) and \(Pr \). Using eqns. (6.31b) and (6.55) to obtain the dependence of \( \delta \) and \( \delta_t \) on \( Re_x \) and \( Pr \), show that stagnant film models give eqns. (6.33b) and (6.55) within a constant on the order of unity. (The constants in these results will differ from the exact results because the effective b.l. thicknesses of the stagnant film model are not the same as the exact values — see eqn. (6.57).)

Heat transfer across a stagnant fluid layer having no horizontal gradients must be by conduction in the vertical direction: \( q = k \Delta T / \delta_t \).

The heat transfer coefficient is then \( h = k / \delta_t \) and the Nusselt number is \( Nu_x = h \chi / k = \chi / \delta_t \).

To model momentum transfer, use the absence of horizontal gradients to infer a linear velocity distribution, \( u = (\chi / \delta) u_\omega \), in the film. (This is the “Couette flow” encountered in your basic fluid mechanics course.) The linear velocity distribution here is like the linear temperature distribution assumed above. Then, \( \tau \omega = \mu \frac{du}{dy} = \mu \frac{u_\omega}{\delta} \) and \( C_{f,x} = \tau \omega / \frac{1}{2} \rho \frac{u_\omega^2}{\delta} = 2 \chi / \delta \cdot Re_x \).

Now introduce (6.33) and (6.55): \( \chi / \delta = 4.64 / \sqrt{Re_x} \), \( \Delta T / \delta_t = Pr^{1/3} \).

\( Nu_x = \chi / \delta_t = (\sqrt{Re_x / 4.64}) \cdot Pr^{1/3} = 0.216 \cdot Re_x^{-1/2} \cdot Pr^{1/3} \)

\( C_{f,x} = 2 \chi / \delta \cdot Re_x = (2/4.64) \cdot Re_x^{1/2} = 0.431 \cdot Re_x^{1/2} \)

The constants obtained this way differ by a factor of 1.54 from those in (6.33) and (6.55). Eq'n (6.57) shows that the appropriate effective boundary layer thickness is not that used here, but \( 7/8 \delta_t \).
11.38 (a) What is the largest value of the mass transfer driving force when species is transferred? What is the smallest value? (b) Plot the blowing factor as a function of $B_m,i$, for one species transferred. Indicate on your graph the regions of blowing, suction, and low-rate mass transfer. (c) Verify the two limits used to show that $g_{m,i}^* = \frac{\rho \frac{D_{i,m}}{S_c}}{m_{i,s}}$.

(a) $B_m,i = \frac{m_{i,e} - m_{i,s}}{m_{i,s}}$. For fixed $m_{i,e}$ take the limits:

\[
\lim_{m_{i,s} \to 0} B_m,i = -\infty \quad \text{for } m_{i,e} > 1 \\
\lim_{m_{i,s} \to 1} B_m,i = \begin{cases} 
+\infty & m_{i,e} < 1 \\
-1 & m_{i,e} = 1
\end{cases}
\]

(Note that when $m_{i,e} = 1$, the driving force is -1 for any $m_{i,s}$.)

We see that: $-1 \leq B_m,i \leq +\infty$

(b) 

(c) First argue that $w'' \to 0$ is identical to $B_m,i \to 0$: Recall that $w'' = g_{m,i} \cdot B_m,i$. From our knowledge of transport coefficients, we know that $g_{m,i}$ is essentially independent of $B_m,i$ in forced convection and varies as $B_m,i$ in natural convection, when $B_m,i$ is small. $g_{m,i}$ is either constant or goes to zero as $B_m,i \to 0$; therefore $w''$ always goes to zero as $B_m,i \to 0$.

Thus:

\[
g_{m,i}^* = \lim_{B_m,i \to 0} g_{m,i} = \left(\frac{\rho \frac{D_{i,m}}{S_c}}{m_{i,s}}\right) \lim_{B_m,i \to 0} \frac{\ln(1 + B_m)}{B_m} = \left(\frac{\rho \frac{D_{i,m}}{S_c}}{m_{i,s}}\right)
\]
Nitrous oxide is bled through the surface of a porous 3/8 in. OD tube at 0.025 liter/s per meter of tube length. Air flows over the tube at 25 ft/s. Both the air and tube are at 18°C and the ambient pressure is 1 atm. Estimate the mean concentration of N₂O at the tube surface.

At 1 atm, 18°C eqn (11.116) gives: $D_{\text{air}, N_2O} = 1.483 \times 10^{-5}$ m²/s. As a first estimate, take all properties as those of pure air and assume low-rate mass transfer—both assumptions are motivated by the low flow rate of N₂O from the tube.

At 18°C, 1 atm: $\rho_{\text{air}} = 1.212 \text{ kg/m}^3$, $\nu_{\text{air}} = 1.488 \times 10^{-5}$ m²/s

$Sc = \nu/D = 1.003$

The velocity of air and tube diameter are $U = 7.62 \text{ m/s}$, $D = 0.009525 \text{ m}$, and $Re_0 = 4878$. This is a uniform wall flux problem and eqn (7.68) is appropriate:

$$Nu_{m,0} = 0.3 + \frac{0.62 \cdot Re_0 \cdot Sc}{\left[1 + (0.4/Sc)^{1/4}\right]^{1/4}} \left[1 + \frac{Re_0}{(82,000)}\right]^{4/5} = 41.64$$

$$\overline{q}_{m, N_2O} = \frac{Nu_{m,0}}{D} \cdot \rho_{\text{air}, N_2O} = \frac{(41.64)(1.212)(1.483 \times 10^{-5})}{0.009525} = 0.07858 \text{ kg/m}^2\text{s}$$

The mass flow rate of N₂O is, with $\rho_{N_2O} = 1.842 \text{ kg/m}^3$ (ideal gas):

$$n_{N_2O, s} = (0.025 \times 10^3 \text{ m}^3/\text{s/m}) (1.842 \text{ kg/m}^3)/\pi (0.009525 \text{ m}) = 1.539 \times 10^{-3} \text{ kg/m}^2\text{s}$$

At low rates, $n_{N_2O, s} = \overline{q}_{m, N_2O} / \rho_{m, N_2O}$ from which $\overline{g}_{m, N_2O} = 0.01958$. With $m_{N_2O, o} = 0$, $\overline{g}_{m, N_2O} = (D - m_{N_2O, s})/(m_{N_2O, s} - 1)$. Solving,

$$m_{N_2O, s} = 0.0192$$

This result justifies the use of air properties and low rates.
11.40 Gases are sometimes absorbed into liquids through "film absorption." A thin film of liquid is run down the inside of a vertical tube, through which flows the gas to be absorbed. Analyze this process under the following assumptions: The film flow is laminar and of constant thickness, \( \delta_o \), with a velocity profile given by eqn. (8.41). The gas is only slightly soluble in the liquid, so the liquid properties are unaffected by it and it does not penetrate far beyond the liquid surface. The gas concentration at the s and u-surfaces does not vary along the length of the tube. The inlet concentration of gas in the liquid is \( m_{1,0} \). Show that the mass transfer is given by

\[
Nu_{m,x} = \left( \frac{u_0}{u_0} \right)^{1/2}, \quad \frac{u_0}{2\mu_f} = \frac{(\rho_f - \rho_g)\delta_o^2}{2\mu_f}
\]

The mass transfer coefficient here is based on the concentration difference between the u-surface and the bulk liquid at \( m_{1,0} \). (Hint: The small penetration assumption can be used to reduce the species equation for the film to the diffusion equation.)

Because the liquid film is thin, we ignore the curvature of the tube. As in Chapter 9, we may apply boundary layer assumptions and the appropriate form of the species conservation equation (11.43) becomes:

\[
u \frac{\partial m_i}{\partial x} + \nu \frac{\partial m_i}{\partial y} = D_{lm} \frac{\partial^2 m_i}{\partial y^2}
\]

The velocity profile is (8.51):

\[
u = \frac{(\rho_f - \rho_g)\delta_o^2}{2\mu} \left[ 2 \left( \frac{y}{\delta_o} \right) - \left( \frac{y}{\delta_o} \right)^2 \right], \quad \nu = 0
\]

Because the gas does not penetrate far into the film, we may approximate \( \nu \) in the species eqn by its surface value, \( u_0 \). This is the important feature of the small penetration assumption. The species eqn reduces to the diffusion eqn:

\[
u_0 \frac{\partial m_i}{\partial x} = D_{lm} \frac{\partial^2 m_i}{\partial y^2}
\]
The boundary and initial conditions in the liquid film are:

\[ m_1(y=\delta_0, x) = m_{1,u}; \quad m_1(x=0, y) = m_{1,o}; \quad m_1(y \to -\infty, x) = m_{1,o} \]

In the last condition, we use \( y \to -\infty \) rather than \( y=0 \) because the small penetration approximation essentially removes all effects of the wall. We may cast this problem into the form (11.52) by transforming the \( y \)-coordinate to \( \tilde{y} = \delta_0 - y \); the d.e. is unaffected and the b.c.'s are

\[ m_1(\tilde{y}=0, x) = m_{1,u}; \quad m_1(x=0, \tilde{y}) = m_{1,o}; \quad m_1(y \to \infty, x) = m_{1,o} \]

From example 11.8, the solution is

\[ \left( \frac{m_1-m_{1,u}}{m_{1,o}-m_{1,u}} \right) = \text{erf} \left( \frac{\tilde{y}}{2\sqrt{D_{m}x/m_{0}}} \right) \]

To find the mass transfer coefficient, compute \( \dot{j}_1 \):

\[ \dot{j}_1 = -pD_{m} \frac{\partial m_1}{\partial \tilde{y}} = -pD_{m} (m_1-u-m_{1,o}) \cdot \frac{2}{\sqrt{\pi}} \cdot \frac{1}{2} \left( \frac{m_{0}}{D_{m}x} \right)^{1/2} \]

(see eqn 5.54). The mass transfer coeff. \( \dot{g}_{m,1} = \dot{j}_1 / (m_{1,u}-m_{1,o}) \)

\[ \dot{g}_{m,1} = \dot{j}_1 \left( \frac{m_{0}D_{m}}{\pi x} \right)^{1/2} \quad \text{so} \quad \text{Nu}_{m,x} = \left( \frac{\dot{u}_0 x}{\pi D_{m}} \right)^{1/2} \]

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11.41 Benzene vapor flows through a 3 cm ID vertical tube. A thin film of initially pure water runs down the inside wall of the tube at a flow rate of 0.3 L/s. If the tube is 0.5 m long and 40°C, estimate the rate (in kg/s) at which benzene is absorbed into water over the entire length of the tube. The mass fraction of benzene at the u-surface is 0.206. (Hint: Use the result stated in Prob. 11.40. Obtain \( \delta_0 \) from the results in Chapter 8.)

The properties of water at 40°C are: \( \mu = 6.501 \times 10^{-4} \) kg/m s, \( \rho = 991.8 \) kg/m³. The gas density will be small compared to \( \rho_{H_2O} \).

The diffusion coefficient is calculated from eqn. (11.124):

\[
D_{c_6H_6-H_2O} = \left( \frac{313.15}{6.501 \times 10^4} \right) \left( 4.4 \times 10^{-15} \right) \left( \frac{0.0187}{0.096} \right) \left( \frac{4062}{30.76} \right) = 1.85 \times 10^{-9} \text{ m}^2/\text{s}.
\]

Next obtain the film thickness: the volume flow rate = 0.3 L/s = 0.0003 m³/s = \( \pi D \left( \frac{\dot{m}}{\rho_f} \right) \) = \( \pi D (\rho_f - \rho_g) \) \( \frac{\dot{m}}{3 \mu_f} \) by eqn. (9.45). Thus:

\[
\delta_0^3 = (0.0003)(3 \mu_f) [\pi D (\rho_f - \rho_g)].
\]

Using the results of Problem 11.40, \( \delta_0 = (\rho_f - \rho_g) \delta_0^2 / 2 \mu_f = 5.545 \text{ m/s} \), and \( \rho \text{m,v} = (\rho_0 \nu / \pi D)^{1/2} \).

To get the overall rate of absorption, we need to integrate:

rate of absorption = \( \pi D \int_0^L \frac{c_{c_6H_6}}{c_{c_6H_6}^0} \, dx = \pi D (\mu_{c_6H_6} - m_{c_6H_6}) \int_0^L \dot{g}_m \, dx \)

\[
= \left( \frac{\dot{m}}{\rho_f D / \pi} \right)^{1/2} \pi D (\mu_{c_6H_6} - m_{c_6H_6}) \int_0^L x \, dx
\]

\[
= 2.15^2 (991.8)(5.545)(1.85 \times 10^{-9}) (0.0187)(0.096)(0.206)(2)(0.5)^2
\]

\[
= 1.557 \times 10^{-3} \text{ kg/s} = 1.56 \text{ g/s}
\]

We can check the small penetration assumption with eqn. (5.51):

penetration depth = \( 3.65 \sqrt{D} t^1 = 3.65 \sqrt{DL/\dot{m}} = 13 \mu \text{m} \ll \delta_0 \).
11.44 Consider the process of "hydrogenolysis", described in Problem 11.9, in which the reactants diffuse toward a catalyst surface and the products diffuse away from it. (a) What is \( \dot{m}^n \)? (b) Reaction rates in catalysis are of the form (see Problem 11.9):

\[
R_{\text{reactant}} = A e^{-\Delta E / R T} (p_{\text{reactant}})^n (p_{\text{product}})^m \text{kg mole/m}^2\text{s}
\]

for the rate of consumption of a reactant per unit surface area. The \( p \)'s are partial pressures and \( A, \Delta E, n \) and \( m \) are constants. Suppose \( n = 1 \) and \( m = 0 \) for the reaction \( B + C \rightarrow D \). Approximate the reaction rate, in terms of \( \dot{m} \) mass, as

\[
\dot{m}_B = A \dot{e}^{-\Delta E / R T} \rho_{B_s} \text{kg/m}^2\text{s}
\]

and find in terms of \( \dot{m}_B \) the mass transfer coefficient for the geometry in question. (c) The ratio \( Da = \rho A \dot{e}^{-\Delta E / R T} / \dot{m} \) is called the Damköhler number. Explain its significance in catalysis. What features dominate the process when \( Da \) approaches 0 or \( \infty \)? What temperature range characterizes each?

(a) \( \dot{m}^n = 0 \): nothing passes through the \( u \)-surface since the catalyst undergoes no net reaction. The net mass transfer through the \( s \)-surface must be zero.
(b) Write \( \dot{r}_B = (A' e^{-\Delta E/k_0 T}) p_{B,s} = k'' p \cdot m_{B,s} \) for convenience. Now form a mass balance on \( B \) at the \( s \)-surface: \( \dot{j}_{B,s} = \dot{r}_B \), that is, rate of consumption equals rate at which \( B \) reaches the surface. This is
\[
q_{m_B}^{**} (m_{B,e} - m_{B,s}) = k'' p \cdot m_{B,s} \quad (*)
\]
Note that in this process \( \dot{w} = 0 \) with non-zero net fluxes of \( B, C, \) and \( D \); thus \( m_{B,s} = \dot{j}_{B,s} = q_{m_B}^{**} (m_{B,e} - m_{B,s}) \).
We may rearrange (*) as:
\[
m_{B,s} = \frac{m_{B,e}}{q_{m_B}^{**}} \cdot \frac{1}{\frac{1}{q_{m_B}^{**}} + \frac{1}{pk''}}
\]
Substitute this into our expression for \( \dot{r}_B \):
\[
\dot{j}_{B,s} = \dot{r}_B = k'' p \cdot m_{B,s} = \left( \frac{m_{B,e}}{q_{m_B}^{**}} \cdot \frac{1}{\frac{1}{q_{m_B}^{**}} + \frac{1}{pk''}} \right)
\]
We may view this as a resistance network for \( j_B \):
\[
\begin{array}{c}
\text{m}_{B,e} \\
\rightarrow \\
\frac{1}{q_{m_B}^{**}} \\
\frac{1}{pk''} \\
\rightarrow \\
\text{j}_B \\
\end{array}
\]
(c) From the result of (b), we see that \( Da = \frac{\text{diffusion resistance}}{\text{reaction resistance}} \) or \( (\text{reaction rate})/(\text{diffusion rate}) \). When \( Da \to \infty \), the rate of consumption of \( B \) is limited by the diffusion resistance; the surface reaction is comparatively fast, as at high temperature. When \( Da \to 0 \), the process is limited by the reaction rate; this corresponds to low temperatures.
11.45 One typical kind of mass exchanger is a fixed bed catalytic reactor. A flow chamber of length \( L \) is packed with a catalyst bed. A gas mixture containing some species \( i \) to be consumed by the catalytic reaction flows through the bed at a rate, \( \dot{m} \). The effectiveness of such an exchanger is (cf. Chapter 3)

\[
\epsilon = 1 - e^{-NTU}, \quad \text{where} \quad NTU = g_{m,oa} PL/\dot{m}
\]

where \( g_{m,oa} \) is the overall mass transfer coefficient for the catalytic packing. In testing a 0.5 m catalytic reactor for the removal of ethane, it is found that the ethane concentration drops from a mass fraction of 0.36 to 0.05 at a flow rate of 0.05 kg/s. The packing is known to have a surface area of 11 m\(^2\). What is the overall mass transfer coefficient in this bed? What is the exchanger effectiveness? \( P = \) surface area per unit length. \( \epsilon \) is defined in terms of mass fractions.

By comparison to Chapter 3 (eqn 3.16), the definition of mass exchanger effectiveness must be

\[
\epsilon \equiv \frac{\text{actual mass transferred}}{\text{maximum mass transfer possible}} = \frac{\Delta m_{\text{C2H6}}}{m_{\text{C2H6, in}}}
\]

\[
= \frac{0.36 - 0.05}{0.36} = 0.861
\]

From this we calculate \( NTU = 1.97 \), so that, with \( P = 11 \text{ m}^2 \), we have

\[
g_{m,oa} = (0.05 \text{ kg/s})(1.97)(11 \text{ m}^2) = 0.00895 \text{ kg/s m}^2
\]
11.46 (a) Perform the integration to obtain eqn. (11.92). Then take the derivative and the limit needed to get eqns. (11.93) and (11.94). (b) What is the general form of eqn. (11.95) when more than one species is transferred?

(a) We have
\[ \frac{d}{dy} \left( -k \frac{dT}{dy} + n_i s C_p i T \right) = 0 \]
which can be integrated once immediately:
\[ -k \frac{dT}{dy} + n_i s C_p i T = C, \text{ an unknown constant} \]
Rearrange and integrate:
\[ \int_{y}^{y_T} dy = \int_{0}^{T} \frac{(-k) dT}{C - n_i s C_p i T} = \frac{k}{n_i s C_p i} \ln \left( \frac{C - n_i s C_p i T}{C - n_i s C_p i T_s} \right) \]
or
\[ \exp \left( \frac{y n_i s C_p i}{k} \right) = \frac{\mu - T}{\mu - T_s} , \text{ where the constant } \mu \text{ is } C/n_i s C_p i \text{ for convenience} \]
Now apply the b.c. \( T(y=\delta) = T_e \):
\[ \frac{\mu - T_e}{\mu - T_s} = \exp \left( \frac{\delta n_i s C_p i}{k} \right) = \eta , \text{ for convenience define } \eta \text{ as shown.} \]
Thus, \( \mu = \frac{(T_e - T_s \eta)}{(1-\eta)} \) and we have found the unknown constant \( \mu \) (i.e. C). Substitute for \( \mu \) in (6) and rearrange:
\[ \frac{(T_e - T_s \eta) - T (1-\eta)}{(T_e - T_s \eta) - T_s (1-\eta)} = \frac{(T_e - T_s) \eta - (T_e - T_s)}{(T_e - T_s)} \]
\[ = \frac{(T_e - T_s)(\eta - 1)}{(T_e - T_s)} + 1 = \exp \left( \frac{y n_i s C_p i}{k} \right) \]
Now substitute for \( \eta \) and rearrange to obtain (11.92):
\[ \frac{(T - T_s)}{(T_e - T_s)} = \frac{\exp \left( \frac{y n_i s C_p i}{k} \right) - 1}{\exp \left( \frac{\delta n_i s C_p i}{k} \right) - 1} \]
To get \( h \), write:

\[
 h = t \left. \frac{d}{dy} \left( \frac{T - T_s}{T_e - T_s} \right) \right|_{y=0} = \frac{n_{i,s} c_{pi} \cdot b/k}{\exp \left( \frac{\delta t \cdot n_{i,s} c_{pi}}{k} \right) - 1}.
\]

Then use L'Hôpital's rule to get \( h^* \):

\[
 h^* = \lim_{n_{i,s} \to 0} h = \lim_{n_{i,s} \to 0} \frac{c_{pi}}{(\delta t c_{pi})/k \cdot \exp(\delta t n_{i,s} c_{pi}/k)} = \frac{k}{\delta t}
\]

(b) This form may be obtained with a minimum of work by recognizing that the sum in (11.90) becomes:

\[
 \sum_i \vec{v}_i \cdot \vec{v}_i = \sum_i n_i \cdot h_i = \left( \sum_i n_i c_{pi} \right) (T - T_{ref})
\]

provided we select a common \( T_{ref} \) for each species (even if we don't, \( \frac{d}{dy} \) will eliminate the \( T_{ref} 's \) below).

Now we just integrate \( \frac{d}{dy} \left( -k \frac{dT}{dy} + \left( \sum_i n_{i,s} c_{pi} \right) T \right) = 0 \)

as before. Thus, the result is:

\[
 \frac{T - T_s}{T_e - T_s} = \frac{\exp \left[ \frac{y \left( \sum_i n_{i,s} c_{pi} \right)}{k} \right] - 1}{\exp \left[ \frac{S_t \left( \sum_i n_{i,s} c_{pi} \right)}{k} \right] - 1}
\]
11.47 (a) Derive eqn. (11.105) from eqn. (11.104). (b) Suppose that 1.5 m² of the wing of a spacecraft re-entering the earth’s atmosphere is to be cooled by transpiration. 900 kg of the vehicle’s weight is allocated for this purpose. The low rate heat transfer coefficient is about 1800 W/m²·K in the region of interest and the hottest portion of re-entry is expected to last 3 min. If the air behind the shock wave ahead of the wing is at 2500°C, which of these gases H₂, He, and N₂ -- keeps the surface coolest? (Of course, the result for H₂ is marred by fact that it would burn under these conditions.) The reservoir is at 5°C.

(a) Substitute for \( h \) in (11.104) with (11.95) and divide through by \( n_i s c_p i \):

\[
T_s = \frac{T_e + T_r (\exp(n_i s c_p i/h^*) - 1)}{1 + (\exp(n_i s c_p i/h^*) - 1)}
\]

\[= T_r + (T_e - T_r) \exp(n_i s c_p i/h^*)\]

(b) If the entire gas reservoir is to be used at an even rate during the 3 min. of hottest reentry, the mass flux is:

\[
n_i s = \frac{(900 \text{ kg})}{(1.5 \text{ m}^2)(4 \text{ min})(3600 \text{ s/min})} = 0.0417 \frac{\text{kg}}{\text{m}^2 \cdot \text{s}}
\]

The specific heat \( c_p \) is not strongly dependent on \( p \) or \( T \) so, using App. A, we estimate

\[
c_{pH_2} \approx 17 \times 10^3 \frac{\text{J}}{\text{kg} \cdot \text{K}}, \quad c_{pHe} = 5200 \frac{\text{J}}{\text{kg} \cdot \text{K}}, \quad c_{pN_2} \approx 1.4 \times 10^3 \frac{\text{J}}{\text{kg} \cdot \text{K}}
\]

Substitute all this into (11.105), with \( T_r = 5°C \) and \( T_e - T_r = 2495°C \) and \( h^* = 1800 \text{ W/m}^2\cdot\text{K} \), to obtain:

\[
T_s = \begin{cases} 
1688°C & \text{for } H_2 \\
2120°C & \text{" He} \\
2420°C & \text{" N}_2
\end{cases}
\]

\( H_2 \) is the coolest. Even when it burns, it may be the coolest.
11.48: We do not presently have a solution for this problem. However, we'd previously solved it for a much lower terminal velocity -- one that might have occurred with a drag-inducing streamer attached to the small sphere. The present terminal velocity is more appropriate for a free-fall of the sphere. The previous solution, below, outlines the solution method.

Dry ice (solid CO$_2$) is used to cool medical supplies transported by a small plane in Alaska. A roughly spherical chunk of dry ice, roughly 5 cm in diameter, falls from the plane through air at 5°C with a terminal velocity of 15 m/s. If steady state is reached quickly, what are the temperature and sublimation rate of the dry ice? The latent heat of fusion is 574,000 J/kg, $r_s = 1550$ kg/m$^3$, and $\log_{10}(\text{mm Hg}) = 9.9082 - 1367.3/\text{TK}$. The temperature will be well below the sublimation point of CO$_2$ which is 78.6°C. Use the heat transfer relation $\text{Nu}_D = 2 + 0.3 \Re?_0 \Pr^{0.8}$. (Hint: first estimate the surface temperature using properties for pure air, then correct the properties as necessary.)

We know that $T_s \leq 78.6°C$ because solid CO$_2$ does not exist at higher temps. To make a first estimate of the surface temperature, use properties of pure air at $T_a = -45°C$, which corresponds to a guess of $T_s = -95°C$. For air:

$\rho_a = 1.225$ kg/m$^3$, $\mu_a = 1.44 \times 10^{-5}$ kg/m-s, $k_a = 0.0205$ W/m-K

$\Pr_a = 0.729$ ; $D_{\text{CO}_2-\text{air}} = 0.9458 \times 10^{-5}$ m$^2$/s, $Sc = 0.62$

Now construct the energy balance:

$\dot{q}_w = 0$ for wet-bulb conditions

$\dot{N}_{\text{CO}_2,s} \cdot \rho_{\text{CO}_2} \cdot \dot{h}_{\text{CO}_2} = \dot{q}_{\text{conv}} = h (T_e - T_s)$

heat absorbed in phase change = heat convected to the surface

Use this balance in conjunction with (11.95) for $h$ and

$\dot{N}_{\text{CO}_2,s} = \frac{\dot{q}_w}{\dot{m}_{\text{CO}_2}} \cdot \ln (1 + B_{\text{w,CO}_2})$. The Reynolds number is $\Re_0 = 7.80 \times 10^5$; compute $\dot{q}_w$ and $\dot{h}_w$ from the given

eqn: $\dot{q}_w = \frac{\rho_{\text{CO}_2} \cdot \dot{h}_{\text{CO}_2}}{D} \left\{ 2 + 0.3 \Re_0 \cdot Sc^{0.6} \right\} = 0.0768$ kg/m$^2$-s

$\dot{h}_w = \frac{k}{D} \left\{ 2 + 0.3 \Pr^{0.8} \cdot \Re_0 \right\} = 96.20$ W/m$^2$-K

To estimate the blowing factor use a guess of $T_s = -95°C$ and
compute $P_v = 171.1 \text{ mmHg}$ from given eqn. this gives $N_{CO_2,s} = 0.2251$ and $M_{CO_2,s} = 0.3063$. The mass transfer driving force is $B_{m,CO_2} = (0 - 0.3063) / (2.056 - 1) = 2.265$. This yields $N_{CO_2,s} = (0.0768) \ln (1 + 2.265) = 0.09077 \text{ kg/m}^2\cdot\text{s}$. With $C_{P_{CO_2}} = 789 \text{ J/kg} \cdot \text{K}$, (11.95) yields $h = 64.73 \text{ W/m}^2\cdot\text{K}$. Solve for $T_s$ from the energy balance: $(0.09077)(5.74 \times 10^5) = (64.73)(5^\circ \text{C} - T_s)$ or $T_s = -802^\circ \text{C}$. Our guess is too high!

Leave properties alone, make a new guess for $T_s$ in calculating $P_v$. Then get a new $B_{m} \Rightarrow N_{CO_2,s} \Rightarrow h \Rightarrow T_s$: Guess $T_s = -100^\circ \text{C}$; $P_v = 102.7 \text{ mmHg}$; $M_{CO_2,s} = 0.192$; $B_{m} = 0.237$; $N_{CO_2,s} = 0.01638 \text{ kg/m}^2\cdot\text{s}$; $h = 09.38 \text{ W/m}^2\cdot\text{K}$; $T_s = -99.6^\circ \text{C}$. Note that the calculation of $T_s$ is very sensitive to the initial guess.

Now repeat the calculation using updated property values. Take $T_s = \frac{1}{2}(100 + 5) = -47.5^\circ \text{C}$. For $h^*$, use $M_{CO_2,s} = 0.09$:

$k_{h} = 0.0202 \text{ W/m} \cdot \text{K}$, $M_{h} = 1.47 \times 10^{-5} \text{ kg/m} \cdot \text{s}$, $\rho_{h} = 1560 \text{ kg/m}^3$

$Pr_{h} = 0.730$, $Re_{h} = 7.98 \times 10^4$

The result is $h^* = 96.12$. For $\frac{g_{m}}{g_{m,s}}$, the appropriate reference state is $T_s = -47.5^\circ \text{C}$ and $M_{CO_2,s} = 0.09$ ($N_{CO_2,s} = 0.061)$:

$M_{g} = 29.08 \text{ kg/mol}$, $\rho_{g} = 1.61 \text{ kg/m}^3$, $C_{P_{g}} = 83.7 \text{ J/kg} \cdot \text{K}$

$D_{CO_2-air} = 9.266 \times 10^{-6} \text{ m}^2/\text{s}$

Now compute the mixture viscosity, using (11.128):

$\mu_{\text{air}} = 1.47 \times 10^{-5} \text{ kg/m} \cdot \text{s}$, $\mu_{CO_2} = 1.14 \times 10^{-5} \text{ kg/m} \cdot \text{s}$

$\phi_{12} = 1.403$, $\phi_{21} = 0.7162$; $\mu_{m} = 1.44 \times 10^{-5} \text{ kg/m} \cdot \text{s}$
We have $Re_0 = 5.9 \times 10^4$, $Sc = 0.965$ so $\bar{q}_m = 0.0802 \text{ kg/h}^2\text{s}$.

We now repeat the guessing process for $T_s$:

Guess $T_s = -100^\circ C$ and $B_m = 0.237$ from above; $N_{CO_2,s} = 0.0171 \text{ kg/m}^2\text{s}^2$; $h = 89.6 \text{ W/m}^2\text{K}$; Solve to find $T_s = -104.5^\circ C$.

Guess $T_s = -101^\circ C$; $N_{CO_2,s} = 0.122; B_m = 0.210$; $N_{CO_2,s} = 0.0153 \text{ kg/m}^2\text{s}^2$; $h = 90.2 \text{ W/m}^2\text{K}$; Solve to find $T_s = -92.4^\circ C$.

It appears that the surface temperature is slightly less than $-100^\circ C$ and the sublimation rate is about $0.016 \text{ kg/m}^2\text{s}$. Additional iteration is probably meaningless because of limited accuracy of property data.

11.49 The following data were taken at a weather station over a period of several months:

<table>
<thead>
<tr>
<th>Date</th>
<th>$T_{dry-bulb}$</th>
<th>$T_{wet-bulb}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/15</td>
<td>15.5°C</td>
<td>11.0°C</td>
</tr>
<tr>
<td>4/21</td>
<td>22.0</td>
<td>16.8</td>
</tr>
<tr>
<td>5/13</td>
<td>27.3</td>
<td>25.8</td>
</tr>
<tr>
<td>5/31</td>
<td>32.7</td>
<td>20.0</td>
</tr>
<tr>
<td>7/4</td>
<td>39.0</td>
<td>31.2</td>
</tr>
</tbody>
</table>

Use eqn. (12.126) to find the mass fraction of water in the air at each date. Compare these values to ones obtained using a psychrometric chart.

Use eq'n (11.63): $B_{m,H_2O} = (T_e - T_{w.b.}) \frac{C_p}{h_{fg}(T_{w.b.})} \cdot \frac{L_e}{L_e}^{2/3}$

Set $L_e = 1.18$ and find $M_{H_2O,s}$ and $h_{fg}(T_{w.b.})$ using a steam table. To get $C_p$, note $C_{p,air} = 1003 \text{ J/kg}$ in the range of interest, estimate $C_{p,H_2O} = 2030 \text{ J/kg}$ and use $C_p = \Xi_{i=1} M_{i} C_{p,i}$. Evaluate $C_p$ at $M_{H_2O,s}$ and correct if necessary.
<table>
<thead>
<tr>
<th>Tw.b.</th>
<th>( h_{fg}(\text{Tw.b.}) )</th>
<th>( m_{r_2o} )</th>
<th>( c_p )</th>
<th>( B_{v(h_2o)} )</th>
<th>( M_{H_2O,e} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.0</td>
<td>2475.4</td>
<td>0.00810</td>
<td>1011</td>
<td>0.00165</td>
<td>0.000647</td>
</tr>
<tr>
<td>16.8</td>
<td>2461.6</td>
<td>0.01184</td>
<td>1015</td>
<td>0.00192</td>
<td>0.000944</td>
</tr>
<tr>
<td>25.8</td>
<td>2440.4</td>
<td>0.02067</td>
<td>1024</td>
<td>0.00056</td>
<td>0.02012</td>
</tr>
<tr>
<td>20.0</td>
<td>2454.1</td>
<td>0.01449</td>
<td>1018</td>
<td>0.00472</td>
<td>0.00985</td>
</tr>
<tr>
<td>31.2</td>
<td>2427.6</td>
<td>0.02841</td>
<td>1032</td>
<td>0.00297</td>
<td>0.02553</td>
</tr>
</tbody>
</table>

Psychrometric charts give \( \gamma \) (kg H\(_2\)O/kg air). To convert to \( m_{H_2O} \) note:

\[
m_{H_2O} = \frac{\rho_{H_2O}}{\gamma} = \left(1 + \frac{\rho_{air}}{\rho_{H_2O}}\right)^{-1} = \left(1 + \frac{1}{\gamma}\right)^{-1}.
\]

We can now use a psychrometric chart to prepare another table:

<table>
<thead>
<tr>
<th>Date</th>
<th>( T )</th>
<th>( M_{H_2O,e} ) from chart</th>
<th>( M_{H_2O,e} ) calculated</th>
<th>% diff. difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/15</td>
<td>0.0064</td>
<td>0.00636</td>
<td>0.00647</td>
<td>1.7%</td>
</tr>
<tr>
<td>4/21</td>
<td>0.0098</td>
<td>0.00970</td>
<td>0.00994</td>
<td>2.4</td>
</tr>
<tr>
<td>5/13</td>
<td>0.0206</td>
<td>0.0202</td>
<td>0.02012</td>
<td>0.3</td>
</tr>
<tr>
<td>5/31</td>
<td>0.0095</td>
<td>0.00941</td>
<td>0.00985</td>
<td>4.6</td>
</tr>
<tr>
<td>7/4</td>
<td>0.0260</td>
<td>0.0253</td>
<td>0.02553</td>
<td>0.7</td>
</tr>
</tbody>
</table>
11.50 Biff Harwell has taken Deb sailing. Deb and Biff’s towel fall into the harbor. Biff rescues them both from a passing dolphin, and then spreads his wet towel out to dry on the fiberglass foredeck of the boat. The incident solar radiation is 1050 W/m²; the ambient air is at 31°C with \( m_{H_2O} = 0.017 \); the wind speed is 8 knots relative to the boat (1 knot = 1.151 mph); \( \alpha_{towel} = 1 \); and the sky has the properties of a black body at 280 K. The towel is 3 ft in the windward direction and 2 ft wide. Help Biff to figure out how rapidly (in kg/s) water evaporates from the towel. (Help Biff to understand why Deb won’t date him anymore.)

Construct an energy balance on the towel:

\[
\begin{align*}
\text{s-\& u surfaces:} & \quad q_{\text{conv}} = n_{H_2O} h_f \\
\text{w \& below:} & \quad q_{\text{solar}} + q_{\text{sky}} + q_{\text{u}} = q_{\text{rad}} \\
\text{Fiberglass is a poor conductor, so the heat conducted from the bottom of the towel is } 0. \\
\text{Combining the two balances, we have:} & \quad q_{\text{solar}} + q_{\text{sky}} - q_{\text{rad}} + q_{\text{conv}} = n_{H_2O} s h_f
\end{align*}
\]

In this eqn:

\[
q_{\text{solar}} = 1050 \text{ W/m}^2, \quad q_{\text{sky}} = 5 T^4 \text{ }, \quad q_{\text{rad}} = 5 T_s^4
\]
and we have taken \( \varepsilon = \alpha = 1 \) for the towel.

To begin the calculations, let’s assume low-rates and properties of pure air at a film temperature \( T_f = 36°C \) (i.e. \( T_s = 41°C \)):

\[
\begin{align*}
p_f &= 1.142 \text{ kg/m}^3, \quad k_f = 0.02680 \text{ W/m} \cdot \text{K}, \\
\mu_f &= 1.895 \times 10^{-5} \text{ kg/m} \cdot \text{s}, \quad Pr_f = 0.710, \\
D_{\text{air-water}} &= 2.697 \times 10^{-5} \text{ m}^2/\text{s}, \quad Sc_f = 0.615
\end{align*}
\]

The wind speed is \( U = 8(1.151)(5280)(0.3048)/(18000) = 4.116 \) m/s.

The Reynolds number is \( Re_L = 2.27 \times 10^5 \) (laminar), so

\[
\overline{Nu} = 0.664 Re_L^{\frac{1}{2}} Pr^{\frac{1}{3}}. \text{ From this, } \overline{Nu} = 282.2 \text{ and } \overline{Nu} = 269.0; \text{ solving, we have } h^* = 8.27 \text{ W/m}^2\text{K and }
\]

\[
\]
\[ q_f = 0.009061 \text{ kg/m}^2\cdot\text{s}. \]

In terms of the unknown, \( T_S \), the low rate energy balance is

\[
(1398 \text{ W/m}^2) - \sigma T_S^4 + \dot{h}_f^*(T_e - T_S) = q_f \frac{B_m h_f g}{g_m}
\]

where \( T_e = 304.2^\circ K \) and \( B_m \) and \( h_f g \) depend on \( T_S \).

Solve by iteration on \( T_S \):

\[
\dot{h}_f^*(T_S - 304.2) = 1398 - \sigma T_S^4 - \frac{q_f}{q_m} B_m h_f g
\]

Guess \( T_S \), evaluate the RHS with a steam table, and solve for the value of \( T_S \) implied on the LHS:

<table>
<thead>
<tr>
<th>( T_S )</th>
<th>( m_{H_2O,s} )</th>
<th>( h_f g )</th>
<th>( B_m )</th>
<th>Implied ( T_S )</th>
</tr>
</thead>
<tbody>
<tr>
<td>314.2^\circ K</td>
<td>0.04924</td>
<td>2404.3</td>
<td>0.03391</td>
<td>317.1^\circ K</td>
</tr>
<tr>
<td>315.2</td>
<td>0.05200</td>
<td>2401.9</td>
<td>0.03692</td>
<td>308.5</td>
</tr>
<tr>
<td>314.7</td>
<td>0.05062</td>
<td>2403.1</td>
<td>0.03541</td>
<td>312.8</td>
</tr>
</tbody>
</table>

The surface temperature is slightly higher than 41.0^\circ C and slightly less than 41.5^\circ C. Evaluate the evaporation rate at 41.3^\circ C: \( m_{H_2O,s} = 0.05007 \), \( h_f g = 2403.6 \text{ kJ/kg} \), \( B_m = 0.03481 \); implied \( T_S = 41.3^\circ C \). \( m_{H_2O,s} = (0.009061) \cdot (0.03481) = 0.0003154 \text{ kg/m}^2\cdot\text{s}. \)

Check the assumptions: \( \ln(1+B_m)/B_m = 0.983 \), low rates alright; \( m_{H_2O,s} = 0.0335 \)-only changes will be for \( \rho, \mu, S_c \) but since both \( \mu \) and \( \rho \) decrease the effect on \( V \) is small: the present property values are adequate for engineering purposes. The overall evaporation rate is \( \dot{m} = 3(0.3040)(2)(0.3040)(m_{H_2O,s}) = 1.76 \times 10^{-4} \text{ kg/s}. \)
$\mu$ and $k$ from (11.125, 127) with $C_p = 2040 \text{ J/kg-K}$:

$\mu_{\text{steam}} = 1.127 \times 10^{-5} \text{ kg/m-s}$, $k_{\text{steam}} = 0.02279 \text{ W/m-K}$ ($\gamma = 1$).

From the ideal gas law, $p_{\text{steam}} = 0.05593 \text{ kg/m}^3$. Take $\rho = 1$. The Reynolds number is $Re_L = 9.925$ : laminar.

Calculate $\overline{h^*}$ from equation (6.68): $\overline{h^*} = 6.083 \text{ W/m-K}$.

Finally, calculate $\overline{\dot{q}^*}$. Evaluate properties at $T_f = 310 \text{ K}$ and $W_{\text{air}, f} = 0.30$, as a guess. Use (11.128) for $\mu$: $\mu_{\text{air}} = 1.899 \times 10^{-5} \text{ kg/m-s}$, $\phi_{f_{1}} = 1.015$, $\phi_{f_{2}} = 0.9680$, $\gamma_{1} = 0.2105$, $\gamma_{2} = 0.7895$: $\mu_{f} = 1.291 \times 10^{-5} \text{ kg/m-s}$. From (12.43), $D_{\text{air}, \text{steam}} = 3.433 \times 10^{-4} \text{ m}^2/\text{s}$. $p_f = 0.06288 \text{ kg/m}^3$ and $Sc_f = 0.5981$. Again using (6.68) find $\overline{\dot{q}^*} = 0.004768 \text{ kg/m}^2\text{-s}$ with $Re_L = 9741$.

The operating equations are:

$\bar{T} = (318.2 - T_s) - \bar{n}_{H_2O,s} (2.438 \times 10^6) = 1.197 \times 10^4 (T_s - 298.2)$

$\bar{n}_{H_2O,s} = 0.004768 \ln (1 + B_m)$

$\bar{T} = (\bar{n}_{H_2O,s} C_p_{H_2O}) / [\exp (\bar{n}_{H_2O,s} C_p_{H_2O}/6.083) - 1]$ + Steam table for $M_{H_2O,s} (T_s)$. We solve iteratively for $T_s$:

<table>
<thead>
<tr>
<th>$T_s$ (°C)</th>
<th>$M_{H_2O,s}$</th>
<th>$B_m$</th>
<th>$\bar{n}_{H_2O,s}$</th>
<th>$\bar{T}$</th>
<th>$\bar{T}_s$ (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.0</td>
<td>0.4131</td>
<td>-0.9318</td>
<td>-0.01281</td>
<td>26.49</td>
<td>28.7</td>
</tr>
<tr>
<td>29.0</td>
<td>0.3845</td>
<td>-0.9350</td>
<td>-0.01303</td>
<td>26.93</td>
<td>28.8</td>
</tr>
<tr>
<td>28.9</td>
<td>0.3818</td>
<td>-0.9353</td>
<td>-0.01305</td>
<td>26.97</td>
<td>28.8</td>
</tr>
</tbody>
</table>
Appears that $T_S = 28.8^\circ C$. Check our assumptions now: $T_g = \frac{1}{2}(45 + 28.8)^\circ C = 310.1 K$ for the vapor, $T_g = \frac{1}{2}(25 + 28.8)^\circ C = 300.1 K$ for the liquid; $M_{air,g} = \frac{1}{2}(0.04 + 0.018) = 0.039$ for the mass transfer coeff., and $M_{air} = M_{air,c} = 0.04$ for the heat transfer coeff. Our guesses for the temperature reference states were right on target. Our choices of composition reference state were close enough that additional iteration is probably unnecessary.

Condensation rate, $m_{H_2O,s} = 0.01305 \frac{kg}{m^2 \cdot s}$

(b) Without air, the liquid surface will be at $T_{sat}(8000 Pa) = 41.51^\circ C$. The film temperature is $T_f = 306^\circ K$, close enough to our previous calculation:

$m'' = \frac{(h_g)^{-1} \cdot h_{Nusselt} \cdot (T_{sat} - T_w)}{(24380 \times 10^3)^{\frac{1}{4}}(1.197 \times 10^4)(41.51 - 25.0)^{\frac{3}{4}}}

= 0.04021 \frac{kg}{m^2 \cdot s}$

This small amount of air cuts condensation by 68%!

Air leaks are a very serious problem for condensers.
11.51 Steam condenses on a 25-cm-high, cold vertical wall in a low-pressure condenser unit. The wall is isothermal at 250°C, and the ambient pressure is 8000 Pa. Air has leaked into the unit and reached a mass fraction of 0.04. The steam-air mixture is at 450°C and is blown downward past the wall at 8 m/s. (a) Estimate the rate of condensation on the wall. (b) Compare the result of part (a) to condensation without air in the steam. What do you conclude?

(a)

The energy balance for the wall is:
\[ \bar{h} (T_e - T_s) - \bar{n}_{H_2O,s} h_{fg} = \frac{(k)}{\delta} (T_s - T_w) \]

where \( \bar{n}_{H_2O,s} = \frac{\dot{m}}{\dot{m}_{H_2O}} \cdot \bar{B}_{m,H_2O} < 0 \)

and \( \frac{(k)}{\delta} = \frac{1}{\bar{h}_{Nusselt}} \) from eqn (8.62b)

This condensation problem will be characterized by high rates of suction which will tend to concentrate air near the liquid surface.

First evaluate \( \bar{h}_{Nusselt} \). Guess a property reference temperature of 300 K for the liquid: \( \rho_f = 996.6 \text{ kg/m}^3 \), \( k = 0.6084 \text{ W/m-K} \), \( \mu = 8.23 \times 10^{-4} \text{ kg/m-s} \), \( h_{fg} = 2438.0 \text{ kJ/kg} \). Approximate: \( \rho_f \approx \rho_f' \), \( h_{fg}' \approx h_{fg} \) since \( \rho_f' \) and \( \Delta T \) are small. Substitute into eqn (8.62b)

\[ \bar{h}_{L} = 4.919 \times 10^3 (T_s - T_w)^{1/4} \]

or

\[ \bar{h}_{Nusselt} = 1.197 \times 10^4 (T_s - T_w)^{-1/4} \]

Next evaluate \( \bar{h}_{air} \) for the vapor flow. Guess a property reference temperature of 310 K and take the composition reference state as pure steam; calculate
\( \mu \) and \( k \) from \((12.5, 55)\) with \( c_p = 2040 \text{ J/kg.K} \):

\[
\mu_{\text{steam}} = 1.127 \times 10^{-5} \text{ kg/m-s}, \quad k = 0.02299 \text{ W/m-K} \quad (\gamma = 1)
\]

From the ideal gas law, \( \rho_{\text{steam}} = 0.05593 \text{ kg/m}^3 \). Take \( \beta = 1 \). The Reynolds number is \( Re_l = 9.925 \) : laminar.

Calculate \( \overline{h}^* \) from equation \((7.68)\): \( \overline{h}^* = 6.083 \text{ W/m.K} \).

Finally, calculate \( \overline{g}^*_{\text{m}} \). Evaluate properties at \( T_f = 310 \text{ K} \) and \( m_{\text{air}, f} = 0.30 \), as a guess. Use \((12.57)\) for \( \mu \): \( \mu_{\text{air}} = 1.899 \times 10^{-5} \text{ kg/m-s} \), \( \phi_{\text{e}} = 1.015 \), \( \phi_{1} = 0.9680 \), \( \gamma_1 = 0.2105 \), \( \gamma_2 = 0.7895 \): \( \mu_f = 1.291 \times 10^{-5} \text{ kg/m-s} \). From \((12.43)\), \( \rho_{\text{air,steam}} = 3.433 \times 10^{-4} \text{ m}^3/\text{s} \). \( \rho_f = 0.06288 \text{ kg/m}^3 \) and \( Sc_f = 0.5981 \). Again using \((7.68)\) find \( \overline{g}^*_{\text{m}} = 0.004768 \text{ kg/m}^2/\text{s} \) with \( Re_l = 9741 \).

The operating equations are:

\[
\overline{h} (318.2 - T_s) - \overline{n}_{\text{H}_2\text{O}, s} (2.438 \times 10^6) = 1.197 \times 10^4 (T_s - 298.2)^{3/4}
\]

\[
\overline{n}_{\text{H}_2\text{O}, s} = 0.004768 \ln (1 + B_m)
\]

\[
\overline{h} = (\overline{n}_{\text{H}_2\text{O}, s} c_p_{\text{H}_2\text{O}}) / \left[ \exp (\overline{n}_{\text{H}_2\text{O}, s} c_p_{\text{H}_2\text{O}} / 6.083) - 1 \right]
\]

+ Steam table for \( m_{\text{H}_2\text{O}, s} (T_s) \). We solve iteratively.

\[ \begin{array}{|c|c|c|c|c|c|}
\hline
T_s \quad (\text{C}) & m_{\text{H}_2\text{O}, s} & B_m, \text{H}_2\text{O} & \overline{n}_{\text{H}_2\text{O}, s} & \overline{h} & \overline{T_s} \quad (\text{C}) \\
\hline
30.0 & 0.4131 & -0.9310 & -0.01281 & 26.49 & 28.7 \\
29.0 & 0.3845 & -0.9350 & -0.01303 & 26.93 & 28.0 \\
28.9 & 0.3818 & -0.9353 & -0.01305 & 26.97 & 28.0 \\
\hline
\end{array} \]
Appears that $T_s = 28.8^\circ C$. Check our assumptions now:

\[ T_g = \frac{1}{2} (45 + 28.8)^\circ C = 310.1 \, K \] for the vapor,  
\[ T_l = \frac{1}{2} (25 + 28.8)^\circ C = 300.1 \, K \] for the liquid;  
\[ \frac{W_{air, g}}{W_{air, l}} = \frac{1}{2} (0.04 + 0.618) = 0.329 \] for the mass transfer coeff. and \[ W_{air} = W_{air, c} = 0.04 \] for the heat transfer coeff. Our guesses for the temperature reference states were right on target. Our choices of composition reference state were close enough that additional iteration is probably unnecessary.

Condensation rate, \[ \dot{m}_{H_2O, s} = 0.01305 \frac{kg}{m^2 \cdot s} \]

(b) Without air, the liquid surface will be at $T_{sat}(8000\, Pa) = 41.51^\circ C$. The film temperature is $T_f = 306^\circ K$, close enough to our previous calculation:

\[
\dot{m}'' = (h_{fg})^{-1} \bar{h}_{Nusselt} \cdot (T_{sat} - T_w)
\]

\[
= (2430 \times 10^3)^{1/3} (1.197 \times 10^4) (41.51 - 25.0)^{3/4}
\]

\[
= 0.04021 \, \frac{kg}{m^2 \cdot s}
\]

This small amount of air cuts condensation by 68%!
Air leaks are a very serious problem for condensers.