A Heat Transfer Textbook

Sixth Edition

Solutions Manual for Chapters 4–10

by

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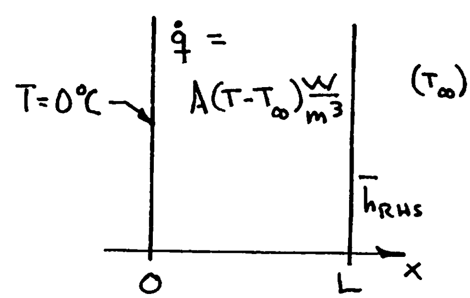
4.1) Make a table listing the general solutions of all steady, uni-dimensional, constant properties, heat conduction problems in Cartesian, cylindrical, and spherical coordinates, with and without uniform heat generation. (This table should prove a very useful tool in future problem work. It should include 18 solutions, all told.)

Geometry		Solution w/o heat generation	Solution with heat generation
cartesian	x-dir.	$T = C_1 x + C_2$	$T = C_1 x + C_2 - \frac{\dot q}{2k} x^2$
	y-dir.	$T = C_1 y + C_2$	$T = C_1 y + C_2 - \frac{\dot q}{2k} y^2$
	z-dir.	$T = C_1 z + C_2$	$T = C_1 z + C_2 - \frac{\dot q}{2k} z^2$
cylindrical	r-dir.	$T = C_1 \ln r + C_2$	$T = C_1 \ln r + C_2 - \frac{\dot q}{4k} r^2$
	θ-dir.	$T = C_1 \theta + C_2$	$T = C_1 \theta + C_2 - \frac{\dot q r^2}{2k} \theta^2$ (where r is some constant value)
	z-dir.	$T = C_1 z + C_2$	$T = C_1 z + C_2 - \frac{\dot q}{2k} z^2$
spherical	r-dir.	$T = C_1 + \frac{C_2}{r}$	$T = C_1 + \frac{C_2}{r} - \frac{\dot q}{6k} r^2$
	θ-dir.	$T = C_1 \ln \tan \frac{\theta}{2} + C_2$	$T = C_1 \ln \tan \frac{\theta}{2} + C_2 + \frac{\dot q r^2}{k} \ln \sin \theta$
	ϕ-dir.	$T = C_1 \phi + C_2$	$T = C_1 \phi + C_1 - \left(\frac{\dot q}{2k} r^2 \sin^2\theta\right) \phi^2$ where in the last two equations r, and r&θ, respectively, must be constants

Some of these solutions will have limited practical value. For example, the θ-dir. solutions will be applicable only to thin cylindrical and spherical shells whose radius is virtually constant. This must also be the case for the ϕ-dir. solution in spherical configurations, but it is also restricted to a narrow longitudinal swath.

4.2 Develop a dimensionless equation for the temperature in the wall shown:



General solution:

$$\frac{d^2T}{dx^2} = -\frac{A}{k}(T - T_\infty)$$

so

$$T - T_\infty = C_1 \cos\sqrt{A/k}\,x + C_2 \sin\sqrt{A/k}\,x$$

b.c.'s: LHS: $\underline{-T_\infty = C_1}$

RHS: $-k\frac{\partial T}{\partial x}\Big|_L = \bar h_{RHS}(T - T_\infty)$ or $T_\infty \sin\sqrt{\frac{A}{k}}L - C_2\cos\sqrt{\frac{A}{k}}L = \underbrace{\frac{\bar h_{RHS}\sqrt{\frac{k}{A}}}{k}}_{Bi_{eff.}}\left(T_\infty \cos\sqrt{\frac{A}{k}}L + C_2 \sin\sqrt{\frac{A}{k}}L\right)$

4.2 (Continued)

$$C_2\left(Bi_e \sin\sqrt{\tfrac{A}{k}}\,L + \cos\sqrt{\tfrac{A}{k}}\,L\right) = Bi_e T_\infty \cos\sqrt{\tfrac{A}{k}}\,L - T_\infty \sin\sqrt{\tfrac{A}{k}}\,L$$

$$\underline{C_2 = T_\infty \frac{Bi_e \cos\sqrt{\frac{A}{k}}\,L - \sin\sqrt{\frac{A}{k}}\,L}{Bi_e \sin\sqrt{\frac{A}{k}}\,L + \cos\sqrt{\frac{A}{k}}\,L}}$$

Thus:

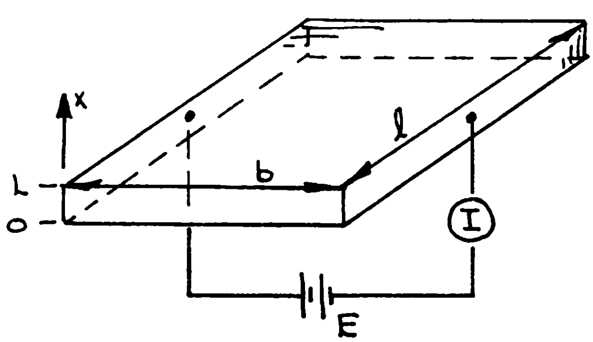
$$\underline{\underline{\frac{T_\infty - T}{T_\infty} = \cos\sqrt{\frac{A}{k}}\,x - \frac{Bi_e \cos\sqrt{\frac{A}{k}}\,L - \sin\sqrt{\frac{A}{k}}\,L}{Bi_e \sin\sqrt{\frac{A}{k}}\,L + \cos\sqrt{\frac{A}{k}}\,L}\sin\sqrt{\frac{A}{k}}\,x}} \longleftarrow$$

Check the limit as $A \Rightarrow 0$:

$$\frac{T_\infty - T}{T_\infty} = 1 - \frac{Bi_e}{Bi_e\sqrt{\frac{A}{k}}\,L + 1}\sqrt{\frac{A}{k}}\,x = 1 - \frac{\frac{\bar{h}_{RHS}L}{k}\frac{1}{L}}{\frac{\bar{h}_{RHS}L}{k}+1}\,x = 1 - \frac{1}{1+\frac{1}{Bi}}\,\xi \quad (ok)$$

4.3) A long wide plate of known size, material, and thickness, L, is connected across the terminals of a power supply and serves as a resistance heater. The voltage, current, and T_∞ are known. The plate is insulated on the bottom and transfers heat out the top by convection. The temperature, T_{tc}, of the bottom is measured with a thermocouple. Obtain expressions for a) temperature distribution in the plate, b) h, at the top, c) temperature at the top. (Note that your answers must depend on known information, only.)

$$\dot{q} = \frac{EI}{Lb\ell} \equiv Bk$$



So: $$\frac{d^2T}{dx^2} + B = 0$$

general solution:

$$T = C_1\left(\frac{x}{L}\right) + C_2 - \frac{BL^2}{2}\left(\frac{x}{L}\right)^2$$

b.c.'s: $T(x=0) = T_{tc}$, the thermocouple reading

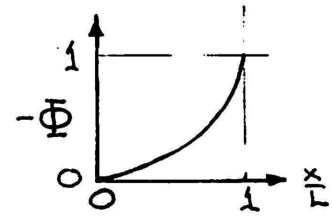
$$\left.\frac{dT}{dx}\right|_{x=o} = 0 \quad \text{, since insulated.}$$

(The b.c.'s are interesting in that both are at x=o. We might have replaced the second one with: $-k\left.\frac{dT}{dx}\right|_{x=\delta} = \frac{EI}{b\ell}$

apply the b.c.'s: $$\begin{cases} T_{tc} = 0 + C_2 - 0; & \underline{C_2 = T_{tc}} \\ 0 = C_1 - 0 & ; \underline{C_1 = 0} \end{cases}$$

4.3 (continued)

Therefore: a.) $\dfrac{T - T_{tc}}{BL^2/2} = -\left(\dfrac{x}{L}\right)^2$



b.) $h = \dfrac{-k(dT/dx)x = L}{T_{x=L} - T_\infty} = \underline{\underline{\dfrac{EI/bl}{T_{top} - T_w}}}$,

c.) $T_{top} = T(x=L) = \underline{\underline{T_{tc} - \dfrac{BL^2}{2}}}$

4.4, 4.5, 4.6 Write the dimensionless functional equation for each of the following situations.

4.4 Heat transfer to a fluid flowing over a plate of length, L.

$\bar{h} = \bar{h}(u_\infty,$	$\mu,$	$\rho,$	$c_p,$	$k,$	$L)$

$\dfrac{W}{m^2\text{-}°C}$	$\dfrac{m}{s}$	$\dfrac{kg}{m\text{-}s}$	$\dfrac{kg}{m^3}$	$\dfrac{J}{kg\text{-}°C}$	$\dfrac{W}{m\text{-}°C}$	m

7 var. in 4 dimensions → 7-4 or 3 pi-groups. We choose:

$$\underline{\underline{\frac{\bar{h}L}{k} = fn\left(\frac{\rho u_\infty L}{\mu}, \frac{\mu c_p}{k}\right)}}$$

see eqn. (6.58) and others that follow it. ←

4.5 Vapor condensing from a pipe. (Call the wavelength, λ.)

$\lambda = \lambda([\rho_f - \rho_g], \quad \sigma, \quad g)$

m $\quad$ kg/m^3 $\quad$ N/m $\quad$ m/s^2; 4 var in 3 dim → 1 pi-group.

$$\underline{\underline{\lambda\sqrt{\frac{g(\rho_f-\rho_g)}{\sigma}} = fn\ (\text{nothing else}) = \text{constant}}}$$ ←

see equation (9.6b)

4.6 Velocity in a condensate film

u m/s $= u(y$ m, g m/s^2, ν m^2/s, δ m)

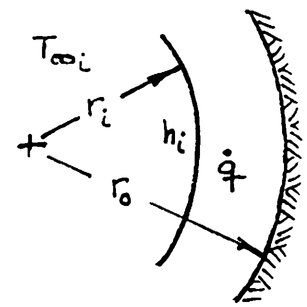
5 variables in 2 dimensions → 3 pi-groups

so

$$\frac{u}{\sqrt{g\delta}} = fn\left(\frac{y}{\delta}, \frac{\nu}{\sqrt{g\delta^3}}\right)$$ ←

We find this situation described by eqn. (8.51) which takes this form when the vapor density is negligible.

4.7 Find the dimensionless temperature distribution in the cylindrical shell shown and plot it for $r_i/r_o = 2/3$. Establish criteria for neglecting convection and internal resistance.



General solution: $T = C_1 \ln r + C_2 - \frac{\dot{q}}{4k} r^2$

if $\Theta \equiv \frac{T - T_{\infty_i}}{\dot{q} r_o^2/4k}$ & $\rho \equiv \frac{r}{r_o}$ this becomes $\underline{\Theta = C_3 \ln \rho + C_4 - \rho^2}$

with b.c.'s: $\left.\frac{\partial T}{\partial r}\right|_{r=r_o} = 0$ or $\underline{\left.\frac{d\Theta}{d\rho}\right|_{\rho=1} = 0}$

and: $\left.\frac{\partial (T - T_{\infty_i})}{\partial r}\right|_{r=r_i} = \frac{h_i}{k}(T - T_{\infty_i})_{r=r_i}$ or $\underline{\left.\frac{d\Theta}{d\rho}\right|_{\rho=\rho_i} = \mathrm{Bi}\,\Theta_{\rho=\rho_i}}$

impose the first b.c. on the gen'l. sol'n.: $0 = C_3 - 2$, $\underline{C_3 = 2}$

impose 2nd b.c.: $\frac{C_3}{\rho_i} - 2\rho_i = \mathrm{Bi}\left(C_3 \ln \rho_i + C_4 - \rho_i^2\right)$, but $C_3 = 2$

so $\underline{C_4 = \frac{2}{\mathrm{Bi}}\left(\frac{1}{\rho_i} - \rho_i\right) - 2\ln\rho_i + \rho_i^2}$

Return to the gen'l solution with these constants:

$$\Theta = 2\ln\rho + \frac{2}{\mathrm{Bi}}\left(\frac{1-\rho_i^2}{\rho_i}\right) - 2\ln\rho_i + \rho_i^2 - \rho^2$$

or

$$\underline{\underline{\Theta = -(\rho^2 - \rho_i^2) + 2\ln\frac{\rho}{\rho_i} + \frac{2}{\mathrm{Bi}}\left(\frac{1-\rho_i^2}{\rho_i}\right)}}$$

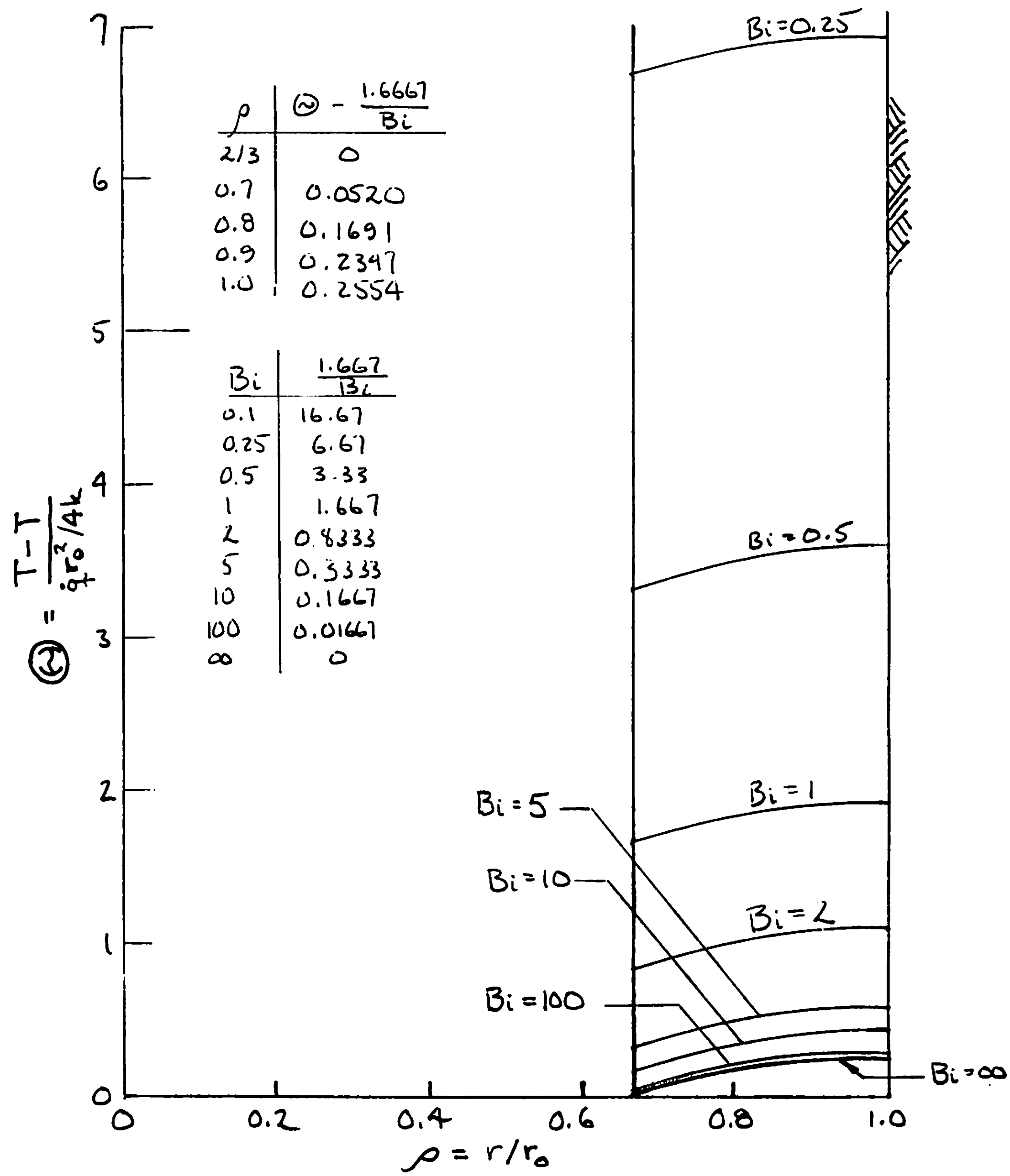
Notice that for Bi large this approaches $\Theta = -(\rho^2 - \rho_i^2) + 2\ln\frac{\rho}{\rho_i}$

and " " small " " $\Theta = \frac{2}{\mathrm{Bi}}\left(\frac{1-\rho_i^2}{\rho_i}\right)$

4.7 (continued)

for $\rho_i = 2/3$ the equation reduces to:

$$\Theta = -(\rho^2 - 0.4444) + 2\ln 1.5\rho + \frac{1.6667}{Bi}$$



ρ	$\Theta - \frac{1.6667}{Bi}$
2/3	0
0.7	0.0520
0.8	0.1691
0.9	0.2347
1.0	0.2554

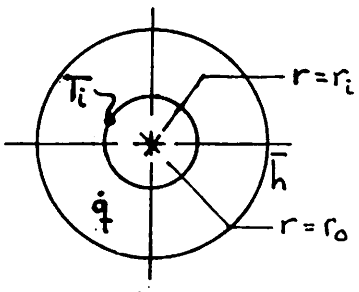
Bi	$\frac{1.667}{Bi}$
0.1	16.67
0.25	6.67
0.5	3.33
1	1.667
2	0.8333
5	0.3333
10	0.1667
100	0.01667
∞	0

When Bi = 0.25, the temperature distribution within the tube wall is within 4 percent of uniform (0.8 percent at Bi = 0.1, etc.)

When Bi = 100, the temperature drop through h_i is 6 percent of that inside the tube wall (3 percent at Bi = 200, etc.)

4.8 Steam condenses in a small pipe keeping the inside at a temperature, T_i. The pipe releases $\dot{q}$ W/m^3 within its walls as a result of electric current flowing through it. The outside temperature is T_∞ and there is a heat transfer coefficient $\bar{h}$ on the outside. a) Evaluate the dimensionless temperature distribution in the pipe. b) Plot the result for an inside radius that is 2/3 of the outside radius. c) Discuss interesting aspects of the result.

a)



(T_∞)

1) $T = T(r)$

2) $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = -\frac{\dot{q}}{k}$

3) $T = -\frac{\dot{q}}{4k}r^2 + C_1 \ln r + C_2$

4) $T(r = r_i) = T_i$

$$\bar{h}(T - T_\infty)_{r=r_o} = -k\frac{\partial T}{\partial r}\bigg|_{r=r_o}$$

5) $\underline{C_2 = T_i + \frac{\dot{q}}{4k}r_i^2 - C_1 \ln r_i}$

$$-\frac{\dot{q}}{4k}r_o^2 + C_1 \ln r_o + C_2 - T_\infty = +\frac{k}{\bar{h}}\left(\frac{\dot{q}r_o}{2k} - \frac{C_1}{r_o}\right)$$

Combine: $$\frac{\dot{q}}{4k}(r_i^2 - r_o^2) + C_1 \ln r_o/r_i + \underbrace{(T_i - T_\infty)}_{\equiv \Delta T} = \frac{\dot{q}r_o}{2\bar{h}} - \frac{C_1}{Bi}$$

$$\therefore\ C_1 = \underline{\frac{\frac{\dot{q}r_o^2}{4k\Delta T}\left[\frac{2}{Bi} - \left(\frac{r_i}{r_o}\right)^2 + 1\right] - 1}{\ln\frac{r_o}{r_i} + \frac{1}{Bi}}\,\Delta T}$$

6) $$\frac{T - T_i}{T_i - T_\infty} = -\frac{\dot{q}r_o^2}{4k\Delta T}\left[\left(\frac{r}{r_o}\right)^2 - \frac{\frac{2}{Bi} - \left(\frac{r_i}{r_o}\right)^2 + 1}{\ln\frac{r_o}{r_i} + \frac{1}{Bi}}\ln r - \left(\frac{r_i}{r_o}\right)^2 + \frac{\frac{2}{Bi} - \left(\frac{r_i}{r_o}\right)^2 + 1}{\ln\frac{r_o}{r_i} + \frac{1}{Bi}}\ln r_i\right] - \frac{\ln r/r_i}{\ln\frac{r_o}{r_i} + \frac{1}{Bi}}$$

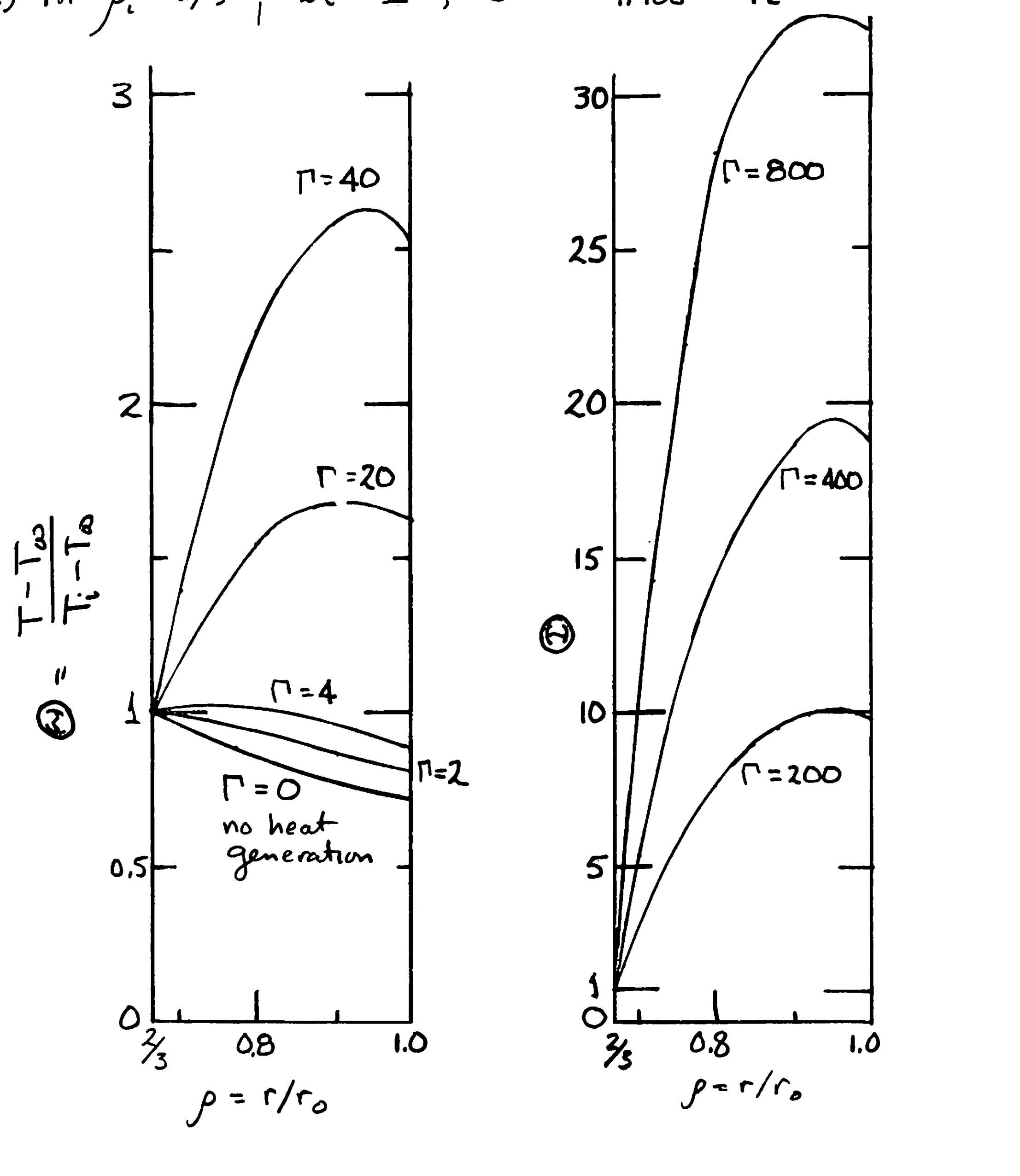
$$\frac{T - T_\infty + T_\infty - T_i}{T_i - T_\infty} = \frac{\Gamma}{4}\left(\rho^2 - \rho_i^2 - \frac{\frac{2}{Bi} - \rho_i^2 + 1}{-\ln\rho_i + \frac{1}{Bi}}\ln(\rho/\rho_i)\right) - \frac{\ln\rho/\rho_i}{\frac{1}{Bi} - \ln\rho_i}$$

4.8 (continued)

This can be rewritten as

$$\Theta = 1 - \frac{\Gamma}{4}\left[\rho^2 - \rho_i^2 + \frac{\frac{2}{Bi} + 1 - \rho_i^2}{\ln \rho_i - \frac{1}{Bi}} \ln \frac{\rho}{\rho_i}\right] - \frac{\ln \rho/\rho_i}{\frac{1}{Bi} - \ln \rho_i}$$

b.) for $\rho_i = 2/3$ & $Bi = 1$, $\Theta = 1 - \frac{\ln 3\rho/2}{1.405} - \frac{\Gamma}{4}\left[\rho^2 - \frac{4}{9} - 1.818 \ln \frac{3\rho_i}{2}\right]$



We see that internal heat generation becomes important for $\Gamma > O(1)$. For $\Gamma \gtrsim 2$, that the temperature maximizes within the shell.

4.9 Solve Problem 2.5, putting it in dimensionless form first. With reference to the Problem 2.5 solution, we repeat the steps as follows.

Step 3) $T = C_1 \ln r + C_2$ becomes $\underline{\dfrac{T - T_{\infty_i}}{T_{\infty_o} - T_{\infty_i}} \equiv \Theta = C_3 \ln \rho + C_4}$, $\rho \equiv \dfrac{r}{r_i}$

step 4) $$\left.\frac{\partial \Theta}{\partial \rho}\right|_{\rho=1} = Bi_i\, \Theta_{\rho=1}$$

$$\left.\frac{\partial \Theta}{\partial \rho}\right|_{\rho_o} = Bi_o(1 - \Theta_{\rho_o})/\rho_o$$

step 5) $C_3 = Bi_i C_4$ & $\dfrac{C_3}{\rho_o} = Bi_o(1 - C_3 \ln \rho_o - \underbrace{C_4}_{= C_3/Bi_i})/\rho_o$

or

$C_4 = \dfrac{C_3}{Bi_i}$ & $C_3 = 1\Big/\left(\dfrac{1}{Bi_o} + \dfrac{1}{Bi_i} + \ln \rho_o\right)$

step 6) $$\Theta = \frac{\ln \rho + 1/Bi_i}{\frac{1}{Bi_o} + \frac{1}{Bi_i} + \ln \rho_o}$$

Same result as in Prob. 2.5 with a lot less algebra

when we allow Bi_i and $Bi_o \Rightarrow \infty$, $\Theta \Rightarrow \dfrac{\ln r/r_i}{\ln r/r_o}$ cf. Ex. 2.4

4.10 Complete the algebra leading to equation (4.41).

we have: $\dfrac{d^2(T-T_\infty)}{dx^2} = \dfrac{\bar{h}P}{kA}(T-T_\infty)$ or $\dfrac{d^2\Theta}{d\xi^2} = (mL)^2\Theta$, so $\underline{\Theta = C_1 e^{mL\xi} + C_2 e^{-mL\xi}}$

subject to: $(T-T_\infty)_{x=0} = (T_o - T_\infty)$ or $\Theta(\xi=0) = 1$, so $\underline{1 = C_1 + C_2}$

and to: $\left.\dfrac{d(T-T_\infty)}{dx}\right|_{x=L} = 0$ or $\left.\dfrac{d\Theta}{d\xi}\right|_{\xi=1} = 0$, so $\underline{0 = C_1 e^{mL} - C_2 e^{-mL}}$

Now put $C_1 = 1 - C_2$ from 1st b.c., in 2nd b.c. & get: $\underline{C_2 = \dfrac{e^{mL}}{e^{mL}+e^{-mL}} = \dfrac{1}{2}\dfrac{e^{mL}}{\cosh mL}}$

Then:
$$\Theta = \frac{2e^{mL\xi}\cosh mL - e^{mL(1+\xi)} + e^{mL(1-\xi)}}{2\cosh mL} = \frac{\cancel{e^{mL(1+\xi)}} + e^{-mL(1-\xi)} - \cancel{e^{mL(1+\xi)}} + e^{mL(1-\xi)}}{2\cosh mL}$$

so:
$$\Theta = \frac{\cosh mL(1-\xi)}{\cosh mL}$$

Problem 4.11 Derive eqn. (4.48)

Solution

We already have the dimensionless form of the general solution of eqn. (4.30) in eqn. (4.35)

$$\Theta = C_1 e^{mL\xi} + C_2 e^{-mL\xi}$$

and the dimensionless form of the b.c.s. (eqn. 4.31a) given in eqn. (4.46).

We put the solution in the two b.c.'s and get:

$$\Theta(\xi=0) = 1 \Rightarrow 1 = C_1 + C_2 \quad \text{or} \quad \underline{C_1 = 1-C_2}$$

$$\left.\frac{d\Theta}{d\xi}\right|_{\xi=1} = -Bi_{ax}\,\Theta(\xi=1) \Rightarrow mLe^{mL}C_1 - mLe^{-mL}C_2 = -Bi_{ax}(C_1 e^{mL} + C_2 e^{-mL})$$

We put $C_1 = 1-C_2$ in this, rearrange it, and get:

$$C_2 = \frac{e^{mL} + \frac{Bi_{ax}}{mL}e^{mL}}{2\left(\cosh mL + \frac{Bi_{ax}}{mL}\sinh mL\right)}$$

Put this C_2 in $\Theta = (1-C_2)^{mL\xi} + C_2 e^{-mL\xi}$ and get:

$$\Theta = \frac{2e^{mL\xi}\left(\cosh mL + \frac{Bi_{ax}}{mL}\sinh mL\right) - \left(e^{mL} + \frac{Bi_{ax}}{mL}e^{mL}\right)e^{mL\xi} + \left(e^{mL} + \frac{Bi_{ax}}{mL}\right)e^{-mL\xi}}{2\left(\cosh mL \quad \frac{Bi_{ax}}{mL}\sinh mL\right)}$$

$$= \frac{\cancel{e^{mL(1+\xi)}} + e^{-mL(1-\xi)} + \frac{\cancel{Bi_{ax}}}{mL}\cancel{e^{mL(1+\xi)}} - \cancel{e^{mL(1+\xi)}} - \cancel{\frac{Bi_{ax}}{mL}e^{mL(1+\xi)}}}{}$$

(continued)

$$\frac{e^{mL(1-\xi)} + \frac{Bi_{ax}}{mL}e^{mL(1-\xi)}}{2\left(\cosh mL + \frac{Bi_{ax}}{mL}\sinh mL\right)}$$

$$\Theta = \frac{\frac{1}{2}\left[e^{mL(1-\xi)} + e^{mL(1-\xi)}\right] + \frac{1}{2}\frac{Bi_{ax}}{mL}\left[e^{mL(1-\xi)} - e^{-mL(1-\xi)}\right]}{\cosh mL + \frac{Bi_{ax}}{mL}\sinh mL}$$

or, finally:

$$\underline{\underline{\Theta = \frac{\cosh mL(1-\xi) + \frac{Bi_{ax}}{mL}\sinh mL(1-\xi)}{\cosh mL + \frac{Bi_{ax}}{mL}\sinh mL(1-\xi)}}}$$

4.12 Obtain the infinite fin result: $\Theta = e^{-mL\xi}$ by starting with the general fin solution: $\Theta = C_1 e^{mL\xi} + C_2 e^{-mL\xi}$ and using the b.c.'s $T(x=0) = T_o$ or $\Theta(\xi=0) = 1$ and $T(x=L) = T_\infty$ or $\Theta(\xi=L) = 0$. Discuss fully.

From the first b.c. we get $C_1 = 1-C_2$ as before. From the second b.c. we obtain: $0 = C_1 e^{mL} + C_2 e^{-mL}$

$$\text{or} \quad 0 = e^{mL} - C_2 \underbrace{(e^{mL} - e^{-mL})}_{\equiv\, 2 \sinh mL}$$

$$\text{or} \quad C_2 = \frac{e^{mL}}{2 \sinh mL}$$

Then the general solution becomes:

$$\Theta = e^{-mL\xi} - C_2 (e^{mL\xi} - e^{-mL\xi}) = \frac{e^{mL\xi}(e^{mL}-e^{-mL}) - e^{mL(1+\xi)} + e^{mL(1-\xi)}}{2 \sinh mL}$$

$$\Theta = \frac{\cancel{e^{mL(1+\xi)}} - e^{-mL(1-\xi)} - \cancel{e^{mL(1+\xi)}} + e^{mL(1-\xi)}}{2 \sinh mL}$$

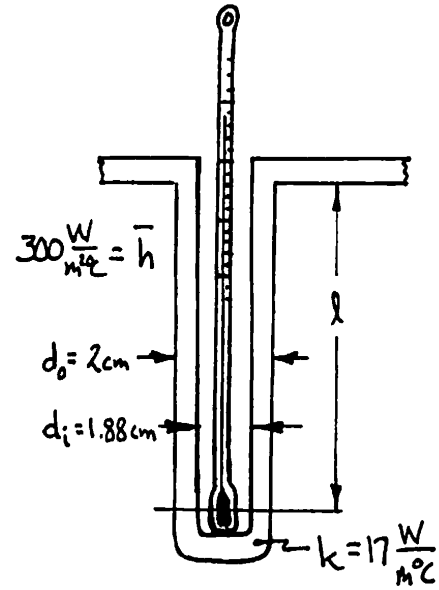
$$\Theta = \frac{\sinh mL(1-\xi)}{\sinh mL}$$

When x is large, $\sinh x = \frac{e^x - e^{-x}}{2} \rightarrow \frac{e^x}{2}$. Therefore, when mL is large

$$\underline{\underline{\Theta \rightarrow \frac{e^{mL(1-\xi)}}{e^{mL}} = e^{-mL\xi}}} \quad \text{which is eqn. (4.50)} \longleftarrow$$

It follows that these b.c.s are not enough, by themselves, to get the infinite fin result. We must also require that the fin <u>be</u> very long.

4.13 How long must ℓ be to guarantee an error less than than 0.5 percent in the thermometer well shown.



Find $m\ell$

$$m\ell = \sqrt{\frac{\bar{h}P\ell^2}{kA}} = \sqrt{\frac{\bar{h}\,\pi d_o \ell^2 4}{k\,\pi(d_o^2 - d_i^2)}}$$

$$= \sqrt{\frac{300(0.02)4}{17(0.0004 - 0.00035344)}}\;\ell = 174\,\ell$$

As long as $\ell > 0.0172$ m, $m\ell$ will be greater than 3 and we can use the "finite fin" approximation.

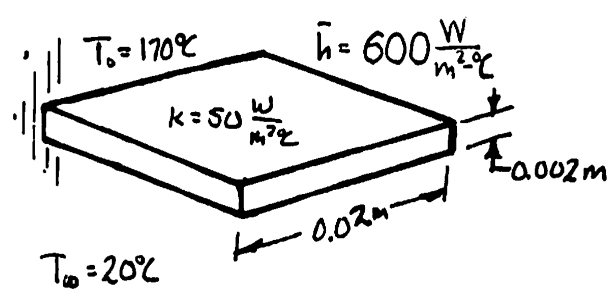
$$\Theta = \frac{1}{\cosh m\ell} \leq 0.005 \quad , \quad \therefore \quad e^{m\ell} + e^{-m\ell} = 400$$

This is true for $m\ell = 5.992$.

Therefore $\quad 5.992 = 174\,\ell \quad$ so $\quad \ell = 0.03444$ m

This means that the well must only be <u>3.44 cm</u> in length to guarantee the required accuracy.

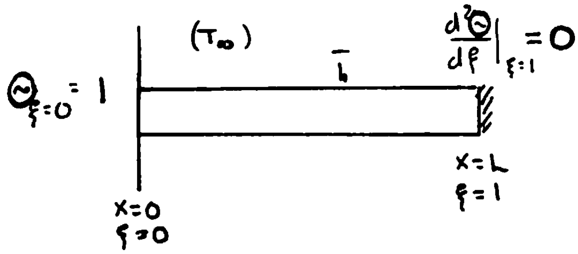
4.14 What is the maximum possible heat flux from the fin shown.



$$Q = \sqrt{kA\bar{h}P}\,(T_o - T_\infty)\underbrace{\tanh mL}_{\text{max value for long L is 1}} = \sqrt{50(.00004)(600)2(.002+.02)}\,(170-20)$$

$$= \underline{\underline{34.5\text{ W}}}$$

4.15) A thin rod is anchored at a wall at $T=T_0$ on one end, and is insulated at the other end. Plot the dimensionless temperature distribution in the rod as a function of dimensionless length: a) if the rod is exposed to an environment at T_∞ through a heat transfer coefficient, and b) if the rod is insulated but heat is consumed in it at the uniform rate $-\dot{q}=\bar{h}P(T_0-T_\infty)/A$. Comment on the implications of the comparison.



$$\frac{d^2\Theta}{d\xi^2} = (mL)^2\Theta \quad \text{in case a)}$$
$$= (mL)^2 \quad \text{in case b)}$$

case a) We already know the solution. It is $\underline{\Theta_a = \dfrac{\cosh mL(1-\xi)}{\cosh mL}}$

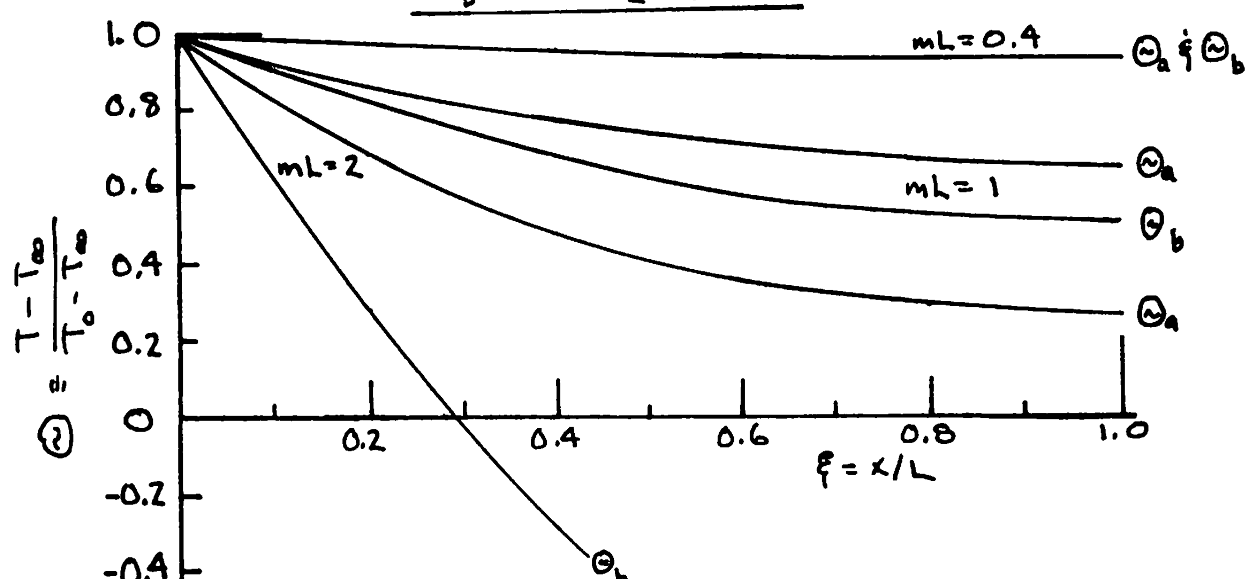
case b) $\dfrac{d^2\Theta}{d\xi^2} = (mL)^2 = \underline{\text{constant}}$ so we integrate twice and get:

the general solution: $\Theta = \dfrac{(mL)^2}{2}\xi^2 + C_1\xi + C_2$

Apply 1st b.c.: $\Theta(\xi=0) = \underline{C_2 = 1}$

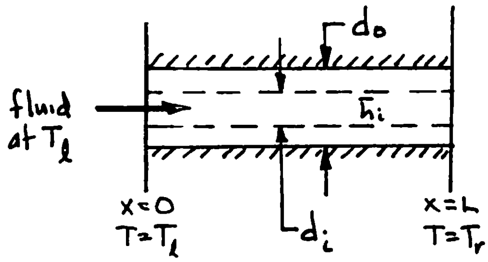
Apply 2nd b.c.: $\left.\dfrac{d^2\Theta}{d\xi^2}\right|_{\xi=1} = (mL)^2 + C_1 = 0$, $\underline{C_1 = -(mL)^2}$

Therefore $\underline{\Theta_b = (mL)^2\left(\frac{\xi^2}{2} - \xi\right) + 1}$



When Θ is close to unity, $\bar{h}P(T_0 - T_\infty)/A \simeq \bar{h}P(T - T_\infty)/A$, or $(mL)^2\Theta \simeq (mL)^2$, and the problems (and their solutions) become identical. As Θ becomes < 1 the solutions diverge. the energy consumption in case b is unabated and Θ_b is generally $< \Theta_a$.

4.16 Consider the tube shown below. Fluid enters the tube on the left at $T = T_\ell$. Assume its temperature to remain constant. Evaluate and plot the temperature distribution in the tube.



$$\frac{d^2(T-T_\ell)}{dx^2} = -\frac{\dot{q}}{k} = \frac{\pi d_i \bar{h}_i (T_\ell - T)}{\frac{\pi}{4}(d_o^2 - d_i^2)\,k}$$

or $$\frac{d^2\Theta}{d\xi^2} = (mL)^2\Theta$$

where $\Theta \equiv \dfrac{T-T_\ell}{T_r-T_\ell}$, $m = \sqrt{\dfrac{4\bar{h}_i}{k d_i\left(\frac{d_o^2}{d_i^2}-1\right)}}$

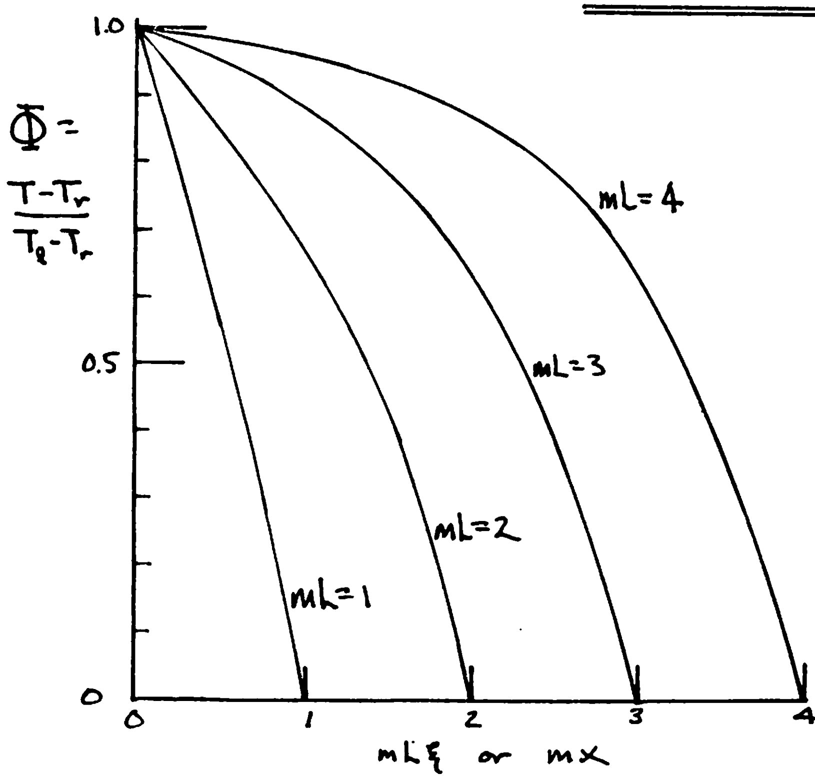
Gen'l. soln.: $\Theta = C_1 e^{mL\xi} + C_2 e^{-mL\xi}$

left b.c.: $\Theta(\xi=0) = 0 = C_1 + C_2$ so $C_1 = -C_2$

right b.c.: $\Theta(\xi=1) = 1 = C_2(e^{-mL} - e^{mL})$ so $C_2 = -1/2\sinh mL$

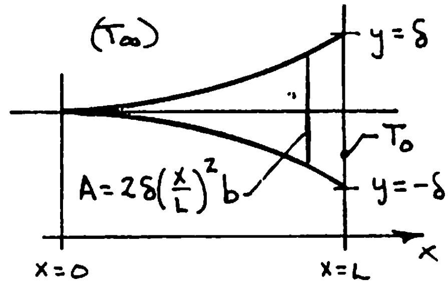
Thus: $\underline{\underline{\Theta = \dfrac{\sinh mL\xi}{\sinh mL}}}$ or we might better define $\Phi \equiv \dfrac{T-T_r}{T_\ell - T_r}$

so: $\underline{\underline{\Phi = 1 - \Theta = 1 - \dfrac{\sinh mL\xi}{\sinh mL}}}$



Notice that when m is small, the influence of convection is also small and the temperature distribution is almost linear as it would be in pure conduction. In the other extreme -- the convection dominated or large m case -- the temperature distribution remains near T_ℓ, except as it approaches the right-hand wall.

4.17 Plot the temperature distribution in the fin shown and evaluate η_t.



Eqn. (4.57) becomes:

$$\frac{d}{d\xi}\left[2\delta(\xi^2)b\,\frac{d\frac{T-T_\infty}{T_0-T_\infty}}{d\xi}\right]=\frac{\bar{h}P}{k}L^2\underbrace{\frac{T-T_\infty}{T_0-T_\infty}}_{\equiv\Theta}$$

or: $$\xi^2\frac{d^2\Theta}{d\xi^2}+2\xi\frac{d\Theta}{d\xi}-\underbrace{\frac{\bar{h}PL^2}{k(2\delta b)}}_{\equiv(mL)^2}\Theta=0$$

To solve this (Euler's d.e.) we look for a solution of the form: $\Theta=c\xi^P$

so: $$P(P-1)\xi^P+2P\xi^P-(mL)^2\xi^P=0 \quad \text{or} \quad \underline{P^2+P-(mL)^2=0}$$

This has two solutions: $$P_1 \text{ and } P_2=\pm\sqrt{\tfrac{1}{4}+(mL)^2}-\tfrac{1}{2}$$

so the general solution is: $$\Theta=C_1\xi^{P_1}+C_2\xi^{P_2}$$

and the usual b.c.'s give: $$\Theta(\xi=1)=1=C_1+C_2$$
$$\Theta(\xi=0)=0=\left[C_1\xi^{P_1}+C_2\xi^{P_2}\right]_{\xi=0}$$

Notice that P_2 must be negative so C_2 must be zero to satisfy this. Therefore $C_2=0$ and $C_1=1$, and:

$$\underline{\underline{\Theta=\xi^{\sqrt{\frac{1}{4}+(mL)^2}-\frac{1}{2}}}}$$

The efficiency is:

$$\eta_f=\frac{\int_0^L\bar{h}(T-T_\infty)b\,dx}{\bar{h}(T_r-T_\infty)bL}=\int_0^1\Theta(\xi)\,d\xi$$

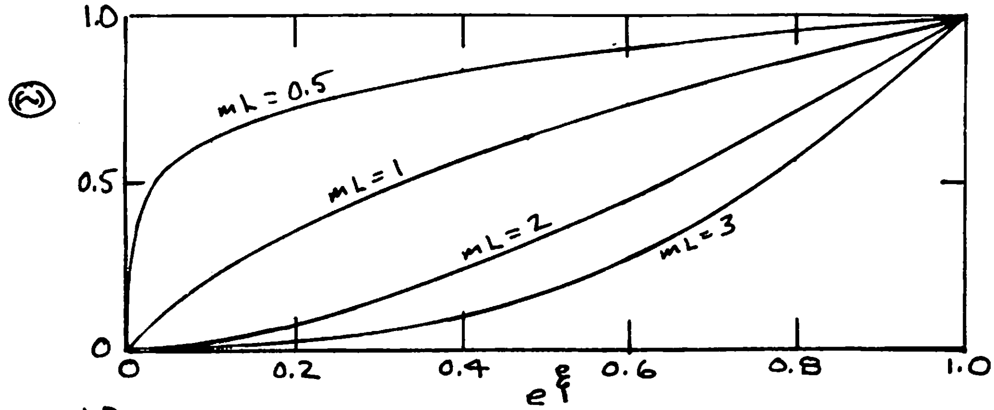
$$=\int_0^1\xi^{\sqrt{\frac{1}{4}+(mL)^2}-\frac{1}{2}}d\xi=\frac{1}{\sqrt{\frac{1}{4}+(mL)^2}+\frac{1}{2}}\,\xi^{\sqrt{\frac{1}{4}+(mL)^2}-\frac{1}{2}}\Bigg]_0^1$$

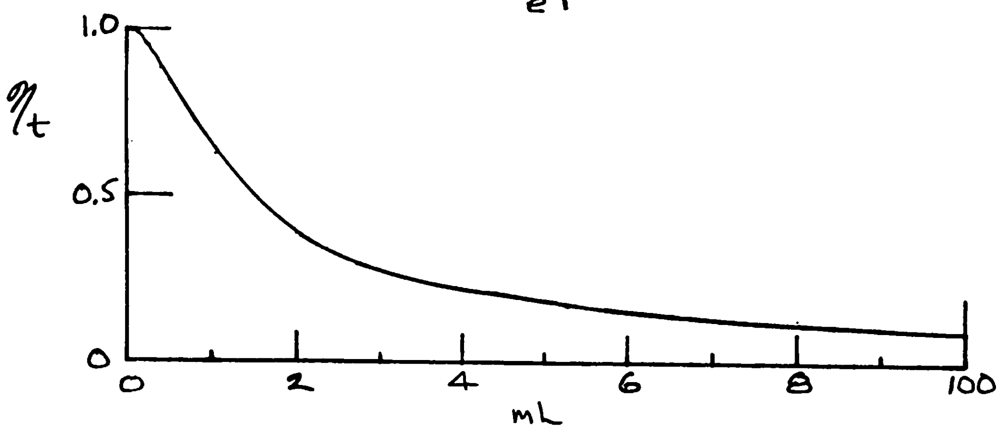
so:

$$\underline{\underline{\eta_f=\frac{2}{\sqrt{1+4(mL)^2}+1}}}$$

(cont'd.)

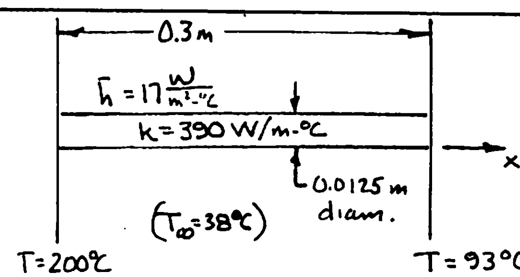
Before we plot these results, we note that mL in this case is the same as $mL(L/P)^{1/2}$ in Fig. 4.13.





4.18 Problem 4.18 was solved under a full nondimensionalization in the solution given for 2.21. We do not repeat it here.

4.19 A fin connects two walls as shown. How much heat is removed from its surface?



$$\Theta = C_1 e^{mx} + C_2 e^{-mx}$$

$$\left[\Theta \equiv \frac{T - T_\infty}{T_{left} - T_\infty}\right] \qquad \text{b.c.'s:} \quad \underline{\Theta_l = 1 = C_1 + C_2} \;;\quad C_1 = 1 - C_2$$

$$\underline{\Theta_r = C_1 e^{ml} + C_2 e^{-ml}}$$

Then: $\Theta_r = (1 - C_2)e^{ml} + C_2 e^{-ml}$; $\underline{C_2 = \dfrac{e^{ml} - \Theta_r}{2 \sinh ml}}$ $\quad \underline{C_1 = 1 - \dfrac{e^{ml} - \Theta_r}{2 \sinh ml}}$

so:

$$\underline{\underline{\Theta = \left(1 - \frac{e^{ml} - \Theta_r}{2 \sinh ml}\right) e^{mx} + \frac{e^{ml} - \Theta_r}{2 \sinh ml}\, e^{-mx}}}$$

4.19 (continued)

$$Q = -kA\Delta T\left[\frac{\partial\Theta}{\partial x}\bigg|_{x=0} - \frac{\partial\Theta}{\partial x}\bigg|_{x=L}\right] = -kmA\Delta T\left[\left(1-\frac{e^{-mL}-\Theta_r}{2\sinh mL}\right)\left(1-e^{mL}\right) + \frac{e^{mL}-\Theta_r}{2\sinh mL}\left(1-e^{-mL}\right)\right]$$

$$= 390\,\pi(.00625)^2(200-38)\underbrace{\sqrt{\frac{17(4)}{390(.0125)}}}_{m=3.735}\left[\Bigg(1-\underbrace{\frac{3.065-\frac{53-38}{200-38}}{e^{1.12}-e^{-1.12}}}_{0.9952}\Bigg)\underbrace{\left(1-e^{1.12}\right)}_{-2.065}+0.9952\underbrace{\left(1-e^{-1.12}\right)}_{0.6705}\right]$$

$$= \underline{\underline{19.13\text{ W}}} \longleftarrow$$

4.20 How much error does the insulated tip assumption giverise to in example 4.8?

Calculate $\%\,\text{error} = \dfrac{Q_{ins.} - Q_{unins.}}{Q_{unins.}}$ using eqns. (4.44) and (4.48):

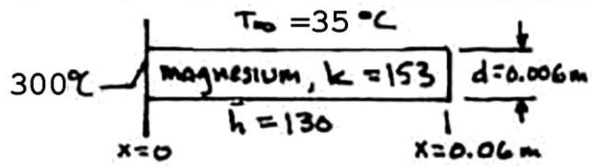
$$\%\,\text{error} = \frac{\tanh mL\left(1+\frac{Bi_{ax}}{mL}\tanh mL\right) - \frac{Bi_{ax}}{mL} - \tanh mL}{\frac{Bi_{ax}}{mL} + \tanh mL}\,100$$

$$= \frac{\frac{Bi_{ax}}{mL}\left(\tanh^2 mL - 1\right)}{\frac{Bi_{ax}}{mL} + \tanh mL}\,100$$

From Example 4.8, $mL = 0.8656$, $\dfrac{Bi_{ax}}{mL} = \dfrac{0.0468}{0.8656} = 0.0541$, so

the error $= \underline{\underline{3.67\,\%}} \longleftarrow$

4.21 Compute the heat removed from the fin shown, con- root depression. Assume the tip to be insulated.



$$mL = \sqrt{\frac{130(\pi)(0.006)}{153(\pi)(.003)^2}}\,(0.06) = 1.428$$

$$Q = \sqrt{k A \bar{h} P}\,(\Delta T)\tanh mL = \sqrt{153\,\pi^2(.003)^2(.006)130}\,(265)(0.8913) = \underline{\underline{24.38\text{ W}}}$$

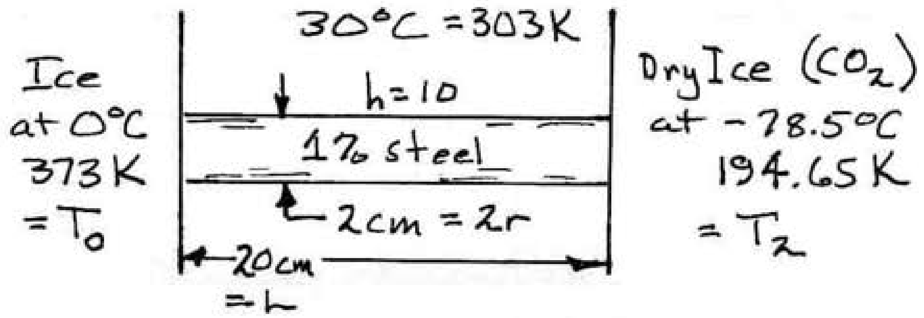
The fin efficiency, $\eta_f = \tanh(mL)/mL = 0.8913/1.428 = 0.624 = \underline{62.4\%}$

The fin effectiveness, $\epsilon = \eta_f$ (fin surface area)/fin cross-sectional area

$\epsilon = 0.624(2\pi rL/\pi r^2) = 1.248L/r = \underline{\underline{25}}$

4.22 A 2 cm dia. horizontal 1.0% steel rod connects a block of ice with a block of dry ice (CO_2) in a 30°C room. The frozen blocks are insulated from the room. The rod is embedded in each block with a 20 cm span between the blocks. The heat transfer coefficient between the rod and the room is 10 W/m²K. Will the ice begin to melt when the rod is at steady state?

Solution We need to determine whether the temperature gradient in rod is positive or negative where it enters the ice on the left. If it is positive, heat will flow into the ice and it will begin to melt. Fortunately, we have already solved for the temperature distribution in a "fin" with specified temperatures at two ends, in Problem 4.19. We need only differentiate that expression for temperature, and determine whether the slope is positive or negative.



$$mL = \sqrt{\frac{hPL^2}{kA}} \quad \text{but } P/A = \frac{2\pi r}{\pi r^2} = \frac{2}{r}$$

$$= \sqrt{\frac{2hL^2}{k_s r}} = \sqrt{\frac{2(10)(0.2)^2}{43(0.01)}} = \underline{1.364}$$

We obtain from the solution of Problem 4.19:

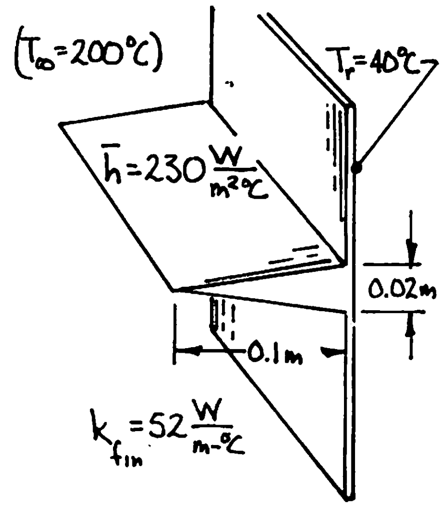
$$\Theta = \left(1 - \frac{e^{mL} - \Theta_2}{2\sinh mL}\right)e^{mx} + \left(\frac{e^{mL} - \Theta_2}{2\sinh mL}\right)e^{-mx}$$

Where $\Theta = \dfrac{T - T_\infty}{T_0 - T_\infty} = \dfrac{T - 303}{273 - 303}$ so $\Theta_0 = 1$, $\Theta_2 = 3.612$

Then $\left.\dfrac{d\Theta}{dx}\right|_{x=0} = m\left[1 - \dfrac{e^{mL} - 3.612}{\sinh(1.364)}\right]$ = 0.836m

The slope is thus positive & the ice will melt.

4.23 Compute the heat removed by the fin shown.



$m = \sqrt{\frac{\bar{h}P}{kA}}$ where $A = \frac{1}{2}(0.1)(0.02)$
$= 0.001 \, m^2$

$mL\sqrt{\frac{L}{P}} = \sqrt{\frac{\bar{h}L}{kA}}\, L = \sqrt{\frac{230(0.1)}{52(0.001)}}\, 0.1 = 2.1$

This gives an efficiency, from Fig. 4.13b, of: $\underline{\eta_t = 0.415}$

$$Q = \eta_t \left(A\bar{h}\,[200-40]\right) = 0.415\left(2\sqrt{0.1^2+0.01^2}\right)(230)(160) = 3070 \, \frac{W}{m}$$

4.24 The initial temperature distribution in a slab of width, L, is:
$\frac{T-T_w}{\dot{q}L^2/k} = \frac{1}{2}\left(\frac{x}{L} - \left(\frac{x}{L}\right)^2\right)$ where $\dot{q}L^2/k$ can be viewed as a constant, A, with the units of temperature. The sides are kept at T_w and the slab is permitted to cool. Predict $(T-T_w)/A$ as a function of x & t.

The maximum amplitude of the parabolic distribution is $\frac{T-T_w}{A} = \frac{1}{8}$ so we approximate the initial distribution with $\frac{T-T_w}{A} = \frac{\sin\pi(x/L)}{8}$.

The heat diffusion equation can be written as: $\frac{d^2\left(\frac{T-T_w}{A}\right)}{dx^2} = \frac{1}{\alpha}\frac{\partial\left(\frac{T-T_w}{A}\right)}{\partial t}$

so the general solution (eqn. (4.11)) becomes:

$$\frac{T-T_w}{A} = (D\sin\lambda x + E\cos\lambda x)\, e^{-\alpha\lambda^2 t}. \quad \text{Then:}$$

b.c. at $x=0$: $\frac{T-T_w}{A} = 0 = (0+E)e^{-\alpha\lambda^2 t}$ so $\underline{E=0}$

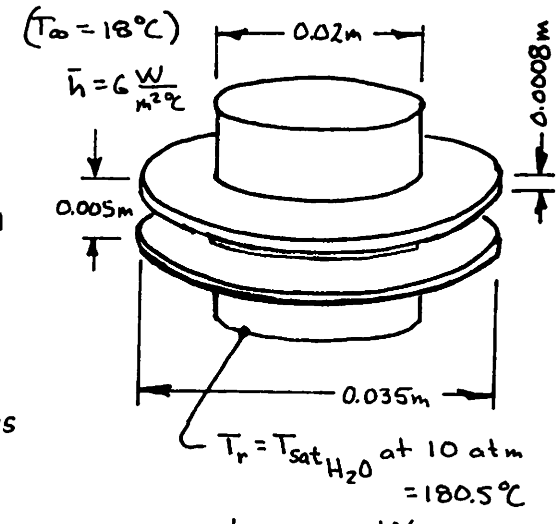
b.c. at $x=L$: $\frac{T-T_w}{A} = 0 = (D\sin\lambda L)e^{-\alpha\lambda^2 t}$ so $\underline{\lambda = \pi/L}$

i.c.: $\frac{T-T_w}{A} \simeq \frac{\sin\pi(x/L)}{8} = D\sin\pi(x/L)$ so $D \simeq \frac{1}{8}$

Thus:

$$\boxed{\frac{T-T_w}{A} \simeq \frac{\sin\pi(x/L)}{8}\, e^{-\pi^2\frac{\alpha t}{L^2}}}$$

4.25 A 1.5 m length of pipe is finned as shown. Find the rate at which steam at 10 atm, within the tube, will be condensed.



First evaluate the heat removal

$r_2/r_1 = 3.5/2 = 1.75$

$$mL\sqrt{\frac{L}{P}} = \sqrt{\frac{\bar{h}L}{k\Delta}}\,L$$

$$= \sqrt{\frac{6(0.0075)}{190(0.0008 \times 0.0075}}\;0.0075$$

$$= 0.04712$$

So from Fig. 4.13a we read, $\eta_f = 0.98$

Then: $Q_{fin} = 0.98\left[(6)\,2\frac{\pi}{4}(0.035^2 - 0.02^2)(180.5-18)\right] = 1.24\text{ W}$

and: $$Q_{pipe} = Q_{fin}\left(\frac{1.5}{0.005}\right) + \bar{h}A_{pipe}\left(\frac{0.50-0.08}{0.50}\right)\Delta T$$

$$= 1.24(300) + 6(\pi[0.02]1.5)(0.84)(180.5-18)$$

$$\underline{= 448\text{ W}}$$

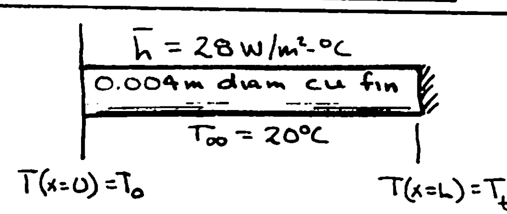
The mass rate of condensate is $\dot{m}_{cond} = \dfrac{Q_{pipe}}{h_{fg}}$

At 10 atm, $h_{fg} = 2.013 \times 10^6$ J/kg so

$$\dot{m}_{cond} = \frac{448\text{ J/s}}{2.013\times10^6\text{ J/kg}} = \underline{\underline{0.000223\,\frac{kg}{s} = 0.802\,\frac{kg}{hr}}}$$

4.26 How long must the fin shown be, if:

$$\left.\frac{T - T_\infty}{T_0 - T_\infty}\right|_{x=L} = 0.2\ ?$$



Using eqn. (4.45): $$0.2 = 1/\cosh\sqrt{\frac{\bar{h}P}{kA}}\,L = 1/\cosh\sqrt{\frac{2\bar{h}}{kR}}\,L$$

or: $$5 = \cosh\sqrt{\frac{2(28)}{398(0.002)}}\,L$$

solving by trial and error we obtain $\underline{\underline{L = 0.2734\text{ m}}}$

4.27 A 2 cm ice cube sits on a shelf of aluminum rods, 3 mm in diam., in a refrigerator at 10°C. How rapidly, in mm/min, does the ice cube melt through the wires if $\bar{h}$ between the wires and the air is 10 W/m^2-°C. (Be careful that you understand the physical mechanism before you make the calculation.) Check your result experimentally. (h_{fs} = 333,300 J/kg.)

<u>Solution</u>. The rods act as infinite fins. Each carries heat off in both directions at a rate given by eqn. (4.42). This is balanced by the rate of melt:

$$2\sqrt{(kA)(\bar{h}P)}\ (T_r - T_{sat}) = (2R)(2\text{cm})\dot{\ell}\ \rho_{ice}\ h_{fs}$$

where $\dot{\ell}$ is the rate the rod advances. Thus

$$\dot{\ell} \simeq \frac{2\sqrt{209\,(0.0015)^3\,10\,(2)\pi^2}(10-0)}{2(0.0015)(0.02)(917)333{,}300} = 12.9\times10^{-6}\ \frac{\text{m}}{\text{s}}$$

$$= \underline{\underline{0.772\ \text{mm/min}}}$$

I did this in my refrigerator and found about 1 cm advance after 15 min. This gave <u>0.667 mm/min</u> which is a reasonable comparison.

4.28 The highest heat flux that can be achieved in nucleate boiling (called q_{max}--see the qualitative discussion in Section 9.1) depends upon: ρ_g, the saturated vapor density; h_{fg}, the latent heat of vaporization; σ, the surface tension; a characteristic length, L; and the gravity force per unit volume, $g(\rho_f - \rho_g)$, where ρ_f is the saturated liquid density. Develop the dimensionless functional equation for q_{max} in terms of a dimensionless length.

$$\underset{\frac{J}{m^2\text{-}s}}{q_{max}} = fn\Big(\underset{\frac{kg}{m^3}}{\rho_g},\ \underset{\frac{J}{kg}}{h_{fg}},\ \underset{\frac{kg}{s^2}}{\sigma},\ \underset{\frac{kg}{m^2\text{-}s^2}}{g(\rho_f-\rho_g)},\ \underset{m}{L}\Big)$$

There are 6 variables in 4 dimensions (J, m, kg, s). This gives 2 Π groups. To find them we first eliminate J from the dimensional functional equation:

only term with J, must go out

$$\underset{kg/m^2\text{-}s}{\frac{q_{max}}{h_{fg}}} = fn\Big(\underset{kg/m^3}{\rho_g},\ \underset{J/kg}{\cancel{h_{fg}}},\ \underset{kg/s^2}{\sigma},\ \underset{kg/m^2\text{-}s^2}{g(\rho_f-\rho_g)},\ \underset{m}{L}\Big)$$

4.28 (continued)

Next, get rid of m:

$$\frac{q_{max}}{h_{fg}\,g(\rho_f-\rho_g)} = fn\left(\frac{\rho_g}{[g(\rho_f-\rho_g)]^{3/2}},\ \sigma,\ \xcancel{g(\rho_f-\rho_g)},\ L\sqrt{g(\rho_f-\rho_g)}\right)$$

delete

$s \qquad s^3/kg^{1/2} \qquad kg/s^2 \qquad kg/m^2\text{-}s \qquad kg^{1/2}/s$

Get rid of kg:

$$\frac{q_{max}}{h_{fg}\,g(\rho_f-\rho_g)} = fn\left(\frac{\rho_g\sqrt{\sigma}}{[g(\rho_f-\rho_g)]^{3/2}},\ \xcancel{\sigma},\ \frac{L}{\sqrt{\sigma/g(\rho_f-\rho_g)}}\right)$$

delete

$S \qquad s^2 \qquad kg/s^2$

Finally we get rid of s and the term ↑, & obtain the result:

$$\underline{\underline{\underbrace{\frac{q_{max}}{\rho_g^{1/2}h_{fg}\sqrt[4]{\sigma g(\rho_f-\rho_g)}}}_{\text{This is called the Kutateladze No. after S. S. Kutateladze, head of the heat transfer lab. in Novosibirsk, Siberia}} = fn\left(\underbrace{\frac{L}{\sqrt{\sigma/g(\rho_f-\rho_g)}}}_{\text{This is the square root of what is called a Bond No.}}\right)}}$$

4.29 You want to rig a handle for a door in the wall of a furnace. The door is at 160°C. You consider bending a 16 in. length of 1/4 in. mild steel rod into a U-shape, and welding the ends to the door. Surrounding air at 24°C will cool the handle ($\bar{h}$ = 12 W/m^2-°C). What is the coolest temperature of the handle? How close to the door can you grasp it without being burned? How might you improve the handle?

This handle is like 2, 8in. (0.2032 m) long fins with insulated tips.

$$(mL)^2 = \frac{\bar{h}PL^2}{kA} = \frac{\bar{h}\pi D L^2}{k\pi D^2/4} = \frac{4(12)(0.2032)^2}{52(0.00635)} = 6.00$$

$$\Theta_{tip} = \frac{1}{\cosh mL} = \frac{1}{\cosh 2.45} = 0.1713\ , \quad T_{tip} = 24 + 0.1713(160-24)$$

$$= T_{coolest} = \underline{\underline{47.3°C}}$$

$$\Theta_{burn} = \frac{65-24}{160-24} = 0.3015 = \frac{\cosh 2.45(1-\xi)}{\cosh 2.45}$$

4.29 (continued)

or $\cosh 2.45(1-\xi) = 1.76,\quad 2.45(1-\xi) = 1.165,\quad \xi \doteq 0.5245$

Therefore if you touch this handle within 0.5245(8) or <u>4.2 inches</u> of the door, you'll be burned.

To improve the design you need a far smaller rod diameter -- maybe a mere wire loop. Better still, weld on two short steel studs and connect them with a (low conductivity) piece of wood. The proposed design is not a good one.

4.30 A 14 cm long, 1 cm by 1 cm square brass rod is supplied with 25 W at its base. The other end is insulated. It is cooled by air at 20°C with h = 68 W/m²-°C. Develop a dimensionless expression for Θ as a function of ξ and other known information. Calculate the base temperature.

We know that: $T - T_\infty = C_3 e^{mL\xi} + C_4 e^{-mL\xi}$ (cf. eqn. (4.35)) with b.c.s:

$$-\left.\frac{d(T-T_\infty)}{d\xi}\right|_{\xi=0} = \frac{Q_{base}L}{Ak}\ ;\qquad \left.\frac{d(T-T_\infty)}{d\xi}\right|_{\xi=1} = 0$$

so:

$$C_3 - C_4 = -\frac{Q_bL}{AkmL}\ ;\qquad C_3 e^{mL} = C_4 e^{-mL}$$

so $\quad C_3 = C_4 e^{-2mL} = C_3 e^{-2mL} + \frac{Q_bL}{AkmL} e^{-2mL}$

or $\quad C_3 = \frac{Q_bL}{AkmL}\underbrace{\frac{e^{-2mL}}{1-e^{-2mL}}}_{=\frac{e^{-mL}}{2\sinh mL}} = C_4 - \frac{Q_bL}{kAmL}$

$$\frac{T-T_\infty}{Q_bL/kAmL} = e^{-mL\xi} + \frac{e^{-mL}\left(e^{mL\xi} + e^{-mL\xi}\right)}{2\sinh mL}$$

$$= \frac{e^{mL(1-\xi)} - \cancel{e^{-mL(1+\xi)}} + e^{-mL(1-\xi)} + \cancel{e^{-mL(1+\xi)}}}{2\sinh mL}$$

$$\underline{\underline{\Theta \equiv \frac{T-T_\infty}{Q_bL/kA} = \frac{\cosh mL(1-\xi)}{mL\sinh mL}}}$$

then:

$$\frac{T_{base}-T_\infty}{Q_bL/kA} = \frac{1}{mL\tanh mL}\ ;\qquad \underline{\underline{T_{base} = T_\infty + \frac{Q_bL}{kAmL\tanh mL}}}$$

so

$$T_{base} = 20 + \frac{25(0.14)}{109(0.01)^2\underbrace{\sqrt{\frac{68\times4}{109(0.01)}}\,0.14}_{2.2116}\ \underbrace{\tanh 2.2116}_{0.9763}}$$

$$= 20 + 148.7 = \underline{\underline{168.7\,^\circ C}}$$

4.31 A cylindrical fin has a constant imposed heat flux of q_1 at one end and q_2 at the other end, and it is cooled convectively along its length. Develop the dimensionless temperature distribution in the fin. Specialize this result for $q_2 = 0$ and $L \to \infty$, and compare it with equation (4.50).

The general solution is $T - T_\infty = C_1 e^{mL\xi} + C_2 e^{-mL\xi}$, with b.c.'s:

$$q_1 = -\frac{k}{L}\frac{d(T-T_\infty)}{d\xi}\bigg|_{\xi=0} \quad \text{\&} \quad q_2 = +\frac{k}{L}\frac{d(T-T_\infty)}{d\xi}\bigg|_{\xi=1}$$

or:

$$\frac{q_1}{km} = -C_1 + C_2 \quad \text{\&} \quad \frac{q_2}{km} = C_1 e^{mL} - C_2 e^{-mL}$$

so

$$C_2 = \frac{q_1}{km} + C_1 \quad \text{\&} \quad \frac{q_2}{km} = -\frac{q_1}{km}e^{-mL} + C_1\, 2\sinh mL$$

thus: $\underline{C_1 = \frac{q_1}{km}\left(e^{-mL} + \frac{q_2}{q_1}\right)\Big/ 2\sinh mL}$

And we have:

$$\frac{T-T_\infty}{q_1/km} = \frac{\left(e^{-mL} + \frac{q_2}{q_1}\right)e^{mL\xi} + 2\sinh mL\, e^{-mL\xi} + \left(e^{-mL} + \frac{q_2}{q_1}\right)e^{-mL\xi}}{2\sinh mL}$$

$$= \frac{(q_2/q_1)\,2\cosh mL\xi + e^{-mL(1-\xi)} + e^{mL(1-\xi)} \cancel{- e^{-mL(1+\xi)}} \cancel{+ e^{-mL(1+\xi)}}}{2\sinh mL}$$

so

$$\underline{\underline{\frac{T-T_\infty}{q_1/km} = \frac{q_2}{q_1}\frac{\cosh mL\xi}{\sinh mL} + \frac{\cosh mL(1-\xi)}{\sinh mL}}}$$

for $q_2 = 0$, the insulated tip, $\frac{T-T_\infty}{q_1/km} = \frac{\cosh mL(1-\xi)}{\sinh mL}$ (which is the solution of Problem 4.30.). As $mL \to \infty$ this becomes:

$$\underline{\underline{\frac{T-T_\infty}{q_1/km} = e^{-mL\xi}}}$$

which is equation (4.50) with q_1/km serving in lieu of the characteristic temperature: $T_o - T_\infty$.

4.32 A thin metal cylinder of radius, r_0, serves as an electrical resistance heater. One axial line in one side is kept at T_1. Another line, θ_2 radians away, is kept at T_2. Develop a dimensionless expressions for the temperature distributions in the two sections.

$$\frac{1}{r_0^2}\frac{d^2(T-T_1)}{d\Theta^2} = -\frac{\dot q}{k} \quad \text{so} \quad T-T_1 = -\frac{\dot q r_0^2}{2k}\Theta^2 + C_1\Theta + C_2$$

b.c.'s: $T-T_1 = 0$ at $\Theta = 0$ so $\underline{C_2 = 0}$

$T-T_1 = T_2-T_1$ at $\Theta = \Theta_2$ so $C_1 = \dfrac{T_2-T_1}{\Theta_2} + \dfrac{\dot q r_0^2}{2k}\Theta_2$

Then: $$\frac{T-T_1}{T_2-T_1} = -\frac{\dot q r_0^2}{2k\Delta T}\Theta^2 + \frac{\Theta}{\Theta_2} + \frac{\dot q r_0^2}{2k\Delta T}\Theta_2\Theta$$

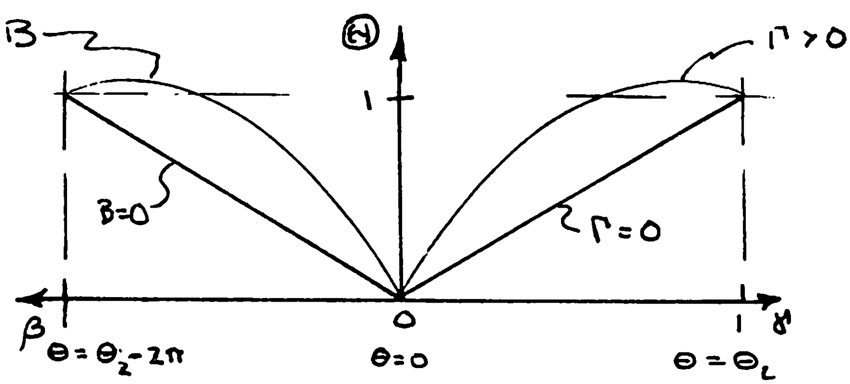
Call: $\Theta \equiv \dfrac{T-T_1}{T_2-T_1}$; $\dfrac{\dot q r_0^2\Theta_2^2}{2k\Delta T} \equiv \Gamma$; $\dfrac{\Theta}{\Theta_2} \equiv \gamma$. Then $\underline{\underline{\Theta = \Gamma(\gamma-\gamma^2) + \gamma}}$
$(0 \leqslant \Theta \leqslant \Theta_2)$

For the other segment $-\Theta = 2\pi - \Theta_2$ at $T-T_1 = T_2-T_1$ so the solution becomes:

$$\Theta = -\frac{\dot q r_0^2(\Theta_2-2\pi)^2}{2k\Delta T}\beta^2 - \beta - \frac{\dot q r_0^2(\Theta_2-2\pi)^2}{2k\Delta T}\beta$$

where $\beta \equiv \Theta/[\Theta_2 - 2\pi]$. so if we define $B \equiv \dot q r_0^2(\Theta_2-2\pi)^2/2k\Delta T$,

$$\underline{\underline{\Theta = B(-\beta-\beta^2) - \beta \qquad (\Theta_2 \leqslant \Theta \leqslant 0)}}$$

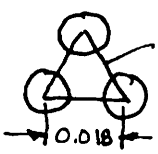


4.33 Heat transfer is augmented, in a particular heat exchanger, with a field of 0.007 m diameter fins protruding 0.02 m into a flow. The fins are arranged in a hexagonal array with a minimum spacing of 1.8 cm. The fins are bronze and $\bar{h}_f$ around the fins is 168 W/m^2-°C. On the wall itself, $\bar{h}_w$ is only 54 W/m^2-°C. Calculate $\bar{h}_{eff}$ for the wall with its fins ($\bar{h}_{eff}$ = Q_{wall} divided by A_{wall} and $[T_{wall} - T_\infty]$.)

In this case: $mL = \sqrt{\dfrac{\bar{h}\pi D}{k\frac{\pi}{4}D^2}}\,L = \sqrt{\dfrac{168 \times 4}{(0.007)}}\,0.02 = \underline{1.215}$

Next define $\bar{h}_A \equiv \dfrac{Q_{fin}}{A}\dfrac{1}{T_w - T_\infty} = k(mL)\tanh(mL)/L = \dfrac{26}{0.02}\,1.215\,\underbrace{\tanh 1.215}_{0.8382}$

$\underline{\bar{h}_A = 1324\ \text{W/m}^2\text{-°C}}$ This characterizes heat removal where the fin replaces the wall.



$A_{triangle} = \frac{1}{2}(0.018)^2\cos 60° = 0.0001403\ \text{m}^2$

$A_{fin\ within\ \Delta} = 3\left[\frac{1}{6}\frac{\pi}{4}(0.007)^2\right] = 1.924 \times 10^{-5}\ \text{m}^2$

$$h_{eff} = \frac{1}{A_\Delta}\left[\bar{h}_A \times A_{fin} + \bar{h}_w \times (A_\Delta - A_{fin})\right]$$

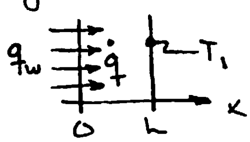
$$= \left[1324 \times 1.924(10)^{-5} + 54(0.0001403 - 1.924\,(10)^{-5}\right]/0.0001403$$

$$\underline{\underline{\bar{h}_{eff} = 228\ \text{W/m}^2\text{-°C}}}$$

The fins therefore yield a considerably improved heat removal.

4.34 An engineer seeks to study the effect of temperature on the curing of concrete by controlling the temperature of curing in the following way. A sample slab of thickness, L, is subjected to a heat flux, q_w, on one side, and it is cooled to temperature, T_1, on the other. Derive a dimensionless expression for the steady temperature in the slab. Plot the expression and offer a criterion for neglecting the internal heat generation in the slab.

general solution: $T - T_1 = -\frac{\dot{q} L^2}{2k}\xi^2 + C_1 \xi + C_2$ where $\xi \equiv \frac{x}{L}$



b.c.'s: $-\frac{k}{L}\frac{d(T-T_1)}{d\xi}\bigg|_{\xi=0} = q_w = \left[\frac{\dot{q} L}{k}\xi - k\frac{C_1}{L}\right]_{\xi=0}$

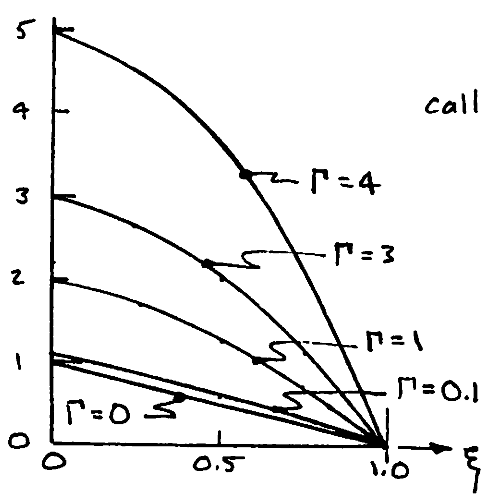
so $C_1 = -q_w L/k$

$\left[T - T_1\right]_{\xi=1} = 0 = -\frac{\dot{q} L^2}{2k} - \frac{q_w L}{k} + C_2$

so $\underline{C_2 = \frac{\dot{q} L^2}{2k} + \frac{q_w L}{k}}$

So: $$T - T_1 = -\frac{\dot{q} L^2}{2k}\xi^2 + \frac{\dot{q} L^2}{2k} - \frac{q_w L}{k}\xi + \frac{q_w L}{k}$$

or: $$\frac{T - T_1}{q_w L/k} \equiv \underline{\underline{\Phi = \frac{\dot{q} L}{2 q_w}(1 - \xi^2) + (1 - \xi)}}$$



call $\underline{\frac{\dot{q} L}{2 q_w} \equiv \Gamma}$

When $\Gamma < 0.1$ we can neglect internal heat generation with only 10 percent error.

4.35 Develop the dimensionless temperature distribution in a spherical shell with the inside wall kept at one temperature, and the outside wall at a second temperature. Reduce your solution to the limiting cases in which $r_{outside} \gg r_{inside}$ and in which $r_{outside}$ is very close to r_{inside}. Discuss these limits.

The general solution is: $T = \dfrac{C_1}{r} + C_2$ with b.c.'s $T(r_i) = T_i$, $T(r_o) = T_o$

Then:
$$\left.\begin{aligned} T_i &= \frac{C_1}{r_i} + C_2 \\ T_o &= \frac{C_1}{r_o} + C_2 \end{aligned}\right\} \quad T_i - T_o = C_1\left[\frac{r_o - r_i}{r_o r_i}\right]; \quad C_1 = \Delta T \frac{r_o r_i}{\Delta r}$$

$$T_i = \frac{C_1}{r_i} + C_2; \quad C_2 = T_i - \Delta T \frac{r_o}{\Delta r}$$

so:
$$T - T_i = \Delta T \frac{r_o}{\Delta r}\left[\frac{r_i}{r} - 1\right]; \quad \underline{\underline{\Theta = \frac{r_o}{r_o - r_i}\left[\frac{r - r_i}{r}\right]}}$$

where $\Theta \equiv (T_i - T)/(T_i - T_o)$ and we have switched the signs to make everything positive. Then:

$$\lim_{r_o \to \infty} \Theta = \frac{r - r_i}{r}$$

which is the result for a a semi-infinite region. (See e.g., the solution to 2.15)

[Note: This could also be written as:
$$-\frac{T - T_o}{T_i - T_o} - \frac{T_i - T_o}{T_i - T_o} = \frac{r - r_i}{r} \quad \text{or as} \quad \underline{\frac{T - T_o}{T_i - T_o} = \frac{r_i}{r}}\;]$$

And:
$$\lim_{r_o \to r_i} \Theta = \frac{r_o}{r_o - r_i}\left[\frac{r - r_i}{r_o}\right] = \underline{\frac{r - r_i}{r_o - r_i}}$$

which is the result for a plane slab of thickness $r_o - r_i$

4.36 Does the temperature distribution during steady heat transfer in an object, with b.c.'s of only the first kind, depend on k? Explain.

For such a problem we have: $\nabla^2 T = 0$ or $-\dot{q}/k$ and $T(x = L_1) = T_1$, $T(x = L_2) = T_2$, etc. Thus $T - T_1 = fn([T_2 - T_1], x, L_1, L_2, \dot{q}/k)$. This gives 6 var. in °C, m, only so with $\dot{q}$ there are 6 − 2 or 4 π groups:

$$\frac{T - T_1}{T_2 - T_1} = fn\left(\frac{x}{L_1}, \frac{L_2}{L_1}, \frac{\dot{q}L}{k(T_2 - T_1)}\right)$$

k can only enter if $\dot{q}$ is <u>in the problem also.</u>

4.37 A long, 0.005 m diameter, duralumin rod is wrapped with an electrical resistor over 3 cm of its length. The resistor imparts a surface flux of 40 kW/m^2. Evaluate the temperature of the rod on either side of the heated section, if $\bar{h}$ = 150 W/m^2-$^\circ$C, and $T_{ambient}$ = 27°C.

Q to either side $= \frac{1}{2}(0.03[\pi(0.005)])40{,}000 = 9.42$ W

q_o at the base of the rod is $\frac{Q}{A} = \frac{9.42}{(\pi/4)0.005^2} = \underline{480 \text{ kW/m}^2}$

From equation (4.51) we have: $q_o = k\,m\,\Delta T$, but

$$m = \sqrt{\frac{4\bar{h}}{kD}} = \sqrt{\frac{4(150)}{164(0.005)}} = 27.05 \text{ m}^{-1} \quad \text{so} \quad \Delta T = \frac{4.8(10)^5}{164(23.85)} = 108.2^\circ\text{C}$$

Therefore the base temperature is $T_o = 108.2 + 27 = \underline{\underline{135.2^\circ\text{C}}}$

4.38 The heat transfer coefficient between a cool surface and a saturated vapor, when the vapor condenses in a film on the surface, depends on: the liquid density and specific heat, the temperature difference, the buoyant force per unit volume ($g[\rho_f - \rho_g]$), the latent heat, the liquid conductivity and kinematic viscosity, and the position (x) on the cooler. Develop the dimensionless functional equation for h.

$$h = h\Big(\rho_f\,,\ c_{p_f}\,,\ \underbrace{(T_{sat.} - T_w)}_{\Delta T}\,,\ h_{fg}\,,\ g(\rho_f - \rho_g),\ k,\ \nu,\ x\Big)$$

$$\frac{J}{m^2\text{-}s\text{-}^\circ C} \quad \frac{kg}{m^3} \quad \frac{J}{kg\text{-}^\circ C} \quad {}^\circ C \quad \frac{J}{kg} \quad \frac{kg}{s^2\text{-}m^2} \quad \frac{J}{m\text{-}^\circ C\text{-}s} \quad \frac{m^2}{s} \quad m$$

We have 9 variables in 5 dimensions (J, m, s, $^\circ$C, kg)
This gives 4 Π-groups. The method in the text will -- if used in the correct sequence -- give:

$$Nu_x = fn(\Pi, Pr, Ja) \quad \text{where: } Pr \equiv \mu c_p/k\,,\ Ja \equiv \frac{c_p \Delta T}{h_{fg}}$$

$$Nu_x = \frac{hx}{k}\,,\quad \Pi = \frac{\rho_f(\rho_f - \rho_g)g\,h_{fg}\,x^3}{\mu k \Delta T}$$

(Of course other combinations are also acceptable. See details in Section 8.5.)

4.39 A duralumin pipe through a cold room has a 4 cm ID and a 5 cm OD. It carries water which sometimes sits stationary. It is proposed to put electric heating rings around the pipe to protect against freezing during cold periods of -7°C. The heat transfer coefficient outside the pipe is $9 W/m^2$-°C. Neglect the presence of the water in the conduction calculation, and determine how far apart the heaters would have to be if they brought the pipe temperature to 40°C, locally. How much heat do they require?

Find m: $m = \sqrt{\frac{\bar{h}P}{kA}} = \left(\frac{9\pi(0.05)}{164 \frac{\pi}{4}(0.05^2 - 0.04^2)} \right)^{1/2} = 3.49$

$\Theta = \frac{0-(-7)}{40-(-7)} = 0.149 = \frac{1}{\cosh mL}$ at the midpoint

so, by trial and error, $\underline{mL = 2.592}$ or $\underline{L = 0.743 \text{ m}}$

Thus the heaters must be spaced every $\underline{\underline{1.486 \text{ m}}}$

and: $\frac{Q}{2} = \sqrt{164 \frac{\pi}{4}(0.05^2 - 0.04^2) 9\pi(0.05)}\,[40-(-7)]\underbrace{\tanh 2.592}_{0.989} = 18.82 W$

For heat flow both left & right, $Q = 2(18.82) = \underline{\underline{37.64}}$

4.40 Evaluate d(tanhx)/dx.

$$\frac{d\tanh x}{dx} = \frac{d}{dx}\left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right) = \frac{e^x}{e^x+e^{-x}} + \frac{e^{-x}}{e^x+e^{-x}} - \frac{e^x - e^{-x}}{(e^x+e^{-x})^2}\left[e^x - e^{-x}\right]$$

$$= \frac{(e^x+e^{-x})^2 - (e^x - e^{-x})^2}{(e^x+e^{-x})^2}$$

$$= \frac{e^{2x}+2+e^{-2x}-e^{2x}+2-e^{-2x}}{(e^x+e^{-x})^2} = \frac{4}{(e^x+e^{-x})^2}$$

$$= \underline{\underline{\frac{1}{(\cosh x)^2}}}$$

4.41 The specific entropy of an ideal gas depends on its specific heat at constant pressure, its temperature and pressure, the ideal gas constant and reference values of the temperature and pressure. Obtain the dimensionless functional equation for the specific entropy and compare it with the known equation.

$$\underset{\frac{J}{kg^\circ K}}{s} = s(\underset{\frac{J}{kg^\circ K}}{c_p}, \underset{{}^\circ K}{T}, \underset{{}^\circ K}{T_{ref}}, \underset{\frac{N}{m^2}}{p}, \underset{\frac{N}{m^2}}{p_{ref}}, \underset{\frac{J}{kg^\circ}}{R})$$

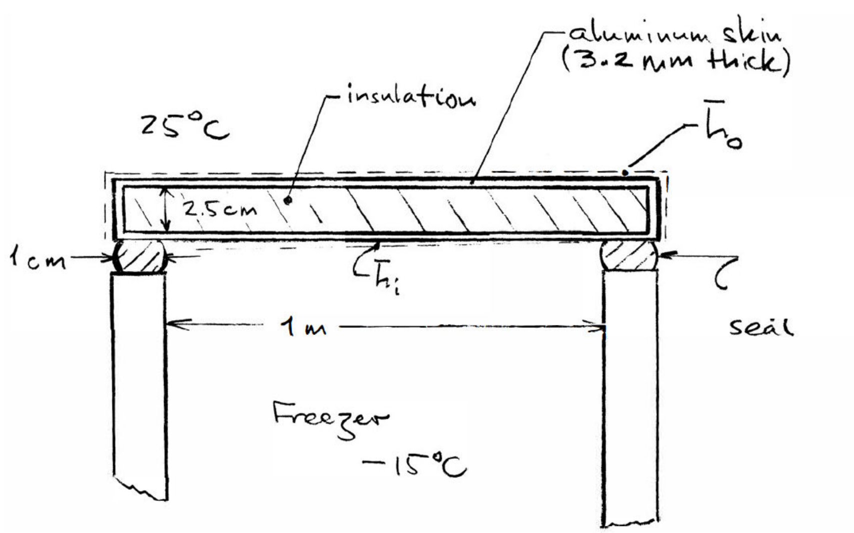
7 var in $\frac{J}{kg}$, $^\circ C$, $\frac{N}{m^2}$ $\Rightarrow$ 7−3 or 4 Π-groups

Thus: $\underline{\underline{\frac{s}{R} = fn\left(\frac{c_p}{R}, \frac{T}{T_{ref}}, \frac{p}{p_{ref}}\right)}}$

This is in the form of the known result: $\underline{\frac{s}{R} = \frac{c_p}{R}\ln\frac{T}{T_{ref}} - \ln\frac{p}{p_{ref}}}$

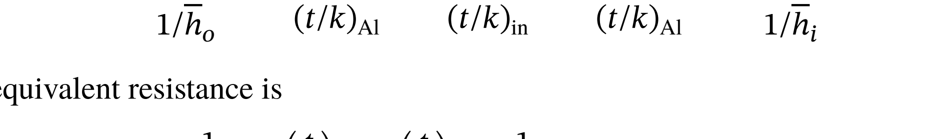
Problem 4.42 A proposed design for a large freezer's door has a 2.5 cm thick layer of insulation ($k_{\text{in}} = 0.04$ W/m·K) covered on the inside, outside, and edges with a continuous aluminum skin 3.2 mm thick ($k_{\text{Al}} = 165$ W/m·K). The door closes against a nonconducting seal 1 cm wide. Heat gain through the door can result from conduction straight through the insulation and skins (normal to the plane of the door) and from conduction in the aluminum skin only, going from the skin outside, around the edge skin, and to the inside skin. The heat transfer coefficients to the inside, $\overline{h}_i$, and outside, $\overline{h}_o$, are each 12 W/m^{2}K, accounting for both convection and radiation. The temperature outside the freezer is 25°C, and the temperature inside is −15°C.

a) If the door is 1 m wide, estimate the one-dimensional heat gain through the door, neglecting any conduction around the edges of the skin. Your answer will be in watts per meter of door height.
b) Now estimate the heat gain through the aluminum skin that wraps the outside and inside of the door. Heat will be conducted from the outside, around the edge of the door, to the inside. For this calculation, assume that the insulation is perfectly adiabatic and ignore the bottom and the top of the door. Your answer will again be in watts per meter of door height.
c) Suggest a few design changes that might reduce the heat conduction around the edges of the door.



Solution

a) In this case, we can make a series of one-dimensional thermal resistances on a per-unit-area basis (1 m width and per meter of height). We assume that all the heat flow is through the aluminum into the insulation and out the opposing side.



The equivalent resistance is

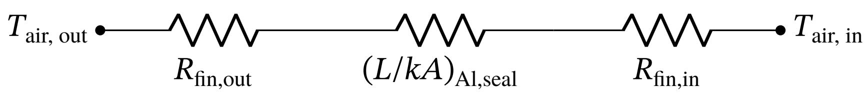
$$R_{\text{equiv}} = \frac{1}{\overline{h}_o} + 2\left(\frac{t}{k}\right)_{\text{Al}} + \left(\frac{t}{k}\right)_{\text{in}} + \frac{1}{\overline{h}_i}$$

$$
\begin{aligned}
&= \frac{1}{12} + 2\left(\frac{0.0032}{165}\right) + \frac{0.025}{0.04} + \frac{1}{12} \\
&= 0.08333 + 2\left(1.939 \times 10^{-5}\right) + 0.6250 + 0.08333 \\
&= 0.7917 \text{ K-m}^2\text{/W}
\end{aligned}
$$

Note that the thermal resistance of the aluminum is entirely negligible. Since the door is 1 m wide, the heat gain per meter of door height is

$$Q_{\text{normal}} = \frac{T_{\text{air, out}} - T_{\text{air, in}}}{R_{\text{equiv}}} = \frac{25 - (-15)}{0.7917} = \underline{50.53 \text{ W/m}}$$

b) Here, we can model the inside and outside surfaces of the door as very long fins. The are separated by a conduction resistance for the aluminum that passes over the nonconducting door seal. For all these resistances, the problem asks us to assume that no heat travels through the insulation.



From eqn. (4.51) and (4.56), noting that only one side of the fin has heat transfer and evaluating A and P per unit width of door ($A = t_{\text{Al}}$, $P = 1$), the fin resistances are

$$R_{\text{fin,out}} = \frac{1}{\sqrt{kA\overline{h}P}} = \frac{1}{\sqrt{(165)(12)(0.0032)(1)}} = 0.3973 \text{ K-m/W}$$

$$R_{\text{fin,in}} = \frac{1}{\sqrt{kA\overline{h}P}} = \frac{1}{\sqrt{(165)(12)(0.0032)(1)}} = 0.3973 \text{ K-m/W}$$

and the equivalent resistance is

$$R_{\text{equiv}} = 2(0.3973) + \frac{0.010}{(0.0032)(1)(165)} = 2(0.3973) + 0.01894 = 0.8135 \text{ K/W}$$

The thermal resistance of the aluminum is 2.3% of the total thermal resistance. The heat gain per meter of door height, accounting for both the left-hand and right-hand sides, is

$$Q_{\text{edge}} = 2\frac{T_{\text{air, out}} - T_{\text{air, in}}}{R_{\text{equiv}}} = 2\frac{25 - (-15)}{0.8135} = \underline{98.34 \text{ W/m}}$$

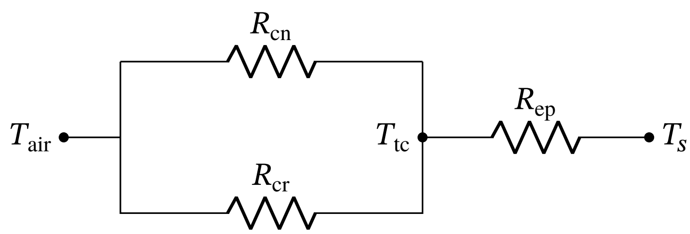
Additional heat gain will be associated with the top and bottom edges, each 1 m in width.

The edge gain substantially exceeds the heat gain in the normal direction.

c) Some improvements could include:
- Change the material used from aluminum ($k = 165$ W/m·K) to a stainless steel ($k \approx 15$ W/m·K).
- Reduce the thickness of the metal skin from 3.2 mm to 1 mm or so.
- Introduce a "thermal break" in the skin at the location of the seal, to interrupt the path of heat conduction. This break could be, e.g., a joint in the material that incorporates a layer of nonconductive material.
- Make the inside skin of the door out of a hard plastic, rather than metal. The plastic might be ABS (acrylonitrile-butadiene styrene) or HIPS (high impact polystyrene).

Problem 4.43 A thermocouple epoxied onto a high conductivity surface is intended to measure the surface temperature. The thermocouple consists of two bare wires of diameter $D_w = 0.51$ mm. One wire is made of Chromel (Ni-10%Cr with $k_{cr} = 17$ W/m·K) and the other of constantan (Ni-45%Cu with $k_{cn} = 23$ W/m·K). The ends of the wires are welded together to create an approximately rectangular measuring junction, with a width $w \approx D_w$ and a length $l \approx 2D_w$. The wires extend perpendicularly away from the surface and do not touch one another. A layer of an epoxy ($k_{ep} = 0.5$ W/m·K) separates the thermocouple junction from the surface by 0.2 mm. The heat transfer coefficient between the wires and the surroundings at 20°C is $\overline{h} = 28$ W/m^{2}K, including both convection and radiation. If the thermocouple reads $T_{tc} = 40$°C, estimate the actual temperature T_s of the surface and suggest a better arrangement of the wires.

Solution The wires act as infinitely-long fins extending away from the surface. The epoxy layer acts as a thermal resistance between the surface and the ends of the wires, which we can approximate as a simple slab resistance. Thus, we may build a resistance network consisting of the epoxy resistance in series with each of the infinite fin resistances, which are parallel to one another.



From eqn. (4.51) and (4.56),

$$R_{cr} = \frac{1}{\sqrt{kA\overline{h}P}} = \frac{1}{\sqrt{(17)(28)\pi^2(0.00051)^3/4}} = 2533.5 \text{ K/W}$$

$$R_{cn} = \frac{1}{\sqrt{kA\overline{h}P}} = \frac{1}{\sqrt{(23)(28)\pi^2(0.00051)^3/4}} = 2178.1 \text{ K/W}$$

$$R_{ep} \simeq \frac{t_{ep}}{k_{ep}(D_w)(2D_w)} = \frac{0.0002}{(0.5)(0.00051)(0.00102)} = 768.9 \text{ K/W}$$

The temperature T_{tc} may calculated from the resistance network in several ways. One way is to use the so-called voltage divider relationship (see Problem 2.48):

$$T_s - T_{tc} = \left(T_s - T_{air}\right)\frac{R_{ep}}{R_{ep} + \left(R_{cr}^{-1} + R_{cn}^{-1}\right)^{-1}}$$

$$(T_s - 40) = (T_s - 20)\frac{768.9}{768.9 + \left[(2533.5)^{-1} + (2178.1)^{-1}\right]^{-1}} = (T_s - 20)(0.3963)$$

Solving,

$$T_s = \frac{40 - (20)(0.3963)}{1 - 0.3963} = \underline{53.13\text{°C}}$$

Thus, the measuring error (13 K) is about 40% of the overall temperature difference (33 K). The thermocouple reading obtained this way is virtually meaningless.

A better arrangement would be to lay the wires onto the surface, rather than putting them perpendicular to it, so that fin heat conduction in the wires will not cool the measuring junction. *Remember*: thermocouples measure the temperature at the junction of the two dissimilar metals. Temperature gradients in other parts of the wires do not matter.

Problem 4.44 The resistor leads in Example 4.9 were assumed to be "infinitely long" fins. What is the minimum length they each must have if they are to be modeled this way? What are the effectiveness, ε_f, and efficiency, η_f, of the wires? Discuss the meaning of your calculated effectiveness and efficiency.

Solution In the example, the fins were considered to be long enough that $\tanh mL \simeq 1$ when calculating the fin thermal resistance from eqn. (4.57). We must make a judgment about how close to 1 we need to be. If we desire no more than 1.00% error, then we need $\tanh mL \geqslant 0.9900$. A calculation gives $mL \geqslant \tanh^{-1} 0.9900 = 2.647$.

For the wires considered in Example 4.9

$$m = \sqrt{\frac{\overline{h}P}{kA}} = \sqrt{\frac{(23)\pi(0.00062)}{(16)\pi(0.00062)^2/4}} = \sqrt{\frac{(23)}{(4)(0.00062)}} = 96.30 \text{ m}^{-1}$$

so that

$$L \geqslant \frac{2.647}{96.30} = 0.02749 \text{ m} = \underline{27.5 \text{ mm}}$$

This length amounts to 44.3 wire diameters.

From eqn. (4.53), the fin efficiency is

$$\eta_\mathrm{f} = \frac{\tanh mL}{mL} = \frac{0.990}{2.647} = \underline{0.3740}$$

This value is significantly less than one because much of the fin is at a temperature closer to the surrounding air temperature than is the base of the wire.

From eqn. (4.55), the fin effectiveness is

$$\varepsilon_\mathrm{f} = \eta_\mathrm{f}\frac{\text{fin surface area}}{\text{fin cross-sectional area}} = (0.3740)\frac{\pi(0.00062)(0.02749)}{\pi(0.00062)^2/4} = (0.3740)(177.4) = \underline{66.33}$$

Thus, the wire manages to remove 66 times more heat than the base area of the wire would remove if the same heat transfer coefficient applied to it. The reason is that the wire has a low resistance to heat flow and can effectively lose heat over much of its surface area, which is 177 times larger than the base area.

Problem 4.45 We use the following experiment to measure local heat transfer coefficients, h, inside pipes that carry flowing liquids. We pump liquid with a known bulk temperature through a pipe which serves as an electric resistance heater, and whose outside is perfectly insulated. A thermocouple measures its outside temperature. We know the volumetric heat release in the pipe wall, $\dot{q}$, from resistance and current measurements. We also know the pipe diameter, wall thickness, and thermal conductivity.

Derive an equation for h. (Remember that, since h is unknown, a boundary condition of the third kind by itself is not sufficient to find $T(r)$.) Then, nondimensionalize your result.

Solution For steady, radial heat conduction in the pipe wall with volumetric heating, the heat conduction equation (eqn. (2.11) with eqn. (2.13)) can be simplified:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\underbrace{\cancel{\frac{\partial^2 T}{\partial \phi^2}}}_{0} + \underbrace{\cancel{\frac{\partial^2 T}{\partial z^2}}}_{0} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\underbrace{\cancel{\frac{\partial T}{\partial t}}}_{0}$$

Here, we have assumed that T_b and h vary only slowly in the z direction. Integrating once:

$$r\frac{\partial T}{\partial r} + \frac{\dot{q}r^2}{2k} = C_1$$

$$\frac{\partial T}{\partial r} = -\frac{\dot{q}r}{2k} + \frac{C_1}{r}$$

Integrate again:

$$T(r) = -\frac{\dot{q}r^2}{4k} + C_1 \ln r + C_2$$

We get C_1 from energy conservation applied at r_i, with the edge at r_o adiabatic and $q_w > 0$ for heat flow in the $+r$ direction:

$$2\pi r_i q_w = -\dot{q}\pi\left(r_o^2 - r_i^2\right)$$

or

$$q_w = -\frac{\dot{q}\left(r_o^2 - r_i^2\right)}{2r_i}$$

$$q_w = -k\frac{\partial T}{\partial r}\bigg|_{r=r_i} = \frac{\dot{q}r_i}{2} - \frac{kC_1}{r_i} = -\frac{\dot{q}\left(r_o^2 - r_i^2\right)}{2r_i}$$

$$C_1 = \frac{\dot{q}r_i^2}{2k} + \frac{\dot{q}\left(r_o^2 - r_i^2\right)}{2k} = \frac{\dot{q}r_o^2}{2k}$$

We need $T(r_o) - T(r_i)$, in which C_2 cancels out:

$$T(r_o) - T(r_i) = -\frac{\dot{q}\left(r_o^2 - r_i^2\right)}{4k} + C_1 \ln\frac{r_o}{r_i}$$

Now we can apply the 3rd-kind boundary condition, noting again that $q_w > 0$ for heat flow in the $+r$-direction:

$$q_w = h\left[T_b - T(r_i)\right]$$

$$+\frac{\dot{q}\left(r_o^2 - r_i^2\right)}{2r_i} = +h\left[T(r_0) - T_b + \frac{\dot{q}\left(r_o^2 - r_i^2\right)}{4k} - \frac{\dot{q}r_o^2}{2k}\ln\frac{r_o}{r_i}\right]$$

$$h = \frac{\dot{q}\left(r_o^2 - r_i^2\right)}{2r_i\left[T(r_0) - T_b + \dfrac{\dot{q}\left(r_o^2 - r_i^2\right)}{4k} - \dfrac{\dot{q}r_o^2}{2k}\ln\dfrac{r_o}{r_i}\right]} \qquad \xleftarrow{\text{Answer}}$$

We can now compute h, since we know k, r_o, r_i, $\dot{q}$, and the two measured temperatures.

Now we recall that we have nondimensionalized h as the Biot number in situations where a conduction resistance is in series with a convection resistance. With the pipe diameter $D_i = 2r_i$, we write $\mathrm{Bi} = h(2r_i)/k$. We also see that the radius ratio appears naturally; call this $\rho \equiv r_o/r_i$. Putting these groups into our result and rearranging

$$\mathrm{Bi} = \frac{\left(\rho^2 - 1\right)}{\left[\dfrac{k\left[T(r_0) - T_b\right]}{\dot{q}r_i^2} + \dfrac{\left(\rho^2 - 1\right)}{4} - \dfrac{\rho^2}{2}\ln\rho\right]} \qquad \xleftarrow{\text{Answer}}$$

The first group in the denominator is also nondimensional. This group is similar to $1/\Gamma$ in Example 4.7. Note, however, that if $\dot{q} \to 0$ then $T(r_0) - T_b$ must also go to zero in a steady-state situation. Similarly, if $\dot{q}$ increases (with other things held fixed), then $T(r_0) - T_b$ must also increase.

Comment 1: Thin walled pipe. If we assume that $t_w = (r_o - r_i) \ll r_i$, the wall behaves like a one-dimensional slab. Then $T_{\text{outside}} - T_{\text{inside}} = \dot{q}t_w^2/2k$ from Example 2.1. From energy conservation, $q_w = \dot{q}t_w = h(T_{\text{inside}} - T_b)$, so that

$$\dot{q}t_w = h(T_{\text{outside}} - \dot{q}t_w^2/2k - T_b)$$

and so

$$h = \frac{\dot{q}t_w}{(T_{\text{outside}} - T_b - \dot{q}t_w^2/2k)} \qquad \xleftarrow{\text{Answer}}$$

Comment 2: This experiment is best for studying fully developed turbulent flow inside the pipe (see Chapter 7), for two reasons. First, in turbulent flow, the fluid away from the pipe wall is well mixed and very close to T_b over a large part of the cross-section, making the bulk temperature measurement less difficult. Second, in fully developed flow, h does not change in the streamwise direction, so that the result will be less susceptible to axial variations. (Note, however, that T_b will always increase in the stream-wise direction if $\dot{q} \neq 0$.)

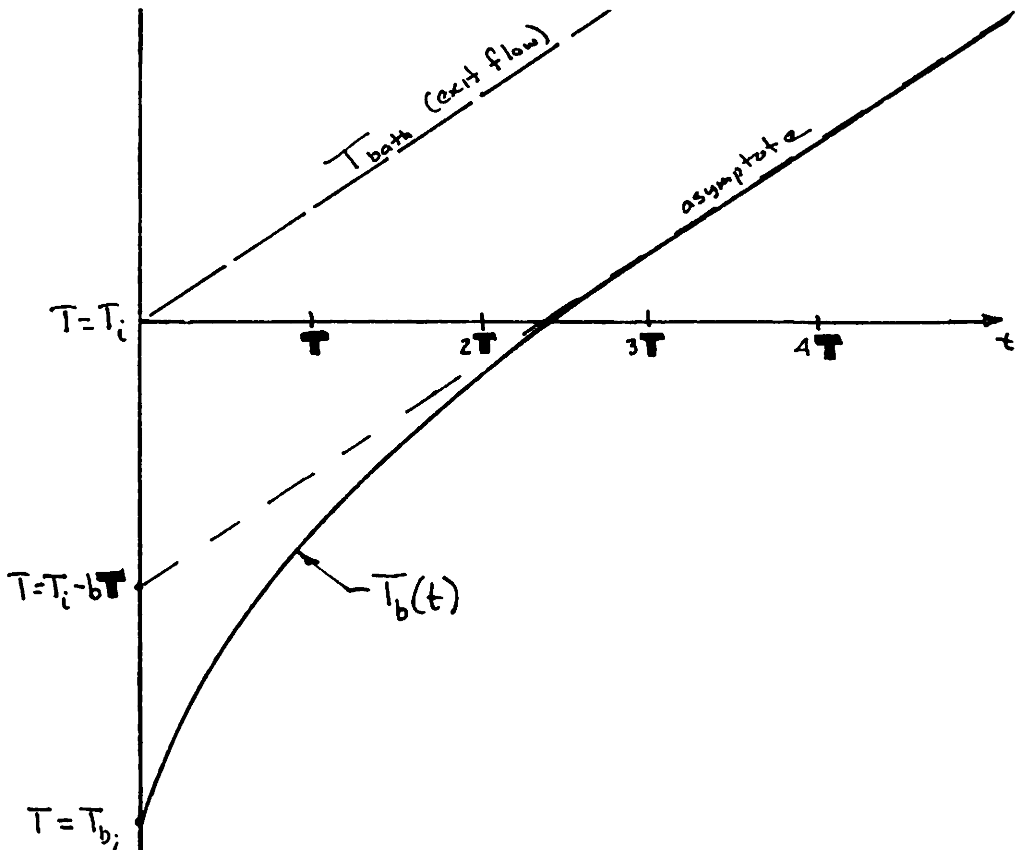
5.1 A body at $T_b = T_i$ with Bi << 1 is immersed in a bath at $t = 0$. If $T_{bath} = T_i + bt$, plot $T_{body} = f(t)$ for the case in which $T_{b_i} < T_i - b\mathbf{T}$

As in Example 5.1, the general solution is given by eqn. (5.13)

$$T_b - T_i = C_1 e^{-t/\mathbf{T}} + b(t - \mathbf{T})$$

and $C_1 = T_{b_i} - T_i + b\mathbf{T}$ so the particular solution is

$$T_b = T_i + (t - \mathbf{T}) + (T_{b_i} - T_i + b\mathbf{T})e^{-t/\mathbf{T}}$$



5.2 A body of known volume and temperature, initially at T_i, is suddenly immersed in a bath for which

$$T_{bath} = T_i + (T_o - T_i)e^{-t/\tau}$$

where $\tau = 10\mathbf{T}$. Plot T_{body} from $t = 0$ to $t = 2\tau = 20\mathbf{T}$.

$$\frac{dT}{dt} = \frac{T - T_b}{\mathbf{T}} \quad \text{or} \quad \frac{d(T-T_i)}{dt} = -\frac{T-T_i}{\mathbf{T}} + (T_o - T_i)e^{t/\tau}$$

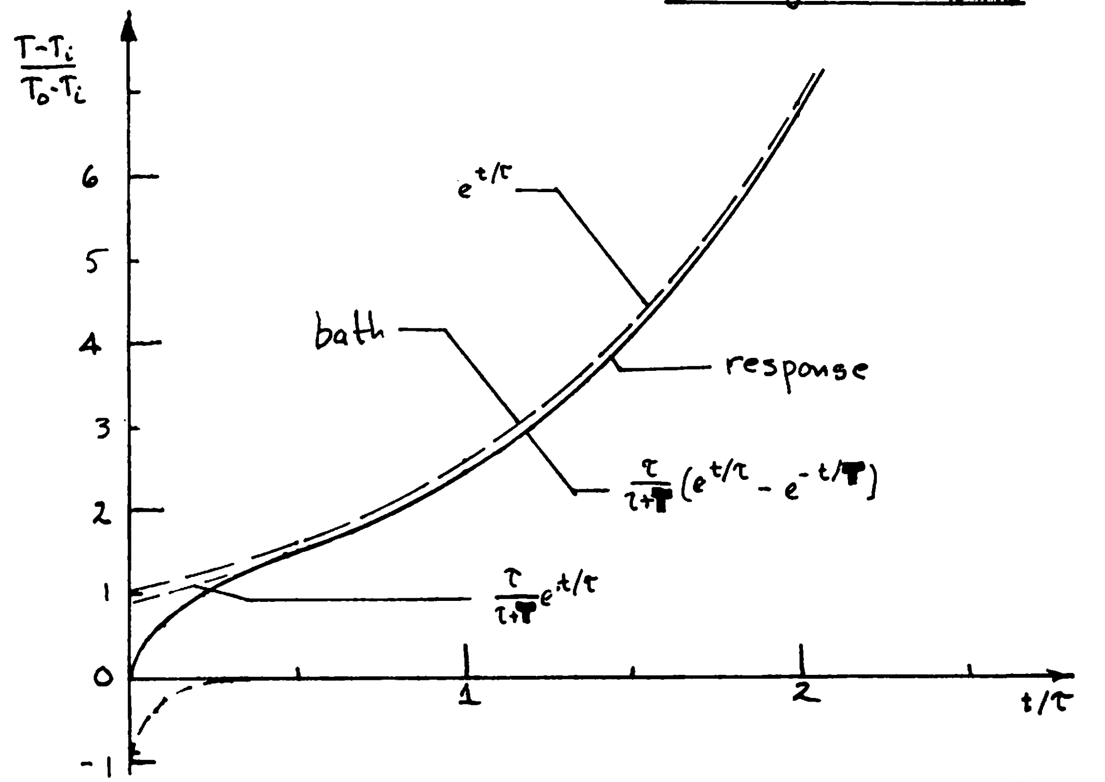
The gen'l. soln. of the homo. eqn. is $T - T_i = C_1 e^{-t/\mathbf{T}}$ and the particular soln. of the complete eqn. might be found by substituting $T - T_i = Ae^{t/\tau} + Be^{-t/\tau}$ in the d.e. and adjusting A & B to satisfy it. We get $B = 0$ and $A = (T_o - T_i)/(\tau^{-1} + \mathbf{T}^{-1})\mathbf{T}$ so

$$\frac{T - T_i}{T_o - T_i} = \frac{T_o - T_i}{\left(\frac{1}{\tau} + \frac{1}{\mathbf{T}}\right)\mathbf{T}} e^{t/\tau} + C_1 e^{-t/\mathbf{T}}$$

5.2 (continued)

The i.c., $(T-T_i)_{t=0} = 0$, gives $C_1 = -\dfrac{T_0 - T_i}{\frac{\mathbf{T}}{\tau} + 1}$

$$\underline{\underline{\frac{T-T_i}{T_0-T_i} = \frac{\tau}{\mathbf{T}+\tau}\left(e^{t/\tau} - e^{-t/\mathbf{T}}\right)}} \quad \& \quad \underline{\underline{\left.\frac{T-T_i}{T_0-T_i}\right|_{\text{long time}} = \frac{\tau}{\mathbf{T}+\tau}\, e^{t/\tau}}} \longleftarrow$$



5.3 A body of known volume and area is immersed in a bath whose temperature varies $T_\infty = T_{mean} + A \sin\omega t$. Find the steady periodic response of the body if its Biot number is small.

Define: $\Theta \equiv \dfrac{T - T_m}{A}$, $\mathbf{T} \equiv \rho c V/\bar{h}A$, $\tau = t/\mathbf{T}$, $\Omega \equiv \omega \mathbf{T}$. Then the d.e.

$$\frac{dT}{dt} = -\frac{T-T_m}{\mathbf{T}} + A\sin\omega t \quad \text{becomes} \quad \underline{\frac{d\Theta}{d\tau} + \Theta = \sin\Omega\tau}$$

The general sol'n of the homogeneous eqn. is: $\underline{\Theta = C_1 e^{-\tau}}$. The particular sol'n. of the complete eqn. can be found by trying $\underline{\Theta = C_2\cos\Omega\tau + C_3\sin\Omega\tau}$ in the complete eqn. This gives

$$-\Omega C_2 \sin\Omega\tau + \Omega C_3\cos\Omega\tau + C_2\cos\Omega\tau + C_3\sin\Omega\tau = \sin\Omega\tau$$

or

$$(-\Omega C_2 + C_3 - 1)\sin\Omega\tau + (\Omega C_3 + C_2)\cos\Omega\tau = 0$$

This will be true if $\underline{C_2 = -\Omega C_3}$ and $\underline{C_3 = \frac{1}{\Omega^2+1}}$. Then the

5.3 (continued)

particular solution of the complete equation is

$$\Theta = C_1 e^{-\tau} - \frac{\Omega}{\Omega^2+1}\cos\Omega\tau + \frac{1}{\Omega^2+1}\sin\Omega\tau$$

or

$$\underline{\Theta = C_1 e^{-\tau} - \frac{1}{\Omega^2+1}\left[\Omega\cos\Omega\tau - \sin\Omega\tau\right]}$$

At time $\tau = 0$, $\Theta = \Theta_0 = C_1 - \frac{\Omega}{\Omega^2+1}$ so $C_1 = \Theta_0 + \frac{\Omega}{\Omega^2+1}$

where Θ_0 might be anything, between 0 and 1, we might wish to specify. Thus

$$\underline{\underline{\Theta = \Theta_0 e^{-\tau} - \frac{1}{\Omega^2+1}\left(\Omega\cos\Omega\tau - \sin\Omega\tau - \Omega e^{-\tau}\right)}} \longleftarrow$$

After a long time ($t > 3T$ or $\tau > 3$) this reduces to the steady periodic solution:

$$\underline{\underline{\Theta \Rightarrow \frac{-1}{\Omega^2+1}\left(\Omega\cos\Omega\tau - \sin\Omega\tau\right)}} \longleftarrow$$

Now use the trigonometric identity

$$A\cos x + B\sin x = \sqrt{A^2+B^2}\,\sin\left(x + \cos^{-1}\frac{1}{\sqrt{A^2-B^2}}\right)$$

In this case:

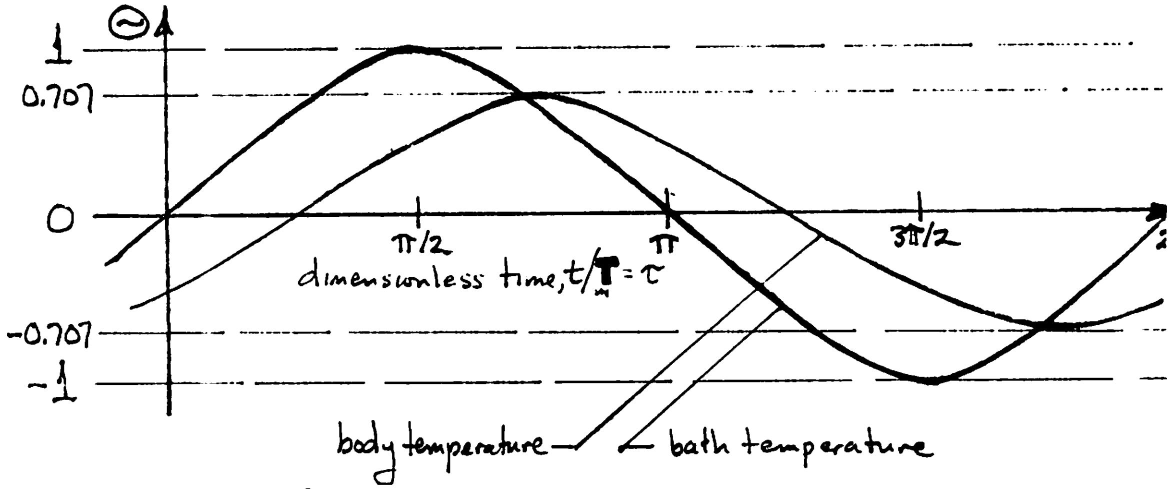
$$\Omega\cos\Omega\tau - \sin\Omega\tau = \sqrt{\Omega^2+1}\,\sin\Big(\Omega\tau - \underbrace{\cos^{-1}\frac{1}{\sqrt{\Omega^2+1}}}_{\beta}\Big)$$

we identify $\beta \equiv$ the phase lag angle. Then

$$\underline{\underline{\Theta_{\text{periodic}} = -\underbrace{\frac{1}{\sqrt{\Omega^2+1}}}_{0<\text{amplitude}<1}\sin(\Omega\tau - \underbrace{\beta}_{0<\beta<90^\circ})}} \longleftarrow$$

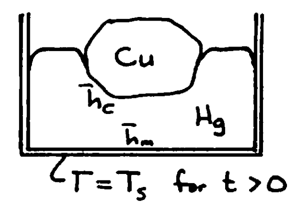
Suppose, for example, that $\omega T = \Omega = 1$. Then $\beta = \cos^{-1} 0.707 = 45^\circ$ or $\pi/4$ radians and the amplitude is 0.707.

5.3 (continued)



Notice that $\Theta_{periodic} = f(\tau \text{ and } \Omega)$. When Ω (or ωT) is large, the process can be regarded as slow and $\Theta \Rightarrow \Theta_{bath} = \sin\Omega\tau$. When Ω is small the process is rapid, $\beta \Rightarrow 90^{\circ}$ and the amplitude of the response goes to zero. In a rapid oscillation $\Theta \Rightarrow 0$.

5.4 A copper block of volume, V, floats in mercury contacting it over an area, A_c, and exchanging heat with it by convection. The mercury container also exchanges heat with the mercury itself by convection. The entire system is initially in equilibrium at temperature, T_i.



Predict the response of the copper if the container temp. is suddenly raised to T_s and if Bi_{cu} and $Bi_{hg} \ll 1$.

For the copper: $(\rho c V)_c \dfrac{dT_c}{dt} = (\bar{h}A)_c (T_m - T_c)$ or $\dfrac{dT_c}{dt} = \dfrac{T_m - T_c}{\mathbf{T}_c}$ ①

for the mercury: $\dfrac{dT_m}{dt} = \dfrac{T_s - T_m}{\mathbf{T}_m}$ ②

This is exactly the second-order lumped capacitance problem solved in the text. The solution is eqn. (5.23) which we paraphrase as follows:

$$\frac{T_c - T_s}{T_i - T_s} = \frac{\frac{b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 - c}}{2\sqrt{\left(\frac{b}{2}\right)^2 - c}}\, e^{\left(-\frac{b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 - c}\right)t} + \frac{-\frac{b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 - c}}{2\sqrt{\left(\frac{b}{2}\right)^2 - c}}\, e^{\left(-\frac{b}{2} - \sqrt{\left(\frac{b}{2}\right)^2 - c}\right)t}$$

where $b = \dfrac{1}{\mathbf{T}_c} + \dfrac{1}{\mathbf{T}_m} + \dfrac{\bar{h}_c}{\bar{h}_m \mathbf{T}_m}$ and $c = 1/\mathbf{T}_c\mathbf{T}_m$

This can be rewritten as $\dfrac{T_c - T_s}{T_i - T_s} = A_1 e^{a_1 t} + A_2 e^{a_2 t}$

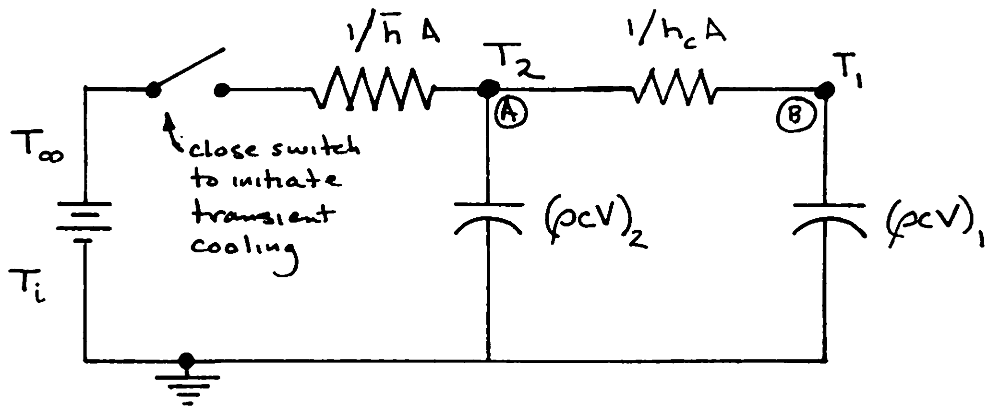
5.4 (continued)

at $t=0$, $\frac{T_c - T_s}{T_i - T_s} = A_1 + A_2 = \frac{2\sqrt{(\frac{b}{2})^2 - c}}{2\sqrt{(\frac{b}{2})^2 - c}} = 1$ $\therefore$ $T_c = T_i$ ✔

at $t=0$, $T_m = T_c$ so from eqn. ① $\frac{dT_c}{dt} = 0 = (T_i - T_\infty)(A_1 a_1 + A_2 a_2)$

Thus: $\frac{A_1}{A_2} = -\frac{a_2}{a_1}$. We see that both sides equal $\frac{\frac{b}{2}+\sqrt{\;}}{-\frac{b}{2}+\sqrt{\;}}$ so the second b.c. is also satisfied. Finally, we expect $T_c \Rightarrow T_s$ or $\frac{T_c - T_s}{T_i - T_s} \Rightarrow 0$ as $t \Rightarrow \infty$. This means that a_1 & a_2 must both be negative. a_2 obviously is. a_1 is also negative because b is a positive number greater than c.

5.5 Sketch the electrical circuit that is analogous to the second order lumped capacity system shown in Fig. 5.5.



To see that this is valid we write the nodal equations for nodes Ⓐ and Ⓑ as an E.E. might.

node Ⓐ

$$(\rho c V)_2 \frac{dT_2}{dt} + \frac{T_2 - T_1}{1/h_c A} + \frac{T_2 - T_\infty}{1/\bar{h} A} = 0$$

node Ⓑ

$$(\rho c V)_1 \frac{dT_1}{dt} + \frac{T_1 - T_2}{1/h_c A} = 0$$

These equations are identical to equations (5.16) and (5.15), respectively, so the circuit is correct.

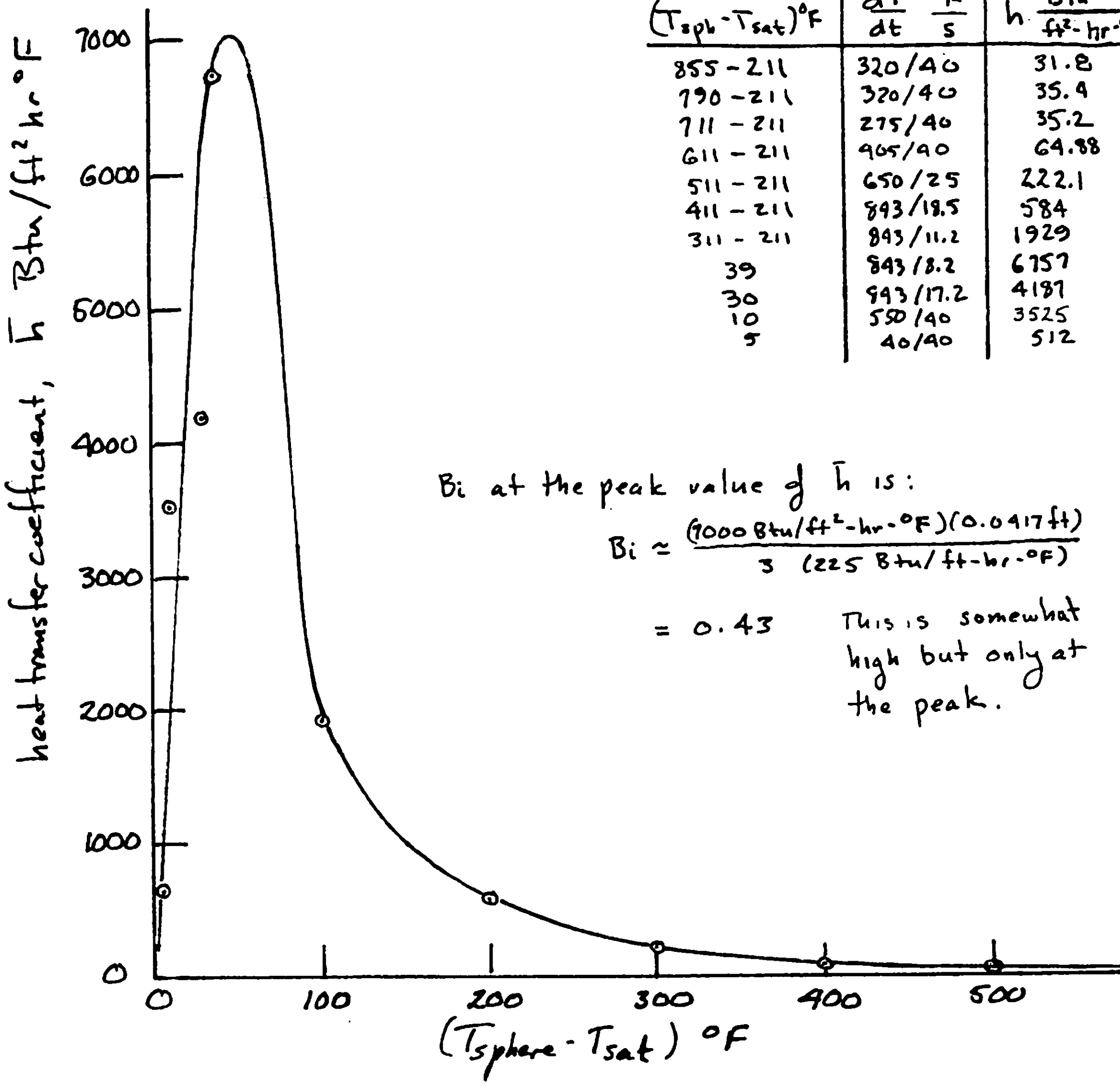
5.6 Plot $\bar{h}$ vs. $(T_{sph} - T_{sat})$ for the sphere quench in the figure with the problem in the text.

$$\frac{dU}{dt} = \rho c V \frac{dT_{sph}}{dt} = \bar{h} A (T_{sph} - T_{sat}) \;; \quad \bar{h} = \rho c \frac{R}{3} \frac{dT_{sph}/dt}{T_{sph} - T_{sat}}$$

$$\rho c \frac{R}{3} = 8954(384) \frac{0.0254}{2(3)} \frac{J}{m^2\text{-}^\circ K} 0.0009478 \frac{Btu}{J} \frac{0.3048^2 m^2}{ft^2} \frac{^\circ K}{1.8^\circ F} = 0.712 \frac{Btu}{ft^{2\circ}F}$$

Then, scaling points from the Figure:

$(T_{sph} - T_{sat})^\circ F$	$\frac{dT}{dt}$ $\frac{^\circ F}{s}$	$\bar{h}$ $\frac{Btu}{ft^2\text{-}hr\text{-}^\circ F}$
855 - 211	320/40	31.8
790 - 211	320/40	35.4
711 - 211	275/40	35.2
611 - 211	405/40	64.88
511 - 211	650/25	222.1
411 - 211	843/18.5	584
311 - 211	843/11.2	1929
39	843/8.2	6757
30	843/17.2	4187
10	550/40	3525
5	40/40	512



Bi at the peak value of $\bar{h}$ is:

$$Bi \simeq \frac{(7000\ Btu/ft^2\text{-}hr\text{-}^\circ F)(0.0417\ ft)}{3\ (225\ Btu/ft\text{-}hr\text{-}^\circ F)}$$

$$= 0.43$$

This is somewhat high but only at the peak.

5.7 The temperature of a butt-welded 36 gage (0.127 mm diam.) thermocouple in a gas flow rises at 20°C/s, and stays 2.4°C below the gas flow temperature. Find h between the wire and the gas if $\rho c = 3800$ kJ/m³-°C.

$$T = \frac{\rho c V}{\bar{h} A} = \frac{\rho c R}{2\bar{h}} = \frac{3.8(10)^6 (0.000127)}{4\bar{h}} = \frac{120.7}{\bar{h}}$$

but

$$\frac{dT_w}{dt} = 20 = \frac{T_g - T_w}{T} = \frac{2.4}{120.7}\bar{h} \;; \quad \boxed{\bar{h} = 1006 \frac{W}{m^2\text{-}^\circ C}}$$

5.8 Predict the temperature at the point Fo = 0.2, Bi = 10 or Bi^{-1} = 0.1, and x/L = 0, and compare it with the graphical value in Fig. 5.7.

To do this we use eqn. (5.34) with (λL) values generated by eqn. (5.35): $\text{ctn}(\lambda L) = \lambda L/Bi = 0.1(\lambda L)$. By trail and error we get: $(\lambda L)_1 = 1.42887$, $(\lambda L)_2 = 4.30580$, $(\lambda L)_3 = 7.22811$, etc. Using these numbers in eqn. (5.34) we get:

$$\Theta = e^{-1.42887^2(0.2)} \frac{2\sin(1.42887)\cos(1.42887[0])}{1.42887 + \sin(1.42887)\cos(1.42887)}$$

$$+ e^{-4.3058^2(0.2)} \frac{2\sin(4.3058)\cos 0}{4.3058 + \sin(4.3058)\cos(4.3058)} + \cdots$$

$$\Theta = 0.8389 - 0.00965 + O\left(e^{-7.23^2(0.2)}\right) \simeq \underline{\underline{0.8293}}$$

From Fig. 7 we read $\Theta \cong$ 0.82 or 0.83 so the results agree within the accuracy with which we can read the graphs.

5.9 Prove that when B_i is large, and the b.c. of the 3rd kind therefore reduces to a b.c. of the 1st kind, eqn. (5.34) reduces to (5.33)

The eigen value eqn. (5.35) becomes $\text{ctn}\lambda L = 0$ or $\tan\lambda L = \infty$, so $\lambda L = \frac{\pi}{2}, 3\frac{\pi}{2}, \ldots, n\frac{\pi}{2}$ where n is odd. Therefore eqn. (5.34) becomes:

$$\Theta = \sum_{n=\text{odd}}^{\infty} e^{-\left(\frac{n\pi}{2}\right)^2 F_o} \frac{2\,\overbrace{\sin\frac{n\pi}{2}}^{=1}\,\overbrace{\cos\frac{n\pi}{2}(\xi-1)}^{=\sin\frac{n\pi}{2}}}{\frac{n\pi}{2} + \sin\frac{n\pi}{2}\underbrace{\cos\frac{n\pi}{2}}_{=0}}$$

$$\underline{\underline{\Theta = \frac{4}{\pi}\sum_{n=\text{odd}}^{\infty} e^{-\left(\frac{n\pi}{2}\right)^2 F_o} \frac{\sin\frac{n\pi}{2}\xi}{n}}} \qquad (5.33)$$

5.10 Check the point $B_i = 0.1$, $F_o = 2.5$ on the graph for slabs in Fig. 5.10.

First we go to eqn. (5.35) ctn $\lambda L = \frac{\lambda L}{0.1}$ and get $\lambda L = 0.31105$, 3.1731, etc., by trial and error. Then we put eqn. (5.34) in eqn. (5.36) and get:

$$\Phi = + \int_0^{F_o} \lambda L e^{-(\lambda L)^2 F_o} \frac{2 \sin\lambda L \sin\lambda L(\xi-1)}{\lambda L+\sin\lambda L \cos\lambda L} \bigg|_{\xi=2} dF_o = - \frac{e^{-(\lambda L)^2 F_o}-1}{\lambda L} \times \frac{2(\sin^2\lambda L)}{\lambda L+\sin\lambda L \cos\lambda L}$$

so

$$\Phi = \frac{1-e^{-.31105^2(2.5)}}{0.31105} \frac{2 \sin^2(.31105)}{.31105+\sin(.31105)\cos(.31105)} + \frac{1-e^{-3.173^3(2.5)}}{3.173} \times \frac{2 \sin^2(3.173)}{3.173+\sin(.31105)\cos(.31105)} + \cdots$$

or

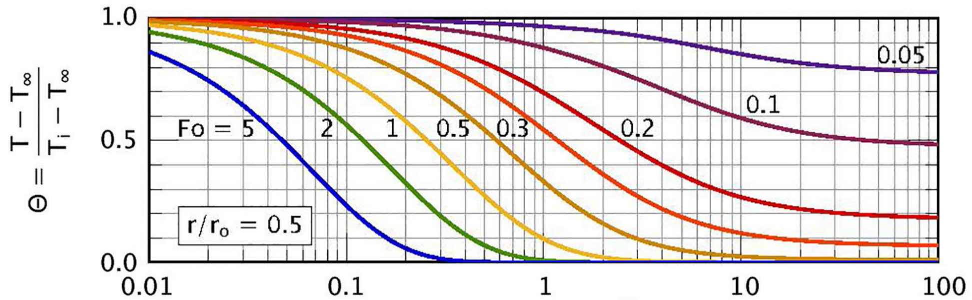
$$\Phi = 0.2148 - 0.0002 + \ldots = \underline{\underline{0.2146}}$$

From Fig. 5.10 we read $\Phi = 0.22$, which agrees within graphical accuracy.

5.11 Show, in Fig. 5.7, 5.8, or 5.9, where b.c.s of the third kind may be replaced with b.c.s of the first kind, where we can assume lumped capacity, and where the solid may be seen as semi-infinite.

Solution We choose the chart for a point midway between the center and surface of a sphere.

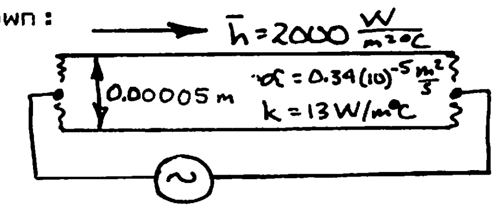
This region will effectively be semi-infinite as long as the change of surface temperature does not penetrate all the way to the center. That will be true for very low values of Fo.



We may treat situation as lumped capacity in this region of low Biot numbers (less than 0.1)

We may treat a situation as a b.c. of the first kind in this region of high Biot numbers (greater than 50 or so.)

5.12 A ribbon is heated by a.c. as shown:



How much does its temperature fluctuate?

$$Bi = \frac{\bar{h}\delta}{k} = \frac{2000(0.00005)}{13} = 0.00769$$

$$\phi = \frac{\omega\delta^2}{\alpha} = \frac{(2\pi 60)(0.00005)^2}{0.34(10)^{-5}} = 0.2772$$

From Fig. 5.11 we then read $\frac{T_{max} - T_{avg}}{T_{avg} - T_\infty} \simeq \underline{\underline{0.014}}$

so the temperature fluctuation is just a little over one percent of the average temperature difference between the wall and the stream.

5.13 Resolve eqn. (5.58) into appropriate dimensionless groups.

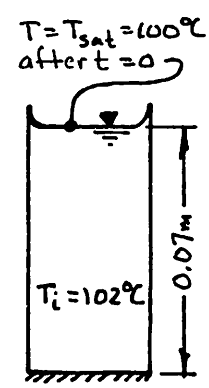
In this case: $R = R(k, \Delta T, \rho_g h_{fg}, \rho_f c_{p_f}, t)$. Thus there are 6 basic variables in J, m, kg, °C so we look for two Π-groups:

$$\frac{R}{\sqrt{\alpha t}} = fn\left(\frac{\rho_f c_{p_f} \Delta T}{\rho_g h_{fg}}, \text{ a modified Ja}\right)$$

The eqn. (5.52) can be rearranged as: $\underline{\underline{\frac{R}{\sqrt{\alpha t}} = \frac{2}{\sqrt{\pi}} Ja_{mod}}}$

which conforms to the dim. analysis.

5.14 The water column shown is initially at 102°C. Then it is suddenly depressurized to 1 atm.



a) When will the temperature reach 101.95 at the bottom?

b) Plot the height of the column vs. time, up to this time.

a) $\Theta = \frac{101.95 - 100}{102 - 100} = 0.975$ and $\frac{k}{\bar{h}L} = \frac{k/L}{\infty} = 0$

from Fig 5.7, $Fo = 0.06 = \frac{\alpha t}{L^2}$; $t = \frac{0.06(0.07)^2}{1.69 \times 10^{-7}}$

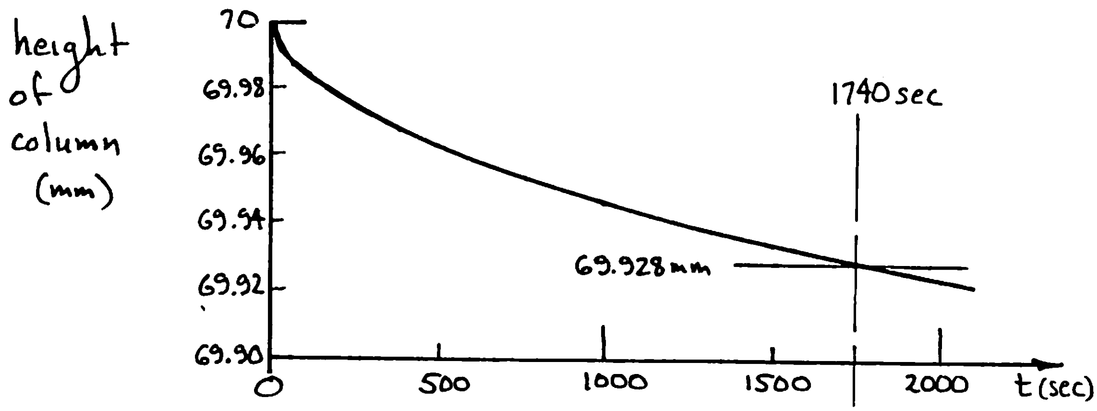
$\underline{\underline{t = 1740 \text{ sec or } 29 \text{ minutes}}}$

b) $\int_0^t q\,dt = \frac{k\Delta T}{\sqrt{\pi\alpha}} \int_0^t \frac{dt}{\sqrt{t}} = \frac{2k\Delta T}{\sqrt{\pi\alpha}}\sqrt{t}$ where we have used eqn.(5.48) for q

$$\text{reduction in height} = \frac{\left(\int_0^t q\,dt\right)\frac{J}{m^2}}{\rho_g h_{fg}\ \frac{J}{m^3}} = \frac{2k\Delta T}{\rho_g h_{fg}\sqrt{\pi\alpha}}\sqrt{t} = \frac{2(0.68)2}{961(2.26)10^6}\ \frac{\sqrt{t}}{\sqrt{\pi(1.68)10^{-7}}}$$

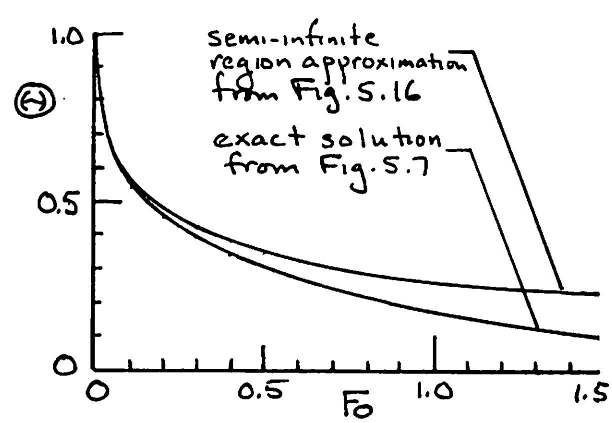
5.14 (continued)

We therefore obtain: $\underline{\underline{\text{height} = \left(0.07 - 1.72(10)^{-6}\sqrt{t}\right) \text{m}}}$ ←



5.15 A slab with Bi = 2 is cooled. Compare the exact and semi-infinite region solutions for Θ, on the surface.

Fo	$\beta^2 = Bi^2 Fo$	$\Theta_{Fig. 5.7}$	$\Theta_{Fig. 5.16}$
0	0	1	1
0.1	0.4	0.54	0.56
0.2	0.8	0.46	0.465
0.3	1.2	0.395	0.41
0.4	1.6	0.345	0.38
0.5	2.0	0.305	0.34
0.75	3.0	0.23	0.30
1.0	4.0	0.175	0.26
1.5	6.0	0.098	0.218



Since the semi-infinite approximation does not reflect the influence of the insulated wall at x/L = 0, it eventually shows a slower cooling than the correct solution.

5.16 Derive eqn. (5.62) from: $\dfrac{1}{2}\dfrac{d^2\Theta}{d\xi^2} = \dfrac{d\Theta}{d\Omega}$, $\Theta(\xi=0) = \cos\Omega$, $\Theta(\xi>0) = \text{finite}$

Assume: $\Theta = f(\xi)\, e^{i\Omega}$ so: $\dfrac{d^2 f}{d\xi^2} = 2i(f)$, hence $f = C_1 e^{\sqrt{2i}\,\xi} + C_2 e^{-\sqrt{2i}\,\xi}$

but $\sqrt{2i} = 1 + i$ & $e^{ix} = \cos x + i\sin x$ } so: $\Theta = \underbrace{(\cos\xi\cos\Omega + \sin\xi\sin\Omega)}_{=\cos(\xi-\Omega)}(C_3 e^{\xi} + C_4 e^{-\xi})$ new consts.

To accomodate the second b.c. we must get $C_3 = 0$.

To accomodate the first b.c.: $\Theta(\xi=0) = \cos(-\Omega) C_4 e^{-0} = \cos\Omega$

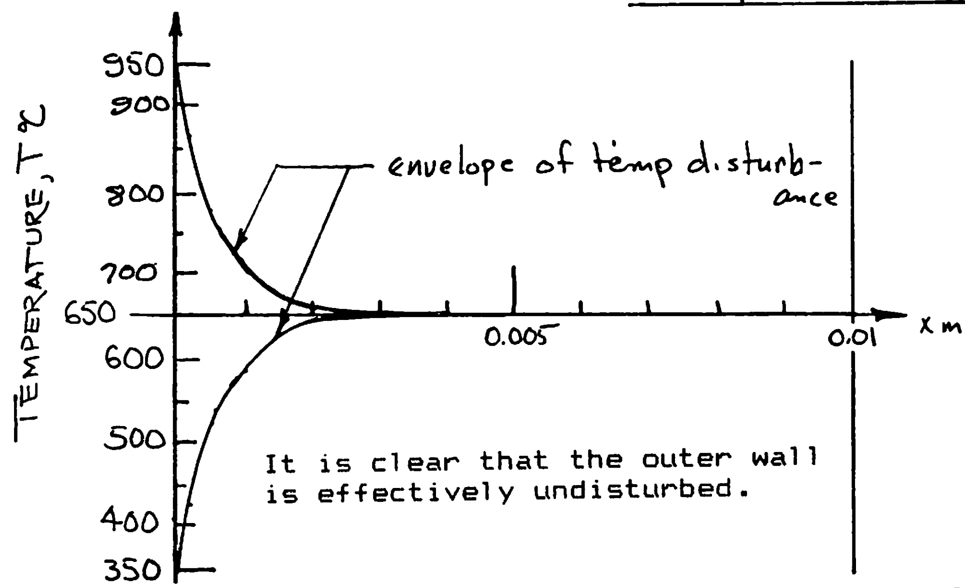
It follows that $C_4 = 1$ so we get: $\underline{\underline{\Theta = e^{-\xi}\cos(\xi - \Omega)}}$ ←

5.17 A "steel" cylinder wall is 1 cm thick. (Take $\alpha = k/\rho c = 32/(7800)(473) = 0.000008.67 m^2/s$.) The inside wall temp. is $(650 + 100\cos\omega t)^{o}C$ and $\omega = 2\pi 8 = 50.26$ rad/sec.
Plot the envelope of the temperature disturbance in the wall.

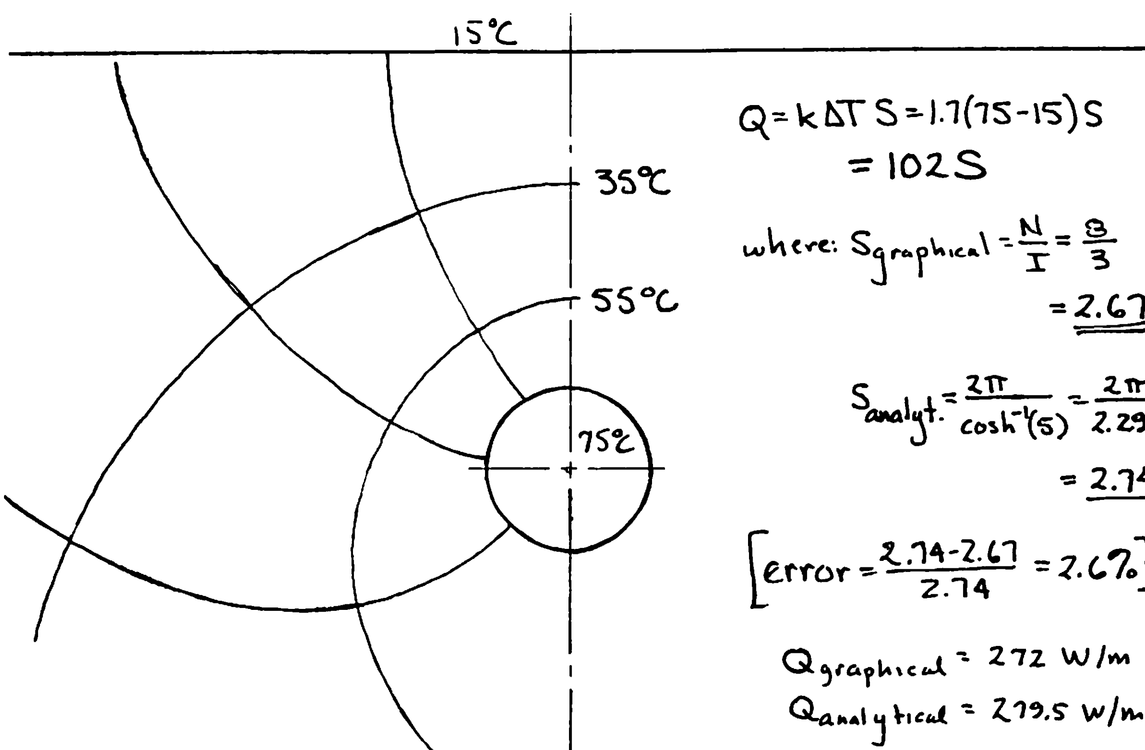
$$\Theta = e^{-\xi}\cos(\Omega - \xi)$$

so the envelope is given by $\Theta = \pm e^{-\xi}$ where $\xi = x\sqrt{\frac{\omega}{2\alpha}} = 170.25x$

and where: $\Theta = \frac{T-\bar{T}}{\Delta T} = \frac{T-650}{300}$, so $\underline{T_{envelope} = 650 \pm 300 e^{-170.25(x\,m)}}$



5.18 A $75^{o}C$, 0.4 m dia. pipe is buried in Portland cement ($k = 17.$) It is parallel to a $15^{o}C$ surface and 1 m away from it. Plot T along a vertical line through the center of the pipe and compute the heat loss per meter of pipe.



$Q = k\Delta T\, S = 1.7(75-15)S$
$= 102 S$

where: $S_{graphical} = \frac{N}{I} = \frac{8}{3}$
$= \underline{\underline{2.67}}$

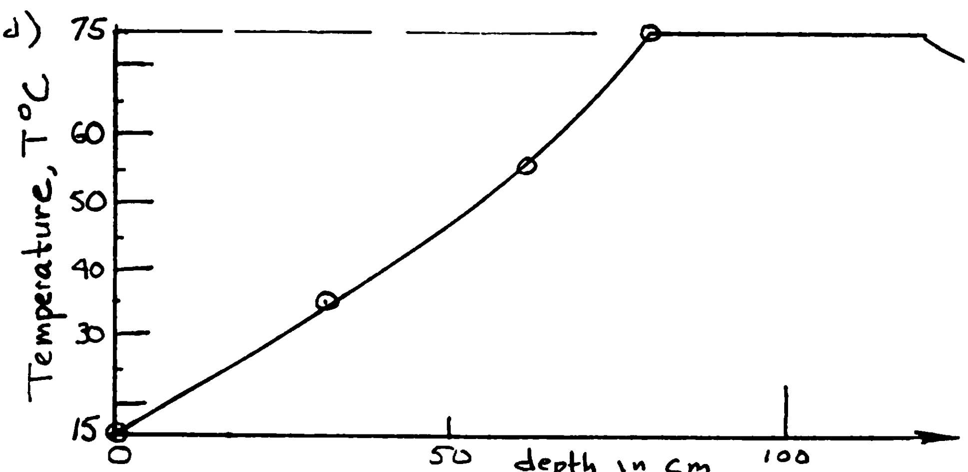
$S_{analyt.} = \frac{2\pi}{\cosh^{-1}(5)} = \frac{2\pi}{2.292}$
$= \underline{2.74}$

$\left[\text{error} = \frac{2.74-2.67}{2.74} = 2.6\%\right]$

$Q_{graphical} = 272$ W/m
$Q_{analytical} = 279.5$ W/m

5.18 (continued)

The variation of temp. with depth obtained from the flux plot.



5.19 Obtain S for a sphere buried in an infinite medium.

general solution for this case (cf. Example 5.10) is $T = C_1/r + C_2$

w/ b.c.'s: $T(r=\infty) = T_\infty$, so $C_2 = T_\infty$; and $T(r=R) = T_w$ so $C_1 = (T_w - T_\infty)R$

Thus: $T - T_\infty = (T_w - T_\infty)\frac{R}{r}$ and $Q = 4\pi R^2\left(-k\left.\frac{\partial T}{\partial r}\right|_{r=R}\right) = +4\pi R^2 k(T_w - T_\infty)\frac{1}{R}$

It follows that $\underline{\underline{S = \frac{Q}{k\Delta T} = 4\pi R}}$

5.20 Find S for parallel cylinders in an infinite medium. One has twice the diameter of the other. The centers are one diameter of the larger cylinder, apart.

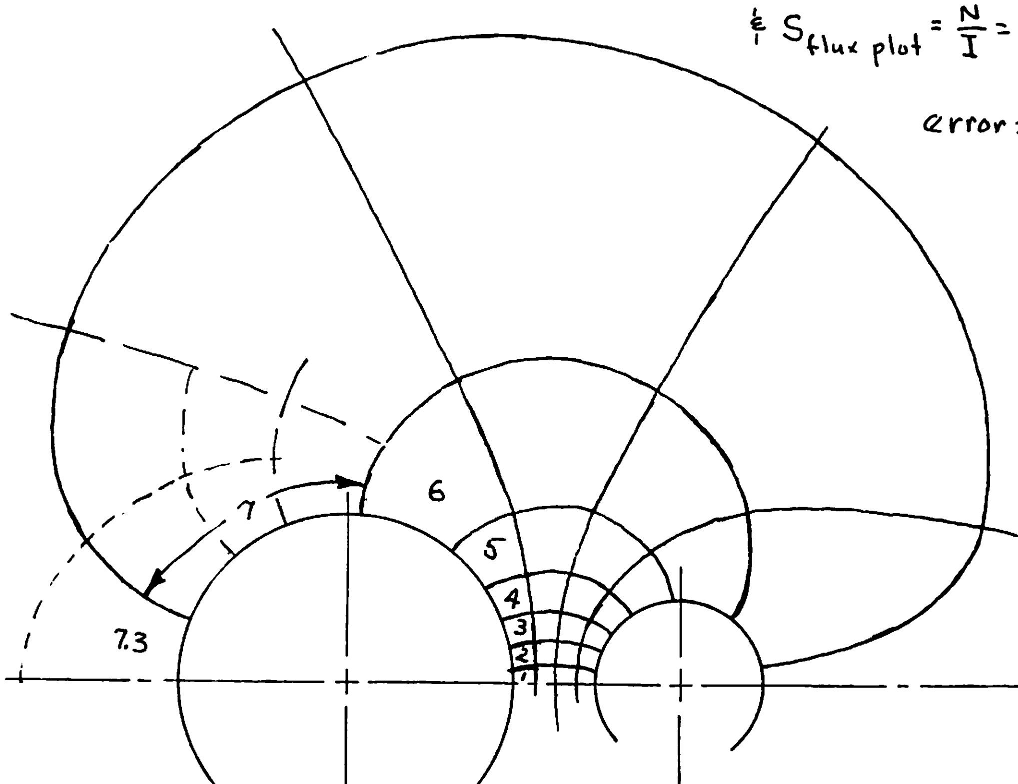
According to #9 in Table 5.2, $m_1/R_1 = 1 - \frac{1}{4} + \frac{1}{16} = \frac{19}{16}$ & $m_2/R_2 = 2\left[1 - \frac{1}{4} + \frac{1}{16}\right] = \frac{13}{8}$

Then $S_{analytical} = \dfrac{2\pi}{\cosh^{-1}\frac{19}{16} + \cosh^{-1}\frac{13}{8}} = \dfrac{2\pi}{0.60 + 1.07} = \underline{\underline{3.76}}$

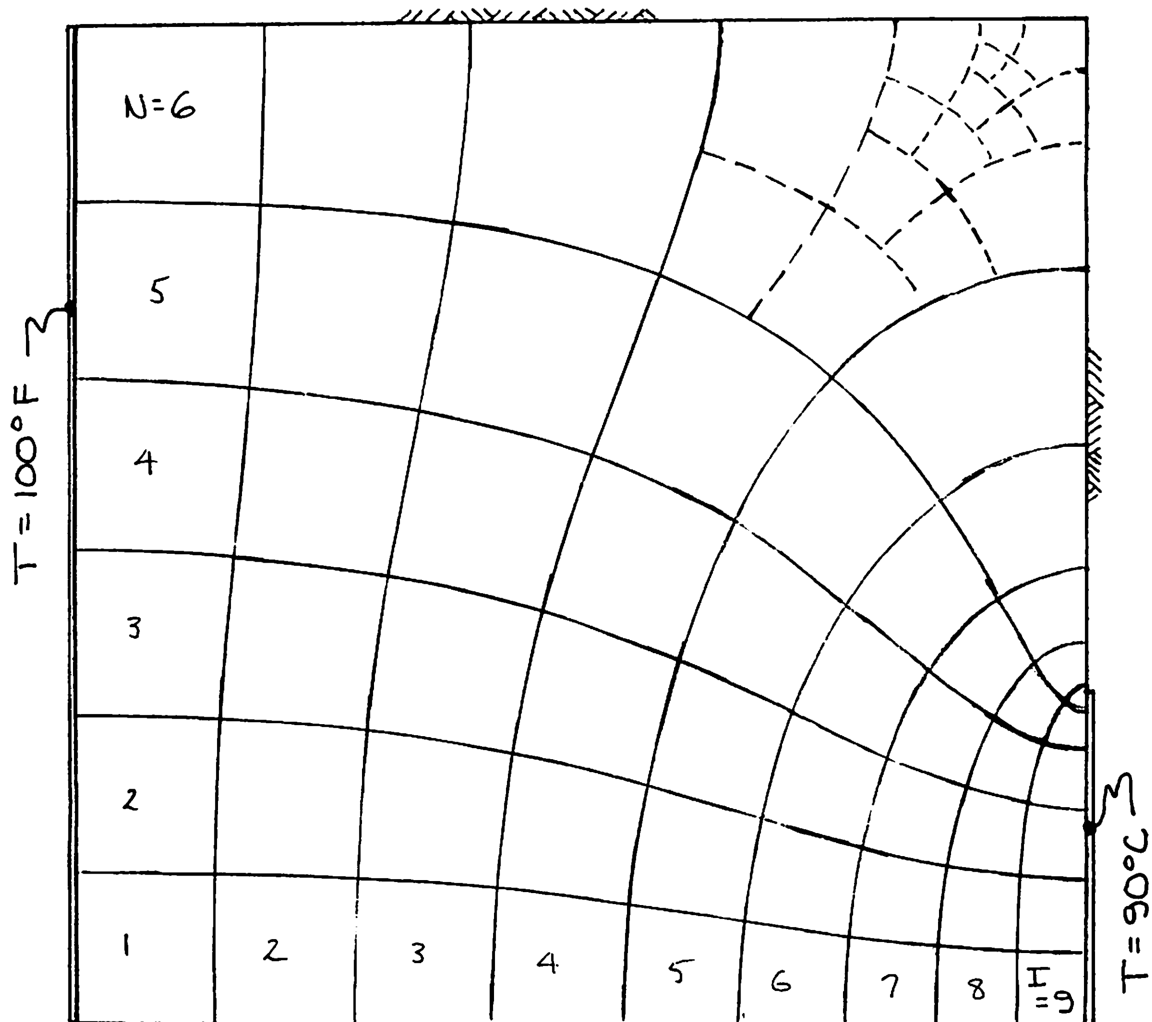
& $S_{flux\ plot} = \frac{N}{I} = \frac{2(7.3)}{4} = \underline{\underline{3.65}}$

error $= \dfrac{3.76 - 3.65}{3.76}$

$= \underline{\underline{2.9\%}}$



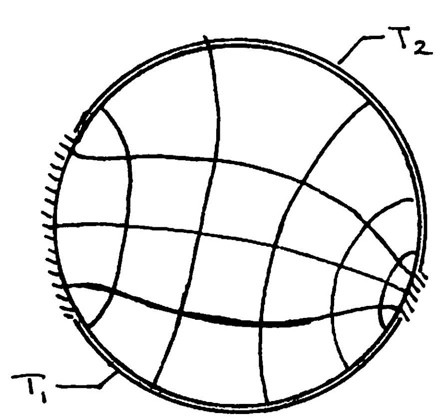
5.21 The 3 in. by 3 in. copper slab (1 in. thick) shown below conducts heat from the 100^{o}F surface on the left to the 1 in. portion of the right side which is kept at 90^{o}F.



$$Q = S k \Delta T = \frac{6}{9}\, 226 \frac{\text{Btu}}{\text{ft}^2\text{-hr-}^\circ\text{F}} \left((100-90)^\circ\text{F}\right) = \underline{1507 \frac{\text{Btu}}{\text{ft-hr}}}$$

$$1507 \frac{\text{Btu}}{\text{ft-hr}} \cdot \frac{\text{ft}}{12} = \underline{\underline{125.6 \frac{\text{Btu}}{\text{hr}}}} = \underline{\underline{38.8\ \text{W}}}$$

5.22a Obtain the shape factor for the following shape.



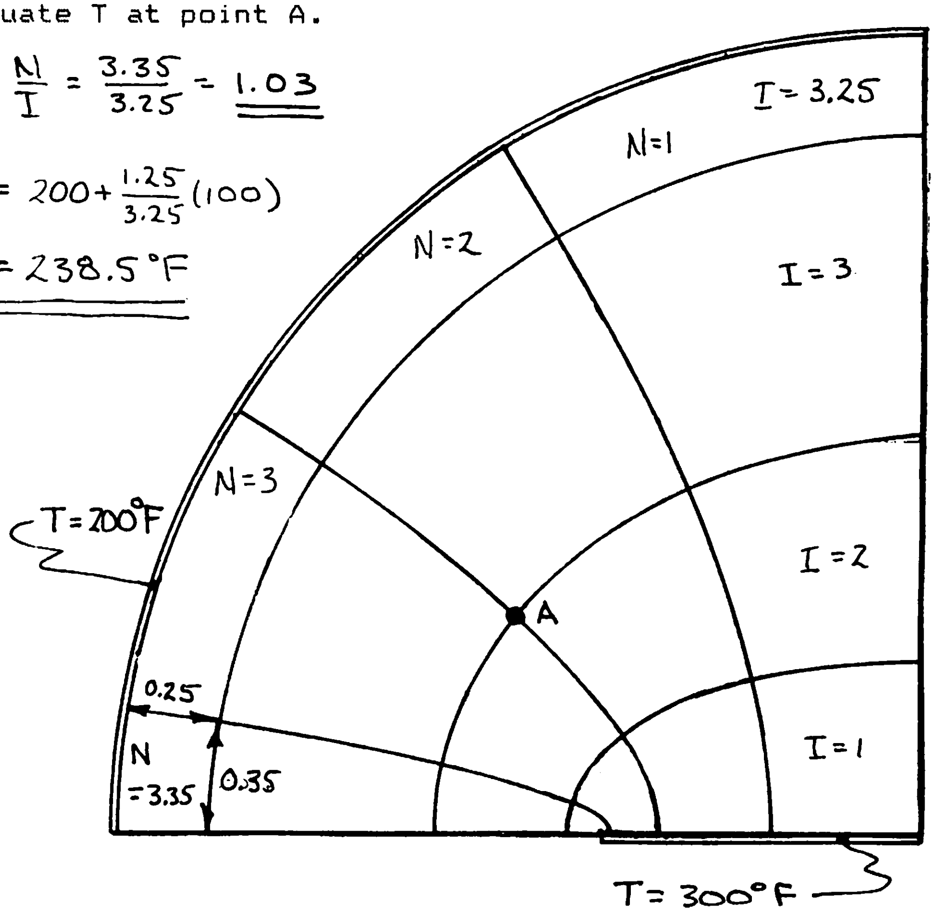
$$S = \frac{N}{I} \approx \frac{6}{4} = 1.67$$

5.22b Obtain the shape factor for the configuration shown. Evaluate T at point A.

$S = \frac{N}{I} = \frac{3.35}{3.25} = \underline{\underline{1.03}}$

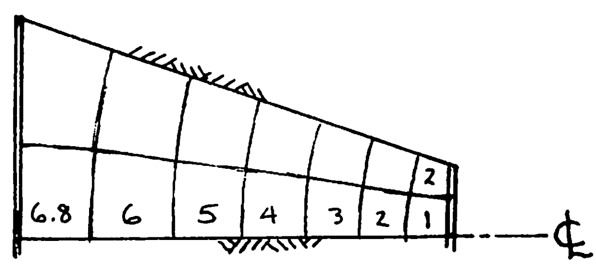
$T_A = 200 + \frac{1.25}{3.25}(100)$

$\underline{\underline{T_A = 238.5\,°F}}$



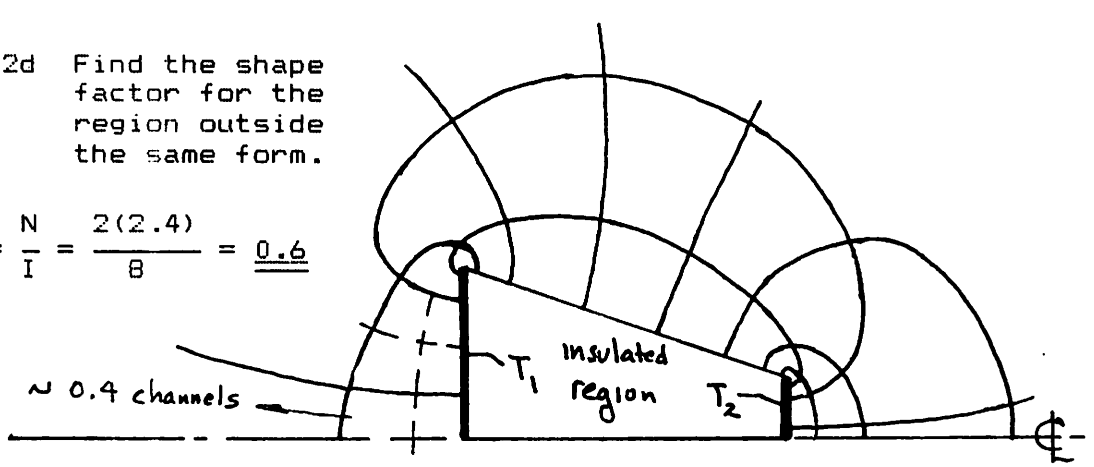
5.22c Find S for the inside of the form shown.

$S = \frac{2(2)}{6.8} = \underline{\underline{0.59}}$



5.22d Find the shape factor for the region outside the same form.

$S = \frac{N}{I} = \frac{2(2.4)}{8} = \underline{\underline{0.6}}$

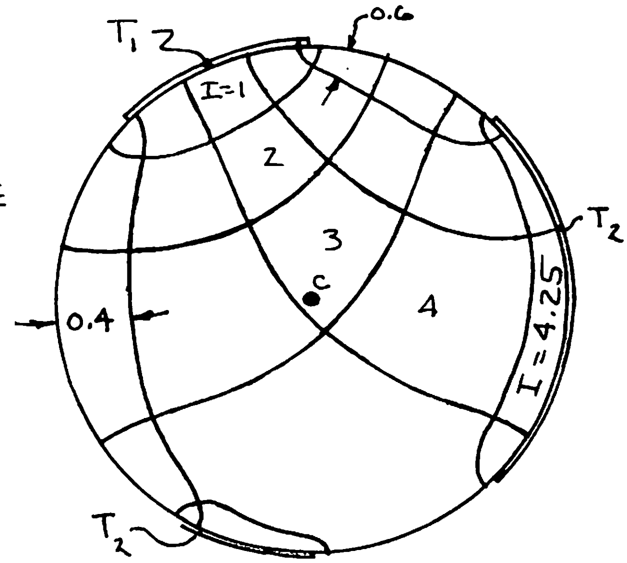


5.22e Find S for the shape shown and the center temperature.

$$S = \frac{N}{I} = \frac{0.4 + 3 + 0.6}{4.25} = \underline{\underline{0.94}}$$

$$T_c = T_1 + \frac{2.75}{4.25}(T_2 - T_1)$$

$$= \underline{\underline{T_1 + 0.65(T_2 - T_1)}}$$

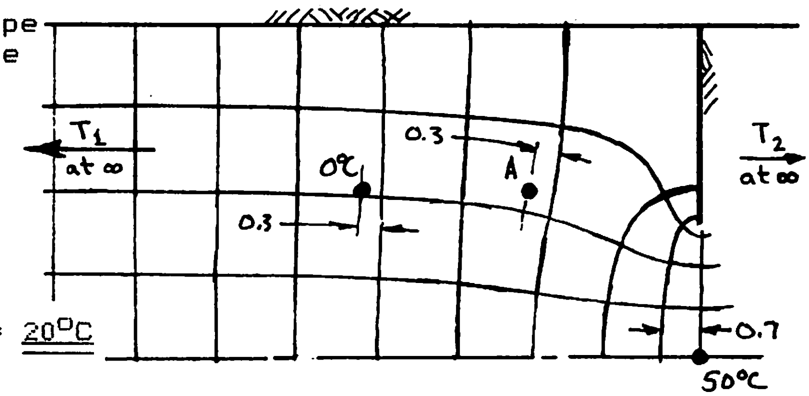


5.22f Find S for the shape below, and evaluate T at point A.

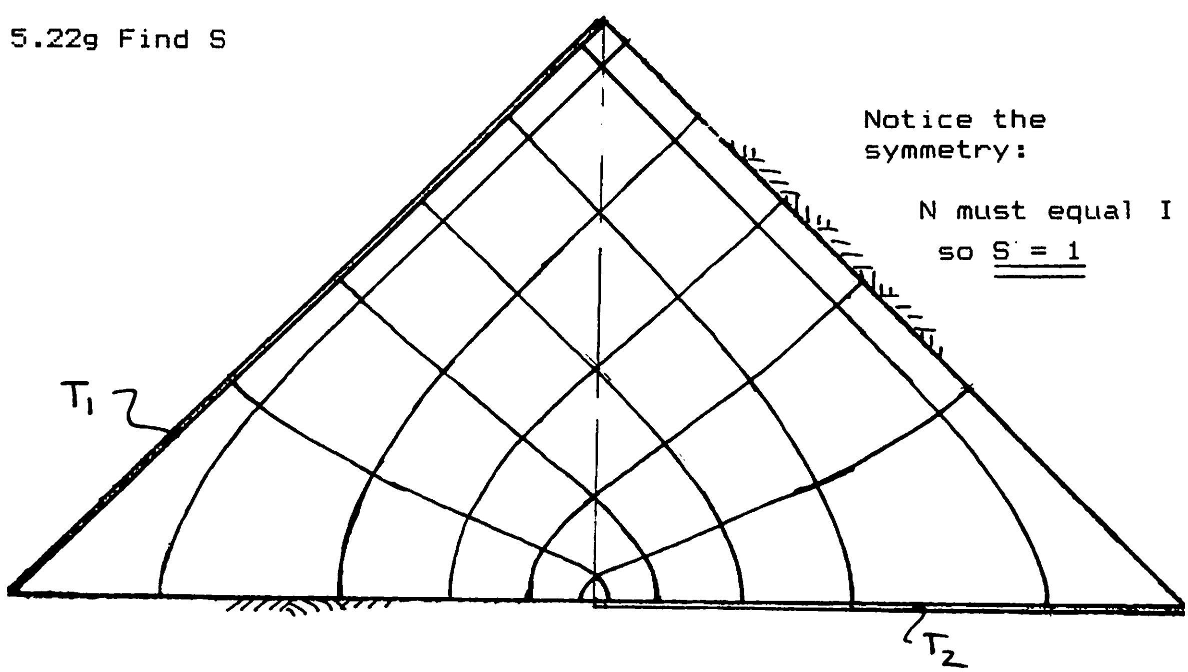
$$S = \frac{N}{I} = \frac{2(4)}{\infty} = \underline{\underline{0}}$$

because the thermal resistance is infinite in length.

$$T_A = 0 + \frac{2}{5}(50 - 0) = \underline{\underline{20^{\circ}C}}$$



5.22g Find S



Notice the symmetry:

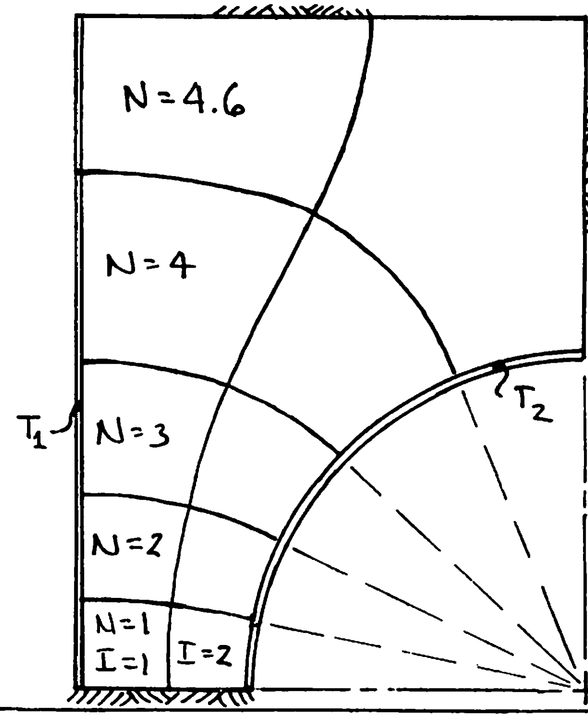
N must equal I so $\underline{\underline{S = 1}}$

(There are some interesting ramifications to this problem. See J. Heat Transfer, Vol. 103, No. 3, 1981, pp. 600-1.)

5.22h Find S

$$S = \frac{N}{I} = \frac{4.6}{2}$$

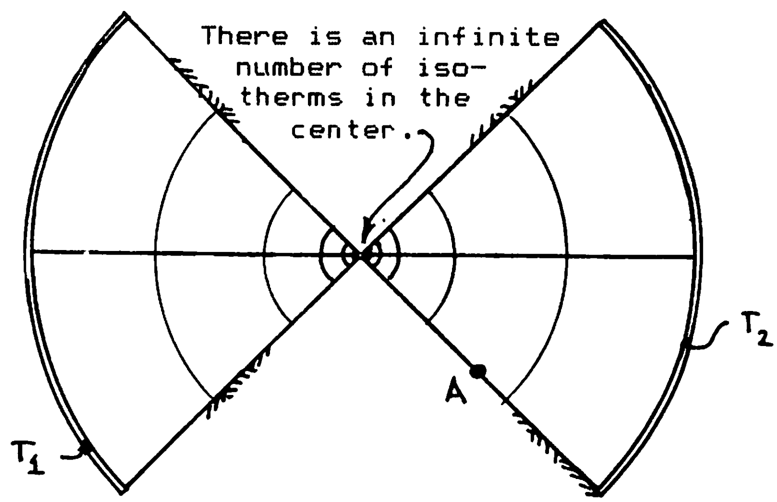
$$\underline{\underline{S = 2.3}}$$



5.22i Find S and T_A

$$S = \frac{N}{I} = \frac{2}{\infty} = \underline{\underline{0}}$$

Thus the singular point of infinite resistance in the center blocks all heat flow from T_1 to T_2.



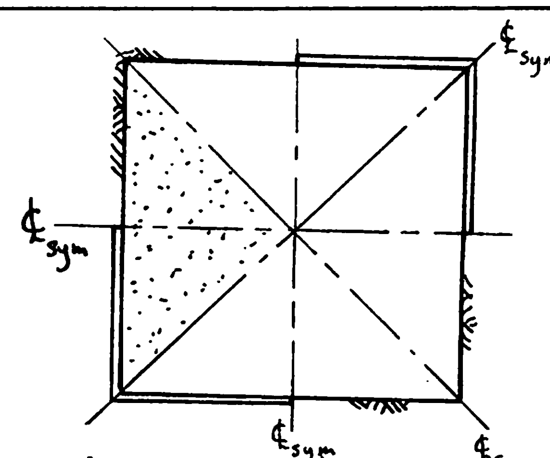
$\underline{\underline{T_A = T_2}}$ since all temperature drop occurs across the point of infinite resistance in the center.

5.22j Find S for the form shown

This form has 4 axes of symmetry. We therefore isolate the stippled area and do a flux plot for it. We get (see Prob. 5.22g)

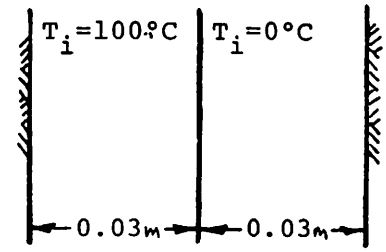
$$S_{\triangleright} = 1.00$$

Then for the total shape



$$S_{\boxtimes} = \frac{\text{twice as many channels}}{\text{twice as many isotherms}} = \frac{2}{2}\, S_{\triangleright} = \underline{\underline{1.00}}$$

5.23 The two copper slabs shown are suddenly laid together as shown. Find the temperature of the left-hand adiabatic side after 2.3 sec elapses.



By symmetry, we see that the interface must immediately assume--and retain--the average temperature of 50°C.

$$F_o = \frac{\alpha t}{L^2} = \frac{11.57(10)^{-5}(2.3)}{0.03^2} = \underline{0.296} \quad \text{and } B_i^{-1} = \frac{k}{\bar{h}L} = \frac{k/L}{\infty} = \underline{0}$$

Then from 5.7 we read $\simeq \dfrac{T_{ad.wall} - T_{int.}}{T_{i_{left}} - T_{int.}} = 0.61$

so $T_{ad.wall} = 0.61(100-50)+50 = \underline{\underline{80.5°C}}$

Problem 5.24 Eggs cook as their proteins denature and coagulate. An egg is considered to be "hard-boiled" when its yolk is firm, which corresponds to a center temperature of 75°C. Estimate the time required to hard-boil an egg if:

a) The minor diameter is 45 mm.
b) k for the entire egg is about the same as for egg white. No significant heat release or change of properties occurs during cooking.
c) $\overline{h}$ between the egg and the water is 1000 W/m^{2}K.
d) The egg has a uniform temperature of 20°C when it is put into simmering water at 85°C.

Solution We approximate the egg as a sphere of diameter 45 mm. From Table A.2, $k_{\text{egg}} =$ 0.56 W/m^{2}K and $\alpha_{\text{egg}} = 1.37 \times 10^{-7}$ m^2/s. Then

$$\text{Bi} = \frac{\overline{h} r_0}{k_{\text{egg}}} = \frac{(1000)(0.045/2)}{0.56} = 40.18$$

Additionally,

$$\Theta = \frac{T - T_\infty}{T_i - T_\infty} = \frac{75 - 85}{20 - 85} = 0.1539$$

The cooking time should be long enough to use the one-term solution, since the temperature change will have strongly affected the center of the egg by the time it is hard boiled. For this value of Bi, interpolation of Table 5.2 gives us

$$\hat{\lambda}_1 \simeq 3.147 \qquad A_1 \simeq 1.991$$

The Fourier number is found with eqn. (5.42)

$$\Theta = A_1 f_1 \exp\!\left(-\hat{\lambda}_1^2 \text{Fo}\right) \tag{*}$$

In this case, we need f_1 for a sphere from Table 5.1, in the limit as $r \to 0$; but recall that

$$\lim_{x \to 0} \frac{1}{x} \sin x = 1$$

so that we will take $f_1 = 1$. Solving eqn. (*) for $\text{Fo} = \alpha t / r_0^2$:

$$\text{Fo} = -(3.147)^{-2} \ln\!\left(\frac{0.1539}{1.991}\right) = 0.2585$$

Finally:

$$t = (0.2585)(0.045/2)^2 \big/ (1.37 \times 10^{-7}) = 955.2 \text{ sec} = 15 \text{ min } 55 \text{ sec} \approx 16 \text{ min} \qquad \xleftarrow{\text{Answer}}$$

To check our answer, we can look at the first panel of Fig. 5.9. For this Bi and Θ, the Fourier number will lie between 0.2 and 0.3, perhaps at 0.27 or so. The chart cannot provide the accuracy of the one-term solution—as noted on page 216, for this value of Fo, the one-term solution has an accuracy of about 0.1% relative to the exact result. Chart reading has an accuracy of 5–10%.

Comment: The cooking time will be less for smaller eggs; this diameter is somewhere between a "large" and an "extra large" egg. The cooking time will be shorter if the water temperature is kept higher (e.g., at a roiling boil). T Shortening the cooking time will lead to a softer, and eventually "soft-boiled" egg. Overcooking the egg will lead to a greenish residue on the yolk, which results from sulfur in the yolk combining with iron in the white to form harmless ferrous sulfide (Ref: University of Nebraska–Lincoln.) Cooling the cooked egg in ice water helps prevent the green tinge.

5.25 Prove that temperature cannot oscillate in a 2nd order lumped capacity system.

If the system is to oscillate, $\sqrt{\left(\frac{b}{2}\right)^2 - c}$ in eqn. (5.23) must be imaginary or

$$b^2 = \left(\frac{1}{T_1} + \frac{1}{T_2} + \frac{h_c}{\bar{h} T_2}\right)^2 < \frac{4}{T_1 T_2}$$

or:

$$\sqrt{\frac{T_2}{T_1}} + \sqrt{\frac{T_1}{T_2}}\left(1 + \frac{h_c}{\bar{h}}\right) - 2 < 0$$

If we call $\sqrt{T_2/T_1} \equiv x$, then this says: $x^2 - 2x + \left(1 + \frac{h_c}{\bar{h}}\right) < 0$

or: $(x-1)^2 + \frac{h_c}{\bar{h}} < 0$

But everything on the left must be positive so this cannot be! Therefore the system cannot oscillate.

5.26 When the isothermal and adiabatic lines in a flux plot are interchanged, N turns into I and I into N. It follows that:

$$\underline{S_{\text{interchanged}} = I/N = 1/S_{\text{original}}}$$

Problem 5.27 A 0.5 cm diameter cylinder at 300°C is suddenly immersed in saturated water at 1 atm. The water boils and $\overline{h} = 10,000$ W/m^{2}K. Find the centerline and surface temperatures for the cases that follow. *Hint:* Evaluate Bi in each case before you begin.

a) After after 0.2 s if the cylinder is copper.
b) After after 0.2 s if the cylinder is Nichrome V. [$T_{\text{sfc}} \simeq 216$°C]
c) If the cylinder is Nichrome V, obtain the most accurate value of the temperatures after 0.04 s that you can [$T_{\text{sfc}} \simeq 259$°C]

Solution

a) For pure copper at 300 °C, Table A.1 gives $k_{\text{copper}} = 384$ W/m·K. Then

$$\text{Bi}_{r_o} = \frac{\overline{h} r_o}{k} = \frac{(10^4)(0.0025)}{384} = 0.06510$$

For this low Biot number, we could use the lumped capacitance solution, but that won't allow us to compute the difference between the surface and centerline temperatures. Instead, we can use the one-term solutions (Section 5.5). The temperatures are found with eqn. (5.42):

$$\Theta = \frac{T - T_\infty}{T_i - T_\infty} = A_1 f_1 \exp\left(-\hat{\lambda}_1^2 \text{Fo}\right)$$

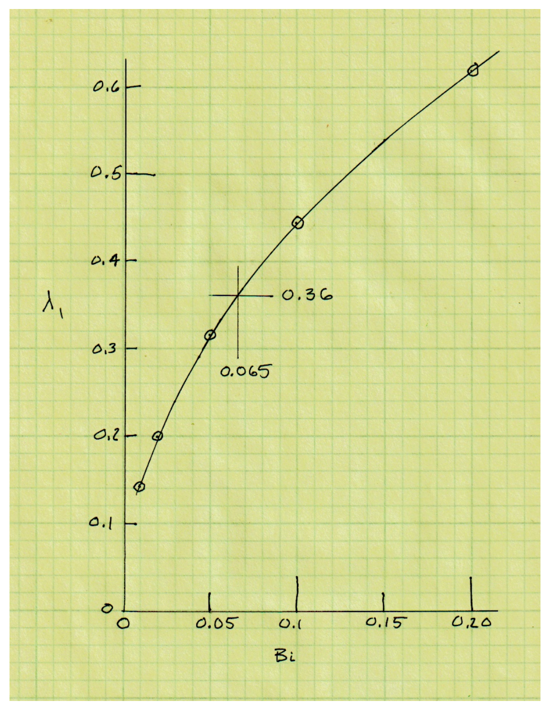
For our value of Bi, linear interpolation of Table 5.2[1] gives us

$$\hat{\lambda}_1 \simeq 0.3528 \qquad A_1 \simeq 1.016$$

The value of $\hat{\lambda}_1$ is deceptively precise, since the variation with Bi is not actually linear. Instead, we could iteratively solve the equation for $\hat{\lambda}_1$ in Table 5.1

$$\hat{\lambda}_1 J_0(\hat{\lambda}_1) = \text{Bi}_{r_o} J_1(\hat{\lambda}_1)$$

with an online Bessel function calculator, such as https://keisan.casio.com/exec/system/1180573474. The iteration converges to $\hat{\lambda}_1 = 0.3579$ with four digit accuracy.[2] Another approach would be to plot the values in Table 5.2 and with a hand-fitted curve find $\hat{\lambda}_1 \approx 0.36$, as shown at right. The iteration is of course most accurate.[3]



The Fourier number, with $\alpha_{\text{copper}} \simeq 11.57 \times 10^{-5}$ m^2/s, is

$$\text{Fo} = \frac{\alpha t}{r_o^2} = \frac{(11.57 \times 10^{-5})(0.2)}{(0.0025)^2} = 3.70$$

[1] Older editions of AHTT give values for Bi = 0.05 and 0.1 only.

[2] Specifically, $J_0(0.3579) = 0.176100$ and $J_1(0.3579) = 0.968232$, and $\text{Bi}_{r_o} J_1/J_0 = 0.3579$.

[3] With that value of $\hat{\lambda}_1$, we could also use the equation for A_1 in Table 5.1 to recalculate, but the result turns out to match what we already have because the relationship for A_1 is nearly linear in this range of Bi.

At the center, we need f_1 for a cylinder from Table 5.1 for $r = 0$, which is $J_0(0)$. A look-up shows that $J_0(0) = 1$. At the surface ($r = r_0$), $f_1 = J_0(\hat{\lambda}_1) = J_0(0.3579) = 0.9682$. With these values,

$$\Theta = \begin{cases} (1.016)(0.9682)\exp[-(0.3579)^2(3.70)] = 0.6124 & \text{at } r = r_o \\ (1.016)(1)\exp[-(0.3579)^2(3.70)] = 0.6325 & \text{at } r = 0 \end{cases}$$

Solving for the temperatures, with $T = T_\infty + \Theta(T_i - T_\infty)$ and $T_\infty = 100°\text{C}$ for saturated water,

$$T = \begin{cases} 100 + (300 - 100)(0.6124) = 223.5°\text{C} \simeq 224°\text{C} & \text{at } r = r_o \\ 100 + (300 - 100)(0.6325) = 226.5°\text{C} \simeq 227°\text{C} & \text{at } r = 0 \end{cases} \quad \xleftarrow{\text{Answer}}$$

As expected, the surface and center temperatures are very close (the lumped solution would make them equal).

b) For Nichrome V at 300°C, Table A.1 gives $k_{\text{NiV}} = 15$ W/m·K. Then

$$\text{Bi}_{r_o} = \frac{\overline{h} r_o}{k} = \frac{(10^4)(0.0025)}{15} = 1.667$$

The Fourier number, with $\alpha_{\text{NiV}} \simeq 0.26 \times 10^{-5}$ m^2/s, is

$$\text{Fo} = \frac{\alpha t}{r_o^2} = \frac{(0.26 \times 10^{-5})(0.2)}{(0.0025)^2} = 0.0832$$

This Fourier number is too low to use the one-term solutions, and this Biot number is too high for a lumped solution. Instead, we can use the temperature-response chart, Fig. 5.8. The author reads:

$$\Theta_{r=r_o} \simeq 0.58 \qquad \Theta_{r=0} \simeq 0.98$$

Solving for the temperatures,

$$T = \begin{cases} 100 + (300 - 100)(0.58) = 216°\text{C} & \text{at } r = r_o \\ 100 + (300 - 100)(0.98) = 296°\text{C} & \text{at } r = 0 \end{cases} \quad \xleftarrow{\text{Answer}}$$

c) For $t = 0.04$ s, the Fourier number is even lower—0.0166. Let us use the semi-infinite body solution shown in Fig. 5.16 and given by eqn. (5.53). Here

$$\beta = \frac{\overline{h}\sqrt{\alpha t}}{k} = \frac{10^4\sqrt{(0.26 \times 10^{-5})(0.04)}}{15} = 0.2150$$

From eqn. (5.53), and either Table 5.3 or an online erfc calculator

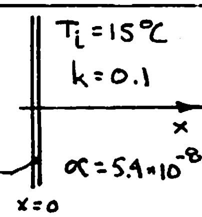
$$\Theta = \exp\left[(0.2150)^2\right]\text{erfc}(0.2150) = 0.7971$$

so that

$$T = T_\infty + (T_i - T_\infty)\Theta = 100 + (300 - 100)(0.7971) = 259°\text{C} \quad \xleftarrow{\text{Answer}}$$

The center temperature is unchanged at this time.

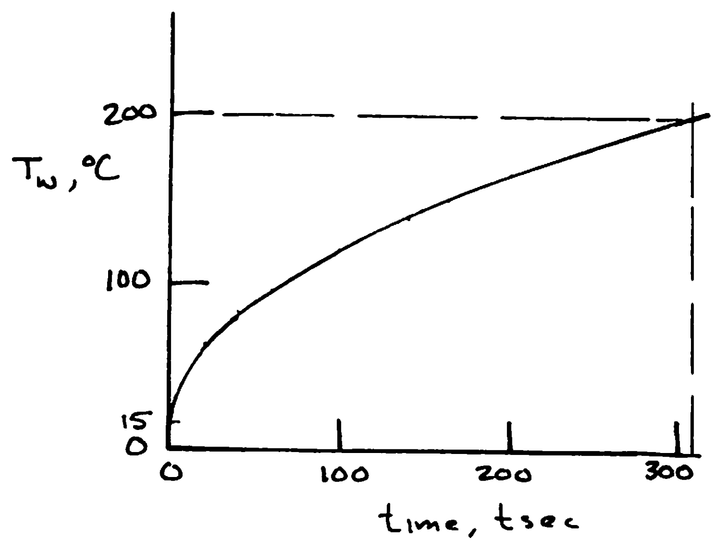
5.28 Plot T(x=0) as a function of time when the strip heater shown is turned on. What is q at x = 0.01 m when $T(x=0) = 200^oC$?



strip heater $4000\ W/m^2$

from eqn. (5.50) $T_w(t) = T_\infty + 2\frac{q_w}{k}\sqrt{\frac{\alpha t}{\pi}} = 15 + \frac{8000}{0.1}\sqrt{\frac{5.4(10)^{-8}}{\pi}}\sqrt{t}$

$$= 15 + 10.49\sqrt{t}$$



$T_w = 200°C$ when $t = 310.4$ sec.

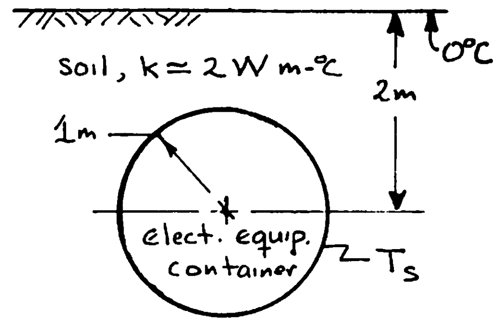
Then from eqn. (5.49):

$$\frac{q_w - q(x,t)}{q_w} = \frac{4000 - q}{4000} = \text{erf}\frac{x}{2\sqrt{\alpha t}} = \text{erf}\ 1.22 = 0.9155$$

so $q(x = 0.01m, t = 310.4s) = 338\ \frac{W}{m^2}$

5.29 There as many answers to this problem as there are students. (The most common error students will make is that of touching items in a room that are not at room temperature -- rings on fingers, window panes, ice cubes, etc.)

5.30 What is the maximum Q from the container shown if T_s cannot exceed 30°C



$$Q = k\Delta T S = 2(30-0)S$$

And from Table 5.2, No.7,

$$S = \frac{4\pi R}{1-\frac{R}{2h}} = \frac{4\pi}{1-\frac{1}{2(2)}} = \underline{16.76}$$

Therefore the maximum Q is 2(30)16.76 = $\underline{\underline{1005\ W}}$.

5.31 A semi-infinite slab of ice at -10°C is exposed to air at 15°C through a heat transfer coefficient of 10 W/m^2-°C. What is the initial rate of melting in kg of water/m^2-s? What is the asymptotic rate of melting? Describe the melting process in physical terms. (The latent heat of fusion of ice h_{fs} = 333,300 J/kg.)

Solution. The surface must first be brought up to the melting temperature. During this period $\underline{\underline{\dot{m}_{melt} = 0\ kg/m^2\text{-}s}}$.

Once the saturation temperature, 0°C, has been reached at the surface, heat will flow into the interior of the slab in accordance with equation (5.48) which shows that $q \sim 1/\sqrt{t}$. Thus, after a long time, that portion of the heat reaching the interface, which flows to the interior, becomes negligible. Then a simple energy belance yields:

$$\bar{h}\ (T_{air} - 0°C) = h_{fs}\ \dot{m}_{melt}$$

or:

$$\dot{m}_{melt} = 10(15-0)/333{,}300 \doteq \underline{\underline{0.00045\ kg/m^2\text{-}s}}$$

5.32 One side of a firebrick wall, 10 cm thick, initially at 20°C is exposed to a 1000°C flame through a heat transfer coefficient of 230 W/m^2-°C. How long will it be before the other side is too hot to touch? (Estimate properties at 500°C).

Solution. $k/\bar{h}L = 0.15/230(0.1) = 0.00652$

$$\frac{T_{burn} - T_\infty}{T_i - T_\infty} = \frac{65-1000}{20-1000} = 0.954$$

Then from Fig. 5.7, upper left, we get

$$\frac{\alpha t}{L^2} = 0.075,\ t \simeq \frac{0.075(0.1)^2}{5.4\times10^{-8}} = 13{,}890\ \text{sec}$$

$$= \underline{\underline{3\ \text{hr}\ 51\ \text{min}}}.$$

Problem 5.33 A lead bullet travels for 0.5 seconds within a shock wave that heats the air near the bullet to 300°C. Approximate the bullet as a cylinder 0.8 mm in diameter. What is its surface temperature at impact if h = 600 W/m²K and if the bullet was initially at 20°C? What is its center temperature?

Solution The Biot number 600(0.004)/35 = 0.0685, so we can first try the lumped capacity approximation. See eqn. (1.22):

$$(T_{sfc} - 300)/(20 - 300) = \exp(-t/\mathbf{T}), \quad \text{where } \mathbf{T} = mc/hA$$

So $\mathbf{T} = \rho c(\text{area})/h(\text{circumf.}) = 11{,}373(130)\pi(0.004)^2/h\pi(0.008) = 4.928$ seconds

And $(T_{sfc} - 300)/(20 - 300) = \exp(-0.5/4.928)$.

So $\underline{\underline{T_{sfc} = 300 - 0.903(280) = 47.0^\circ C}}$

In accordance with the lumped capacity assumption,

<u>47.0°C is also the center temperature.</u>

Now let us see what happens when we use the exact graphical solution, Fig. 5.8:

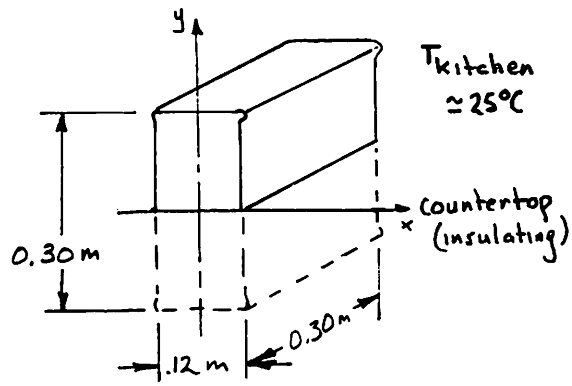
for $Fo = \alpha t/r_o^2 = 2.34(10^{-5})(0.5)/0.004^2 = 0.731$ and $r/r_o = 1$, we get:

$(T_{sfc} - 300)/(20 - 300) = 0.90,$ So $\underline{\underline{T_{sfc} = 48.0^\circ C}}$

And at $r/r_o = 0$, $(T_{ctr} - 300)/(20 - 300) = 0.92,$ & $\underline{\underline{T_{ctr} = 42.4^\circ C}}$

We thus have good agreement within the limitations of graph-reading accuracy. It also appears that the lumped capacity assumption is accurate within around 6 degrees in this situation.

5.34 A loaf of bread as shown, is at 125°C when it is removed from and oven and put to cool on an insulating counter. $k = 0.05$ W/m-°C and 5×10^{-7} m²/s. $\bar{h} = 10$ W/m-°C. When will the bottom center reach 60°C.



$$\Theta = \frac{60-25}{125-25} = 0.35$$
$$= \Theta_x \Theta_y \Theta_z$$

Let us guess times, calculate Θ_x, Θ_y, Θ_z, and see how close to 0.35 we come.

$$F_o = \frac{\alpha t}{L^2} \quad ; \quad k/\bar{h}L = 0.005/L$$

	for x, L = 0.06 m			for y, L = 0.15 m			$\Theta_z = \Theta_y$	$\Theta_x \Theta_y \Theta_z$
Guess t	F_o	$k/\bar{h}L$	Θ_x	F_o	$k/\bar{h}L$	Θ_y		
3600 sec	0.5	0.0833	0.42	0.08	0.0333	0.95	0.95	0.38
4000	0.556	"	0.38	0.089	"	0.95	0.95	0.34

It looks like about 3900 seconds or <u>1hr and 5 min.</u>

5.35 A lead cube, 50 cm on each side, is initially at 20°C. The surroundings are suddenly raised to 200°C and h around the cube is 272 W/m^2–°C. Plot the cube temperature along a line from the center to the middle of one face, after 20 minutes have elapsed.

$$Bi^{-1} = (\bar{h}L/k)^{-1} = \frac{34}{272(0.25)} = \underline{0.5} \quad ; \quad Fo = \frac{\alpha t}{L^2} = \frac{2.35\times10^{-5}(20\times60)}{0.25^2} = \underline{0.451}$$

Then:

$$\Theta = \underbrace{\frac{T-T_\infty}{T_i-T_\infty}}_{(T-200)/-180} = \underbrace{\Theta^2\left(Bi=0.5,\ Fo=0.451,\ \tfrac{x}{L}=0\right)}_{0.7^2} \times \Theta\left(Bi=0.5,\ Fo=0.451,\ \tfrac{x}{L}\right)$$

so:

$T = 200 - 88.2 \times 0.70 = 138.3°C$ at $x/L = 0$

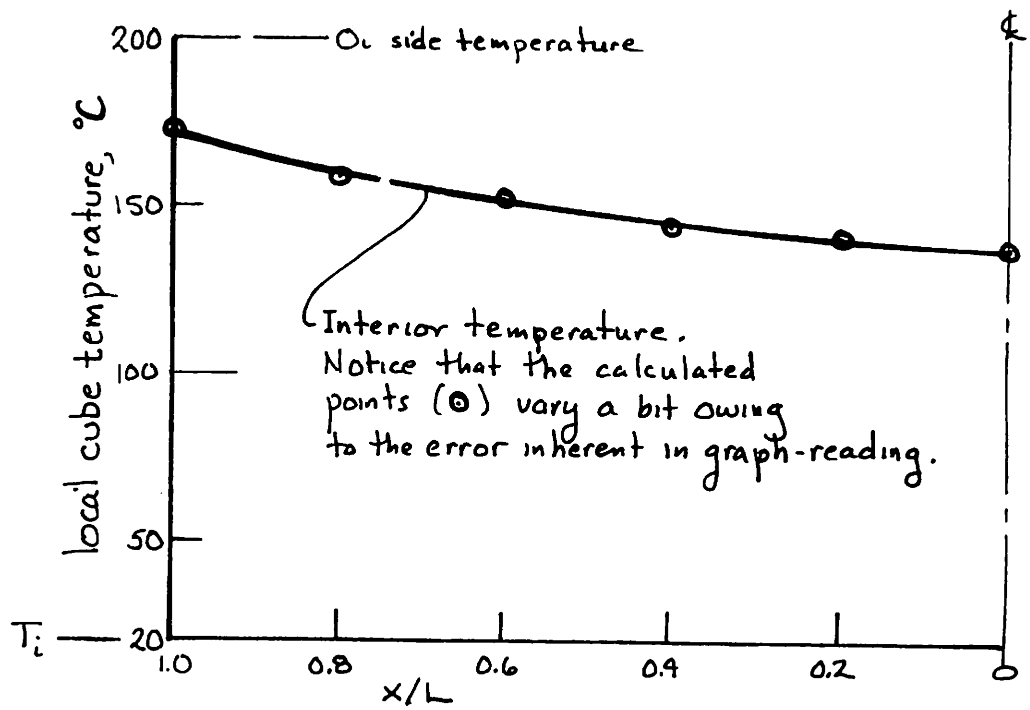
$T = 200 - 88.2 \times 0.67 = 140.9°C$ at $x/L = 0.2$

$T = 200 - 88.2 \times 0.63 = 144.4°C$ at $x/L = 0.4$

$T = 200 - 88.2 \times 0.55 = 151.5°C$ at $x/L = 0.6$

$T = 200 - 88.2 \times 0.46 = 159.4°C$ at $x/L = 0.8$

$T = 200 - 88.2 \times 0.32 = 171.8°C$ at $x/L = 1.0$

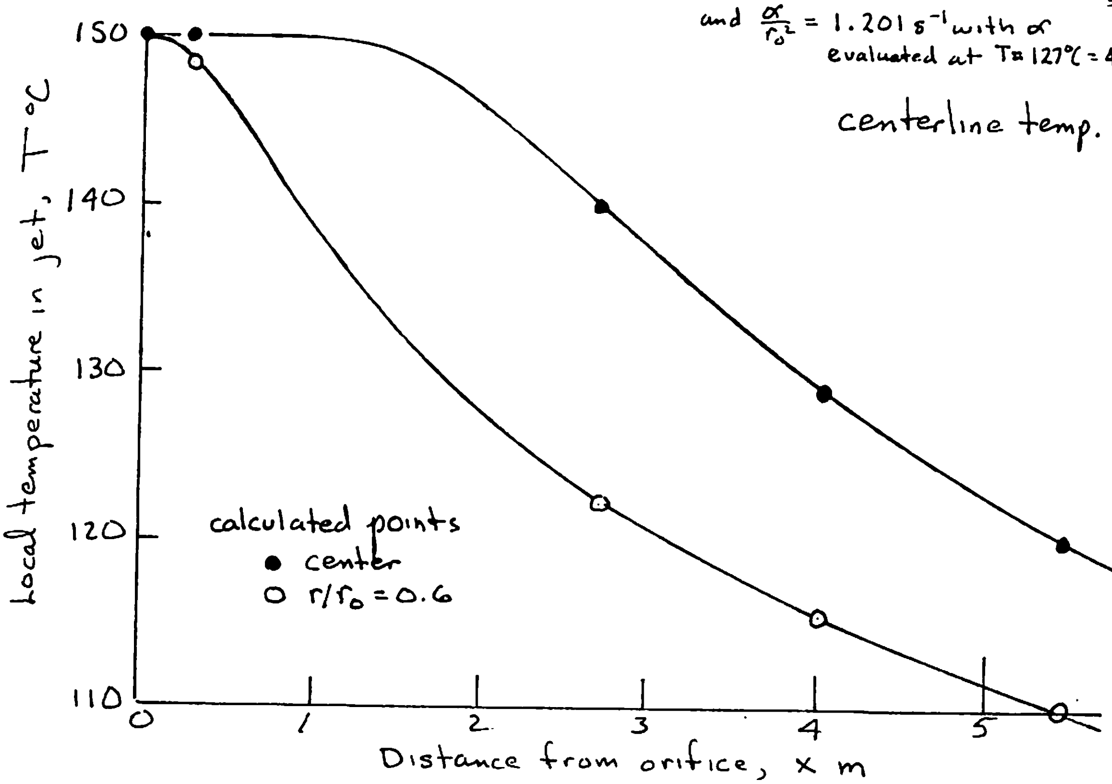


5.36 A jet of clean water superheated to 150°C issues from a (1/16) in. diameter sharp-edged orifice into air at 1 atm., moving at 27 m/s. The coefficient of contraction of the jet is 0.611. Evaporation at $T = T_{sat}$ begins immediately the outside of the jet. Plot the center-line temperature of the jet, and $T(r/r_o = 0.6)$, as functions of distance from the orifice, up to about 5 m. Neglect any axial conduction and any dynamic interactions between the jet and the air.

Any element of the jet cools approximately as an infinite cylinder would, while it moves. Therefore we can use Fig. 5.8 with $r/r_o = 0$ and $r/r_o = 0.6$, and (since we have a b.c. of the first kind, $T(r=r_o) = T_{sat}$) Bi^{-1} is 0. Then:

					$T = [(T_i - T_\infty)\Theta + T_\infty]$ °C	
t sec	x m $= u_{jet}\, t$	$Fo = \frac{\alpha t}{r_o^2}$	$\Theta_{\text{¢}}$	$\Theta(\frac{r}{r_o} = 0.6)$	$\Theta_{\text{¢}}$	$\Theta(\frac{r}{r_o} = 0.6)$
0.01	0.27	0.0120	1.00	0.99	150°C	148 °C
0.1	2.7	0.1201	0.80	0.44	140	122
0.15	4.05	0.1801	0.158	0.29	129	114.5
0.2	5.4	0.2402	0.42	0.20	121	110

where $r_o = \frac{1}{2}(0.0254/16)\sqrt{0.611} = 0.000$; $u_{jet} \doteq 27$ m/s ; $T_\infty \doteq T_{sat} = 100$°C ; $T_i - T_\infty = 150 - 100 = 50$°C ; and $\frac{\alpha}{r_o^2} = 1.201\ s^{-1}$ with α evaluated at $T \cong 127$°C $= 400$°K



5.37 A 3 cm thick slab of aluminum (initially at 50°C) is slapped tightly against a 5 cm slab of copper (initially at 20°C). The outsides are both insulated and the contact resistance is negligible. What is the initial interfacial temperature? Estimate how long the interface will keep its initial temperature.

In accordance with equation (5.60), we get:

$$\frac{T_i - T_c}{T_a - T_c} = \frac{k_a/\sqrt{\alpha_a}}{k_c/\sqrt{\alpha_c} + k_a/\sqrt{\alpha_a}}$$

or

$$\frac{T_i - 20}{50-20} = \frac{237/\sqrt{9.61(10)^{-5}}}{398/\sqrt{11.57\times10^{-5}} + 237/\sqrt{9.61(10)^{-5}}} = 0.39$$

The solution is $\underline{\underline{T_i = 31.86°C}}$

and this will be valid as long as the slabs behave as though they were semi-infinite regions. This will be as long as $\xi > 3.65$ (See eqn. (5.45))

$$\xi = \frac{x}{\sqrt{\alpha t}} \quad \text{so} \quad 3.65 = \xi_a = \frac{0.02}{\sqrt{9.61(10)^{-5} t}} \quad ; \quad t < 0.703 \text{ sec.}$$

$$3.65 = \xi_{Cu} = \frac{0.03}{\sqrt{11.57(10)^{-5} t}} \quad ; \quad t < 1.622 \text{ sec.}$$

The shorter of the two times dictates how long it will take the first insulated wall to be felt. Consequently the interface temperature of $T_i = 31.86°C$ will remain constant for:

$\underline{\underline{0.703 \text{ sec}}}$
(at least)

5.38 A cylindrical underground gasoline tank, 2m in diameter and 4m long, is embedded in 10°C soil with k=0.8 W/m-°C and $\alpha = 1.3(10)^{-6}$ m^2/s. Water at 27°C is injected into the tank to test it for leaks. It is well-stirred with a submerged, (1/2)kW pump. We observe the water level in a 10cm ID transparent standpipe and measure its rate of rise or fall. What rate of change of height will occur after one hour if there is no leakage? Will the level rise or fall? Neglect thermal expansion and deformation of the tank which should be complete by the time the tank is filled. (Hint: see eqn. (8.7)

$$\text{Area} = \frac{4}{3}\pi(2)^2 + 2\pi(4) = \underline{58.64\ m^2}, \qquad V = \frac{4}{3}\pi(2)^2 4 = \underline{67.02\ m^3}$$

There are two energy transfers: $\underline{500\ J/s}$ of work to the water.

and $$Q = \frac{kA\Delta T}{\sqrt{\pi\alpha t}} = \frac{0.8(58.64)(27-10)}{\sqrt{\pi(1.3)(10)^{-6}(3600)}} = 6577\ \frac{J}{s}$$ of heat ~~from~~ the water.

Now, using eqn. (8.7),

$$\beta = \frac{1}{v}\left.\frac{\partial v}{\partial T}\right|_p = \frac{1}{V}\left.\frac{\partial V}{\partial T}\right|_p\ ; \quad \frac{dV}{dT} = 0.000275(67.02) = \underline{0.01843\ \frac{m^3}{^\circ C}}$$

And:
$$\frac{dt}{dT} = \frac{\rho c V}{Q_{net}} = \frac{996.6(4177)(67.02)}{(500-6577)} = \underline{-45{,}909\ \frac{sec}{^\circ C}}$$

so
$$\frac{dV}{dt} = \frac{dV}{dT}\frac{dT}{dt} = -\frac{0.01843}{45{,}909} = -0.401\times10^{-6}\ \frac{m^3}{s} = \underline{-0.001445\ \frac{m^3}{hr}}$$

Then:
$$\frac{dh}{dt} = \frac{d(V/A_{pipe})}{dt} = \frac{-0.001445}{\frac{\pi}{4}(0.1)^2} = -0.184\ \frac{m}{hr} = \underline{\underline{-18.4\ \frac{cm}{hr}}}$$

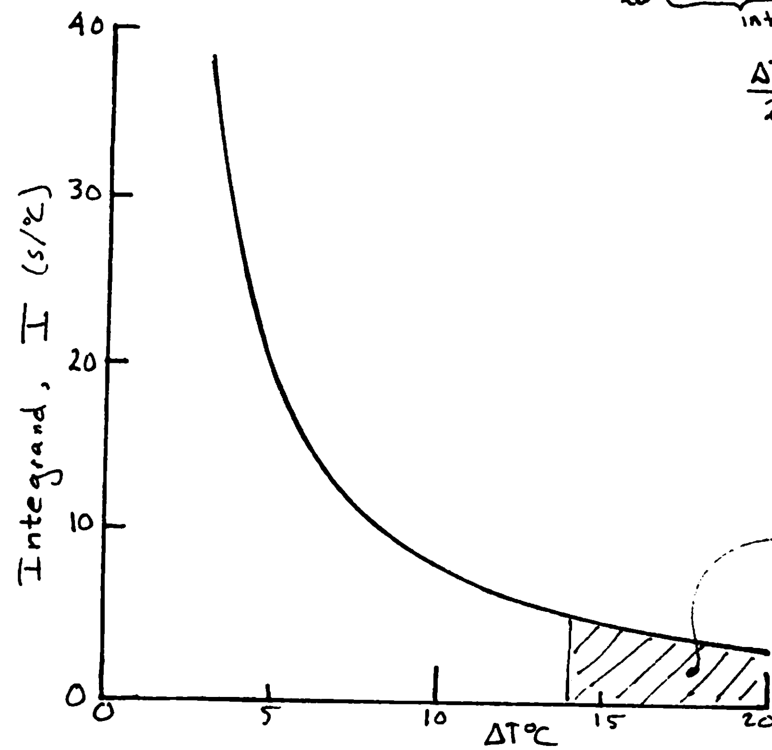
Thus the tank is leak-free when the standpipe drops at the rather substantial rate of 18.4 cm/hr. The drop is the result of thermal contraction.

5.39 A 47°C copper cylinder, 3 cm in diameter, is suddenly immersed horizontally in water at 27°C. Plot $T_{cyl.}$ as a function of time if $g = 0.76\ m/s^2$, $\bar{h} = [2.753+10.448(\Delta T°C)^{1/6}]^2\ W/m^2\text{-}°C$. (Do it numerically if you cannot integrate the resulting equation analytically.)

IN THIS CASE: $\dfrac{d(T_{cyl}-T_\infty)}{dt} = -\dfrac{\bar{h}A}{\rho c V}(T_{cyl}-T_\infty)$ or $\dfrac{\rho c D}{4}\dfrac{1}{\bar{h}\Delta T}\dfrac{d\Delta T}{dt} = -1$

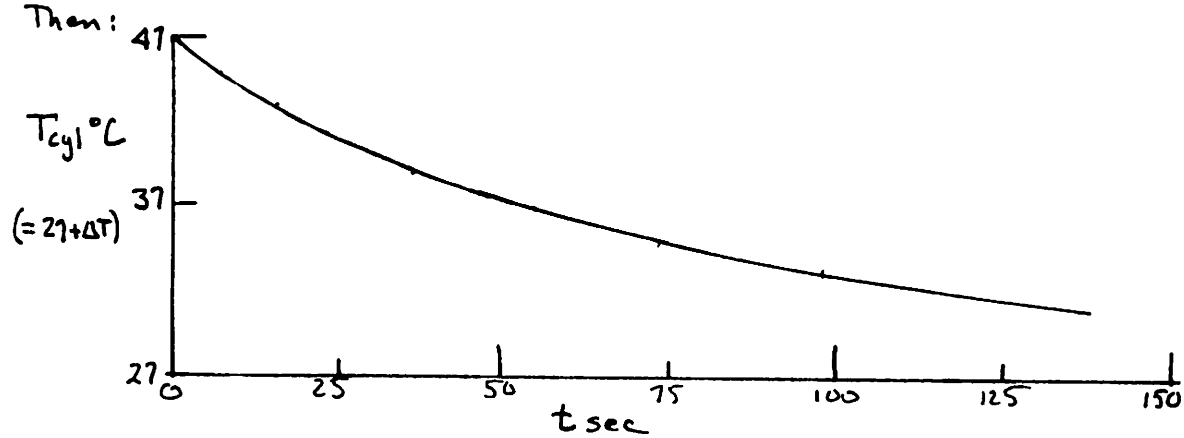
SO: $$\int_{\Delta T_i}^{\Delta T} \frac{8954(384)(0.03)}{4\Delta T(2.733+10.448\Delta T^{1/6})^2}\, d\Delta T = \int_{20}^{\Delta T} \underbrace{\frac{236.2\, d\Delta T}{\Delta T(0.2616+\Delta T^{1/6})^2}}_{\text{integrand} \equiv I} = -t$$

ΔT°C	I	$-\int_{20}^{\Delta T} I\, d\Delta T = t$ sec
20	3.24	0
18	3.71	7.32
16	4.32	15.44
14	5.13	24.84
12	6.25	36.44
10	7.90	54.44
8	10.51	72.64
6	15.20	97.36
4	25.50	136.4
3	61.66	167.4



Area between 20 & 14°C (at 2 sec/⅕ in. square) is 24.84 sec

Then:



5.40 The mechanical engineers at the University of Utah end Spring semester by roasting a pig and having a picnic. The pig is roughly cylindrical and about 26 cm in diameter. It is roasted over a propane flame, whose products have properties similar to those of air, at 280°C. The hot gas flows across the pig at about 2 m/s. If the meat is cooked when it reaches 95°C, and if it is to be served at 2:00 P.M., what time should cooking commence? Assume Bi to be large, but note Problem 7.40. The pig is initially at 25°C.

In this case, $\Theta = \dfrac{95-280}{25-280} = 0.725$, $Bi^{-1} \simeq 0$, so from Fig 5.8 we read: $F_o = \underline{0.13}$

using α for beef we get: $t = F_o \dfrac{r_o^2}{\alpha} = 0.13 \dfrac{0.13^2}{1.35(10)^{-7}} = \underline{16274 \text{ sec}}$

So the pig must be cooked for 4½ hrs. Cooking should begin at about 9:30 AM in the morning. If Bi is not large, this will be low.

5.41 People from cold Northern climates know not to grasp metal with their bare hands in subzero weather. A very slightly frosted piece of, say, cast iron will stick to your hand like glue in, say, -20°C weather, and you can tear off patches of skin. Explain this <u>quantitatively</u>.

Equation (5.60) tells us what to expect for the interfacial temperature when we touch ice. (Take $k_{body} \simeq k_{H_2O}$, $\alpha_{body} \simeq \alpha_{beef}$)

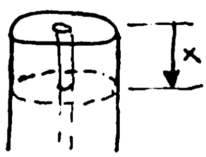
$$\frac{T_{sfc}-(-20)}{37-(-20)} = \frac{0.6/\sqrt{1.35(10)^{-7}}}{2.215/\sqrt{1.15(10)^{-6}} + 1850.7} = 0.473 \qquad \text{so } \underline{T_{sfc} \simeq 7°C}$$

Thus, on immediate contact the ice (or frost) will melt to water (very quickly because it is thin.) Then:

$$\frac{T_{sfc}-(-20)}{0°C-(-20)} = \frac{2.215/\sqrt{1.15(10)^{-6}}}{\underbrace{52/\sqrt{1.7(10)^{-5}}}_{\text{iron}} + 2065} = 0.128 \qquad \text{so } \underline{T_{sfc} = -17.4°C}$$

Thus the iron will immediately refreeze the water, causing it to "glue" your hand to the iron.

5.42 A 4 cm dia. No. 304 stainless steel rod has a very small hole down its center. The hole is clogged with wax that has a melting point of 60°C. The rod is at 20°C. In an attempt to free the hole, a workman swirls the end of the rod -- and about a meter of its length -- in a tank of water at 80°C. If $\bar{h}$ is 688 W/m^2-C° on both the end and the sides of the rod, plot the depth of the melt front as a function of time, up to, say, 4 cm.



$$\Theta(r=0, x) = \underset{\text{semi-infinite}}{\Theta(x)} \cdot \underset{\text{cyl.}}{\Theta(r=0)}$$

At melt front $\Theta = \dfrac{40-80}{20-80} = 0.667$

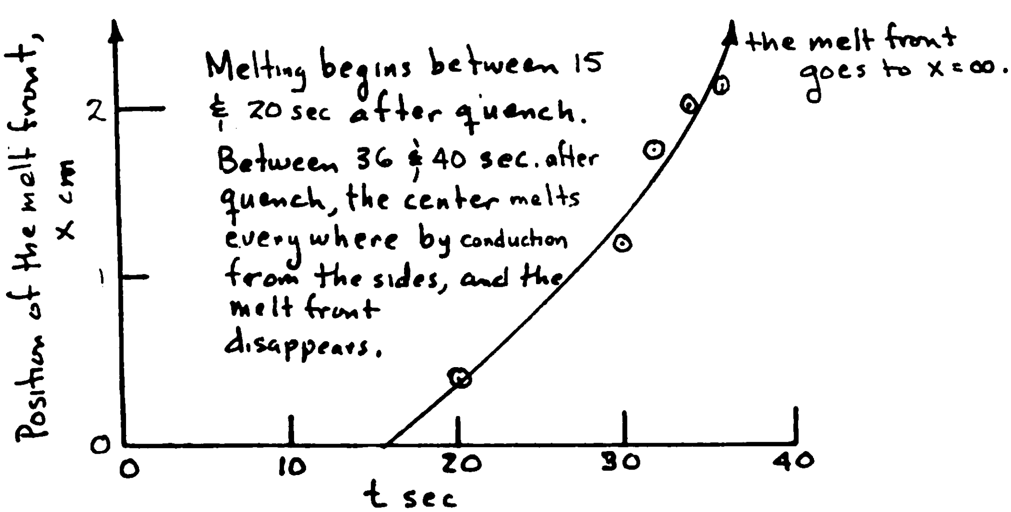
The rod: $Bi^{-1} = \dfrac{k}{\bar{h} r_o} = \dfrac{13.8}{688(0.02)} = 1.00$, $F_o = \dfrac{\alpha t}{r_o^2} = \dfrac{0.000004}{0.02^2} t = 0.01t$

The semi-inf. region: $\beta^2 = \dfrac{\bar{h}^2 \alpha t}{k^2} = \dfrac{688^2(0.000004)}{13.8^2} t = 0.00994t$

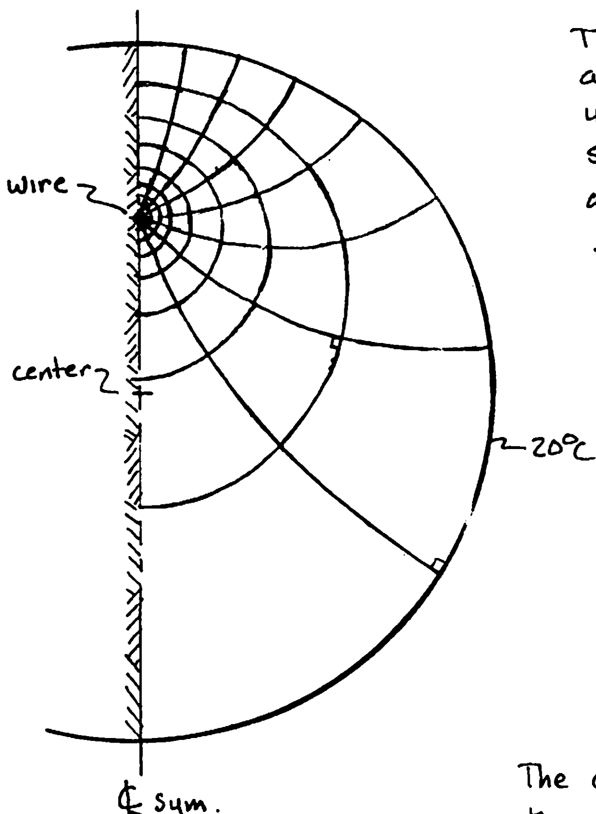
$$\beta\xi = \bar{h}x/k = \frac{688}{13.8}x = 49.86x$$

Then

t (s)	Fo_{cyl}	from Fig. 5.8 $\Theta_{cyl}(\frac{r}{r_o}=0)$	$\Theta_{semi\text{-}inf} = \dfrac{0.667}{\Theta_{cyl}}$	$\beta^2 = 0.00994t$	from Fig. 5.16 $\beta\xi$	x (m) $= \dfrac{\beta\xi}{49.86}$
15	0.15	0.94	0.71	off-scale, Melt has not begun		
20	0.20	0.88	0.76	0.20	0.20	0.004
30	0.30	0.79	0.894	0.30	0.56	0.0112
32	0.32	0.73	0.913	0.32	0.87	0.0174
34	0.34	0.715	0.932	0.338	~1.00	0.020
36	0.36	0.690	0.966	0.358	1.04	0.021
40	0.40	0.640	1.04 off-scale. Melt is complete for all x's.			



5.43 A cylindrical insulator contains a single, very thin, electrical resistor wire that runs along a line halfway between the center and the outside. The wire liberates 480 W/m. The thermal conductivity of the insulation is 3 W/m-°C, and the outside perimeter is held at 20°C. Develop a flux plot for the cross section, considering carefully how the field should look in the neighborhood of the point through which the wire passes. Evaluate the temperature at the center of the insulation.



The wire emits heat equally in all directions. Therefore we set up 16 adiabatic lines -- 8 on one symmetrical side -- all converging at equal angles (22.5°) on the wire.

There are an infinite number of isotherms -- T goes to infinity at the wire. (A real wire with finite diameter would alleviate this problem.)

Now if the wire liberates Q W Then through each square:

$$\frac{Q}{16} = k\Delta T \quad , \quad \Delta T = \frac{480}{16k} = \underline{10°C}$$

The center is 1.85 squares in from the perimeter. Therefore it is at

$$T_{center} = 20 + 1.85(10) = \underline{\underline{38.5°C}}$$

Problem 5.44 A long, 10 cm square copper bar is bounded by 260°C gas flows on two opposing sides. These flows impose heat transfer coefficients of 46 W/m^{2}K. The two intervening sides are cooled by natural convection to water at 15°C, with a heat transfer coefficient of 525 W/m^{2}K. What is the heat flow through the block and the temperature at the center of the block? *Hint:* This could be a pretty complicated problem, but take the trouble to calculate the Biot numbers for each side before you begin. What do they tell you? [34.7 °C]

Solution

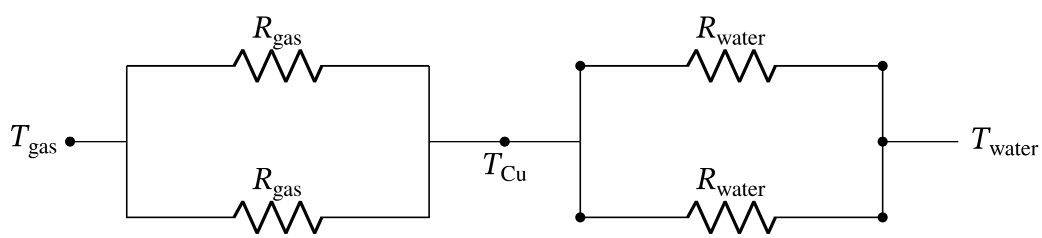
Let's take the advice given in the hint. From Table A.1, the thermal conductivity of [pure] copper at 400°C is $k_{\text{copper}} = 378$ W/m·K. Let $L = 5$ cm. Then

$$\text{Bi}_{\text{gas side}} = \frac{\overline{h}L}{k} = \frac{(46)(0.05)}{378} = 0.00608$$

$$\text{Bi}_{\text{water side}} = \frac{\overline{h}L}{k} = \frac{(525)(0.05)}{378} = 0.0694$$

The Biot number compares the internal conduction resistance to the external convection resistance. For both cases, the internal conduction resistance is small relative to the convection resistance. In addition, the temperature gradients inside the block will be small.

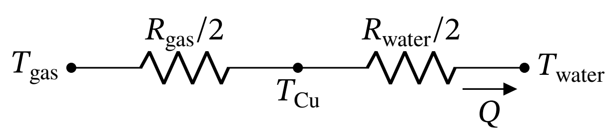
If we neglect the conduction resistance, the remaining resistances form a network as shown:



Then,

$$R_{\text{gas}} = \frac{1}{\overline{h}_{\text{gas}}} = \frac{1}{46} = 0.02174 \text{ K-m}^2\text{/W} \qquad R_{\text{water}} = \frac{1}{\overline{h}_{\text{water}}} = \frac{1}{525} = 0.00190 \text{ K-m}^2\text{/W}$$

The equivalent resistance of two equal resistances in parallel is easily seen to be one half of either resistance. Then the network simplifies:



The total heat flow is

$$Q = \frac{T_{\text{gas}} - T_{\text{water}}}{R_{\text{gas}}/2 + R_{\text{water}}/2} = \frac{260 - 15}{0.02174/2 + 0.00190/2} = 20.7 \text{ kW/m}^2 \xleftarrow{\text{Answer}}$$

The nearly uniform temperature of the copper is

$$\begin{aligned} T_{\text{Cu}} &= T_{\text{water}} + (R_{\text{water}}/2)(Q) \\ &= 15 + (0.00190/2)(20.7 \times 10^{+3}) = 34.7 \text{ °C} \xleftarrow{\text{Answer}} \end{aligned}$$

5.45 Lord Kelvin made an interesting estimate of the age of the earth in 1864. He assumed the earth originated as a mass of molten rock at $4144^{o}K$ ($7000^{o}F$) and that it has been cooled by outer space at $0^{o}K$, ever since. To do this, he assumed that Bi for the earth is very large, and that cooling has thus far penetrated only through a relatively thin (one-dimensional) layer. Using $\alpha_{rock} = 1.18 \times 10^{-6}$ m/s^2 and the measured surface temperature gradient of the earth, $(1/27)^{o}C/m$, find Kelvin's value of Earth's age.

(Kelvin's result turns out to be much less than the accepted value of 4 billion years. His calculation fails because internal heat generation by radioactive decay of the material in the surface layer causes the surface temperature gradient to be higher than it would otherwise be.)

Solution. Since we take the problem to be unidimensional and since, with a large Bi, we may approximate the earth's surface as $0^{o}K$ (with respect to $4144^{o}K$ core), we may therefore use eqn. (5.48) for the heat flux

$$q = k\frac{\partial T}{\partial x}\bigg|_{x=0} = k(T_i - T_\infty)erf(\pi\alpha t)^{-1/2}$$

where the derivative is given as $(1/27)^{o}C/m$. Then

$$t = \frac{(T_i - T_\infty)^2}{(1/27)^2}\frac{1}{\pi\alpha} = 27^2(4144 - 0)^2/\pi(1.18 \times 10^{-6})$$

$$= 3.38 \times 10^{15} \text{ seconds} = \underline{107 \text{ million years}}$$

It is interesting that, though Kelvin used $4144^{o}K$ as the temperature of molten rock, he revised this estimate downward to $1473^{o}K$ in the late 1890's, giving an even smaller age of the earth. Further discussion can be found in Carslaw and Jaeger [1.14] or various geophysics textbooks.

Problem 5.46 A pure aluminum cylinder, 4 cm diam. by 8 cm long, is initially at 300°C. It is plunged into a liquid bath at 40°C with $\overline{h} = 500$ W/m^{2}K. Calculate the hottest and coldest temperatures in the cylinder after one minute. Compare these results with the lumped capacity calculation, and discuss the comparison.

Solution

We begin by looking up the thermal properties and computing the Biot and Fourier numbers. From Table A.1, for pure aluminum at 300°C, $k = 234$ W/m·K; at 20°C, $\alpha = 9.61 \times 10^{-5}$ m^2/s. The conductivity does not vary much with T in this range. Then:

$$\mathrm{Bi}_{r_o} = \frac{\overline{h} r_o}{k} = \frac{(500)(0.02)}{234} = 0.04274$$

$$\mathrm{Fo}_{r_o} = \frac{\alpha t}{r_o^2} = \frac{(9.61 \times 10^{-5})(60)}{(0.02)^2} = 14.42$$

The Biot number is certainly small enough for a lumped capacity solution. The Fourier number is very large ($\gg 1$); for a higher Biot number (> 0.2 or so) this would imply that steady state had been reached. However, for a very low Bi, that need not be the case.

Let us start with the lumped capacity solution. The lumped capacity solution requires us to compute the time constant. With the density and heat capacity of aluminum, $\rho = 2707$ kg/m^3, $c = 905$ J/kg-K

$$\boldsymbol{T} = \frac{\rho c V}{\overline{h} A} = \frac{(2707)(905)(0.08)\pi(0.02)^2}{(500)[2\pi(0.02)(0.08) + 2\pi(0.02)^2]} = 39.20 \text{ s}$$

Then, with eqn. (1.22),

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-t/\boldsymbol{T}} = e^{-60/39.20} = 0.2162$$

$$T = 40 + (300 - 40)(0.2162) = 96.20 \text{ °C} \quad \xleftarrow{\text{Answer}}$$

The cylinder has clearly not finished cooling.

Because the cylinder has a finite length, a solution that is not lumped requires a product solution, i.e., Fig. 5.27a with the product expression eqn (5.70b):

$$\Theta_{\text{finite cyl.}} = \frac{T(r,z,t) - T_\infty}{T_i - T_\infty} = \Theta_{\text{inf. slab}}(z/L, \mathrm{Fo}_s, \mathrm{Bi}_s) \times \Theta_{\text{inf. cyl}}(r/r_o, \mathrm{Fo}_c, \mathrm{Bi}_c)$$

For the slab component, of thickness $2L = 8$ cm:

$$\mathrm{Bi}_L = \frac{\overline{h} L}{k} = \frac{(500)(0.04)}{234} = 0.08547 \qquad \mathrm{Fo}_L = \frac{\alpha t}{L^2} = \frac{(9.61 \times 10^{-5})(60)}{(0.04)^2} = 3.604$$

The highest temperature is in the center, at $(r,z) = (0,0)$. The lowest is on either outside corner, at $(r,z) = (r_o, L)$.

For the slab component, we can read the needed values from Fig. 5.7:

$$\Theta_{\text{inf. slab}}(0, 3.604, 0.08547) \simeq 0.75 \qquad \Theta_{\text{inf. slab}}(1, 3.604, 0.08547) \simeq 0.70$$

The temperature response charts for the cylinder do not extend to such high Fo; our only recourse is to use the one-term solution. To obtain f_1 and A_1, one approach is to make an approximation by interpolating the values in Table 5.2 (by linear interpolation, or more accurately, by plotting some of data in the table and hand-fitting a curve through it). A more accurate approach is to use the equations in Table 5.1 with an online Bessel function calculator[1] and to find a result iteratively.

For $\mathrm{Bi}_{r_o} = 0.04274$, an iterative solution leads to $\hat{\lambda}_1 = 0.2908$ (to 4 digit accuracy), and $A_1 = 1.011$. Then, with eqn. (5.42) and $\mathrm{Fo}_{r_o} = 14.42$:

$$r = 0 \quad f_1 = J_0(0) = 1 \qquad \Theta_{\text{inf. cyl.}} = A_1 f_1 \exp\left[-(\hat{\lambda}_1)^2 \mathrm{Fo}_{r_o}\right] = 0.2974$$

$$r = r_0 \quad f_1 = J_0(\hat{\lambda}_1) = 0.9890 \quad \Theta_{\text{inf. cyl.}} = A_1 f_1 \exp\left[-(\hat{\lambda}_1)^2 \mathrm{Fo}_{r_o}\right] = 0.2941$$

We can now find $\Theta_{\text{finite cyl.}}$: so that

$$\Theta_{\text{finite cyl.}} = \begin{cases} (0.70)(0.2941) = 0.206 & \text{at outside corners} \\ (0.75)(0.2974) = 0.223 & \text{at center} \end{cases}$$

Then, with $T_i - T_\infty = (300 - 40) = 260°\mathrm{C}$,

$$T_{\text{finite cyl.}} = \begin{cases} (0.206)(260) + 40 = 93.6°\mathrm{C} & \text{at outside corners} \\ (0.223)(260) + 40 = 98.0°\mathrm{C} & \text{at center} \end{cases} \quad \xleftarrow{\text{Answer}}$$

The lumped solution lies between the two values obtained from the multidimensional conduction solution. This outcome is not surprising.

Comment: Our solution for $\Theta_{\text{inf. cyl.}}$ is significantly more accurate than that for $\Theta_{\text{inf. slab}}$. If we instead use the one-term solution for the slab and iterate the equation for λ_1 in Table 5.1, we find $\lambda_1 \simeq 0.28825$, $A_1 = 1.014$, $f_1(0) = 1$, $f_1(0.28825) = 0.9587$, and then

$$\Theta_{\text{inf. slab}}(0, 3.604, 0.08547) = A_1 f_1 \exp\left[-(\hat{\lambda}_1)^2 \mathrm{Fo}_L\right] = 0.7516$$

$$\Theta_{\text{inf. slab}}(1, 3.604, 0.08547) = A_1 f_1 \exp\left[-(\hat{\lambda}_1)^2 \mathrm{Fo}_L\right] = 0.7206$$

The latter value is 3% higher than what the author got in reading the chart. These values result in

$$T_{\text{finite cyl.}} = \begin{cases} (0.7206)(0.2941)(260) + 40 = 95.1°\mathrm{C} & \text{at outside corners} \\ (0.7516)(0.2974)(260) + 40 = 98.1°\mathrm{C} & \text{at center} \end{cases}$$

The lumped solution (96.2°C) still lies between these values, but is a bit closer to the outside corners than to the center.

[1] Here's a Bessel function calculator from Casio Computer Co.: https://keisan.casio.com/exec/system/1180573474.

Problem 5.47 When Ivan cleaned his freezer, he accidentally put a large can of frozen juice into the refrigerator. The juice can is 17.8 cm tall and has an 8.9 cm I.D. The can was at −15°C in the freezer, but the refrigerator is at 4°C. The can now lies on a shelf of widely-spaced plastic rods, and air circulates freely over it. Thermal interactions with the rods can be ignored. The effective heat transfer coefficient to the can (for simultaneous convection and thermal radiation) is 8 W/m^{2}K. The can has a 1.0 mm thick cardboard skin with $k = 0.2$ W/m·K. The frozen juice has approximately the same physical properties as ice.

a) How important is the cardboard skin to the thermal response of the juice? Justify your answer quantitatively.
b) If Ivan finds the can in the refrigerator 30 minutes after putting it in, will the juice have begun to melt?

Solution

a) The thermal resistance of the cardboard is

$$R_{\text{cardboard}} = \frac{t}{k} = \frac{0.001}{0.2} = 0.005 \text{ K-m}^2\text{/W}$$

The thermal resistance from the exterior heat transfer coefficient is

$$R_{\text{ext}} = \frac{1}{\overline{h}} = \frac{1}{8} = 0.125 \text{ K-m}^2\text{/W}$$

Thus, $R_{\text{ext}}/R_{\text{cardboard}} = 25 \gg 1$, and the cardboard's thermal resistance can be neglected.

b) We may treat the transient conduction problem as the intersection of a cylinder and a slab, as shown in Fig. 5.27a, using eqn. (5.70b):

$$\Theta_{\text{can}} = \frac{T(r,z,t) - T_\infty}{T_i - T_\infty} = \Theta_{\text{slab}}(z/L, \text{Fo}_s, \text{Bi}_s) \times \Theta_{\text{cyl}}(r/r_o, \text{Fo}_c, \text{Bi}_c)$$

From Table A.2, ice has $k = 2.215$ W/m·K and $\alpha = 1.15 \times 10^{-6}\text{m}^2\text{/s}$. After 30 minutes, the Fourier numbers of the slab and cylinder are:

$$\text{Fo}_s = \frac{\alpha t}{L^2} = \frac{1.15 \times 10^{-6}(30)(60)}{(0.178/2)^2} = 0.261$$

$$\text{Fo}_c = \frac{\alpha t}{r_o^2} = \frac{1.15 \times 10^{-6}(30)(60)}{(0.089/2)^2} = 1.045$$

The Biot numbers are:

$$\text{Bi}_s = \frac{\overline{h}L}{k} = \frac{(8)(0.178/2)}{2.215} = 0.3214$$

$$\text{Bi}_c = \frac{\overline{h}r_o}{k} = \frac{(8)(0.089/2)}{2.215} = 0.1607$$

Melting will occur first at the corners of the can, $r = r_0$ and $z/L = \pm 1$.

These Fourier numbers are large enough for us to use either the one-term solutions or the charts Figs. 5.7 and 5.8. (Note that the charts can only be read to an accuracy of about ±5%, so that different students may come up with slightly different numbers.) With the charts, the author reads:

$$\Theta_{\text{slab}} \simeq 0.84$$

$$\Theta_{\text{cyl}} \simeq 0.78$$

so that

$$\Theta_{\text{can}} = (0.84)(0.78) = 0.655$$

Then, with $T_i - T_\infty = -14 - 4 = -18°\text{C}$,

$$T_{\text{can}} = (0.655)(-18) + 4 = -7.8°\text{C}$$

Thus, the juice has not started melting when Ivan finds it. $\xleftarrow{\text{Answer}}$

Comment 1: We could get better accuracy using the one-term solutions, but the can is a long way from melting. A 5–10% shift in Θ_{can} will not change the answer.

Comment 2: These Biot numbers are almost low enough to use a lumped capacitance solution.

Problem 5.48 A cleaning crew accidentally switches off the heating system in a warehouse one Friday night during the winter, just ahead of the holidays. When the staff return two weeks later, the warehouse is quite cold. In some sections, moisture that condensed has formed a layer of ice 1 to 2 mm thick on the concrete floor. The concrete floor is 25 cm thick and sits on compacted earth. Both the slab and the ground below it are now at 20°F. The building operator turns on the heating system, quickly warming the air to 60°F. If the heat transfer coefficient between the air and the floor is 15 W/m^{2}K, how long will it take for the ice to start melting? Take $\alpha_{\text{concr}} = 7.0 \times 10^{-7}$ m^2/s and $k_{\text{concr}} = 1.4$ W/m·K, and make justifiable approximations as appropriate.

Solution We have transient heat conduction from the air to the ice, concrete, and possibly the ground below the concrete.

The ice layer is very thin in comparison to the concrete, so that it will contribute very little heat capacitance or thermal resistance. To check the size of the ice resistance, with $k_{\text{ice}} = 2.215$ W/m·K, we have

$$R_{\text{ice}} = \frac{t}{k} \approx \frac{0.002}{2.215} = 0.000903 \text{ K-m}^2/\text{W}$$
$$R_{\text{conv}} = \frac{1}{\overline{h}} = \frac{1}{15} = 0.0667 \text{ K-m}^2/\text{W}$$

so that $R_{\text{conv}}/R_{\text{ice}} \simeq 74 \gg 1$. We can neglect the thermal resistance of the ice. Neglecting the heat capacitance of the thin ice layer as well, we may simply treat the slab as if the ice were not present.

Because the concrete layer is thick and not highly conductive, we may attempt to treat it as a semi-infinite body. We will need to check that the temperature change has not reached the bottom of the concrete when the concrete surface reaches the melting temperature.

We can use Fig. 5.16 or eqn. (5.53). At the slab surface, $\zeta = 0$. We seek the time at which the dimensionless temperature is

$$\Theta = \frac{T - T_\infty}{T_i - T_\infty} = \frac{32 - 60}{20 - 60} = 0.700$$

From Fig. 5.16, this value corresponds to $\beta \simeq 0.35$. Since the chart is not easy to ready up in that corner, we can check the result with Table 5.3 and eqn. (5.53):

$$\Theta = \exp\left[(0.35)^2\right] \operatorname{erfc}(0.35) = 0.703$$

which is close enough (within 0.4%). Then, from $\beta = \overline{h}\sqrt{\alpha t}\big/k$

$$t = \left[\frac{(0.35)(1.4)}{(15)}\right]^2 \frac{1}{7.0 \times 10^{-7}} = 1524 \text{ sec} = 25.4 \text{ min} \qquad \xleftarrow{\text{Answer}}$$

Now, let's check whether the concrete remains a semi-infinite body after 25 minutes. We need Θ for

$$\beta\zeta = \frac{\overline{h}x}{k} = \frac{(15)(0.25)}{1.4} = 2.68$$

Figure 5.16 shows that $\Theta \simeq 1$, so that the bottom of the slab remains at the initial temperature. The slab can indeed be modeled as a semi-infinite body.

PROBLEM 5.49 A thick wooden wall, initially at 25°C, is made of fir. It is suddenly exposed to flames at 800°C. If the effective heat transfer coefficient for convection and radiation between the wall and the flames is 80 W/m^{2}K, how long will it take the wooden wall to reach an assumed ignition temperature of 430°C?

SOLUTION The maximum temperature of the wood is at the surface, $x = 0$. The wall thickness is not given, so we will treat it as semi-infinite and then check to see whether that assumption is reasonable.

Referring to eqn. (5.53) or Fig. 5.16, $\zeta = x/\sqrt{\alpha t} = 0$ while $\beta = \overline{h}\sqrt{\alpha t}/k$ is unknown until t is found. We seek the value of t at which the surface reaches 430°C, so that

$$\Theta = \frac{T_{\text{ign}} - T_\infty}{T_i - T_\infty} = \frac{430 - 800}{20 - 800} = 0.4774$$

Reading from Fig. 5.16, we find $\beta \simeq 0.8$.

To get better accuracy, we could use eqn. (5.53):

$$\Theta = \cancelto{0}{\text{erf}\,0} + \exp(\beta^2)\,\text{erfc}(\beta)$$

Then, using Table 5.3 for erfc, we could linearly interpolate a bit:

Guess β	erfc(β)	$\exp(\beta^2)\,\text{erfc}(\beta)$
0.8	0.25790	0.48740
0.9	0.20309	0.45653
0.85	0.23050	0.47472
0.84	0.23598	0.47787

At this point, we are exceeding the accuracy of the interpolation. An online erfc calculator gives erfc(0.84) = 0.23486 and, with more iteration, erfc(0.8346) = 0.23788 resulting in $\Theta = 0.4774$. From either value, with $k = 0.12$ W/m·K, $\alpha = 7.4 \times 10^{-8}$ and $\overline{h} = 80$ W/m^{2}K,

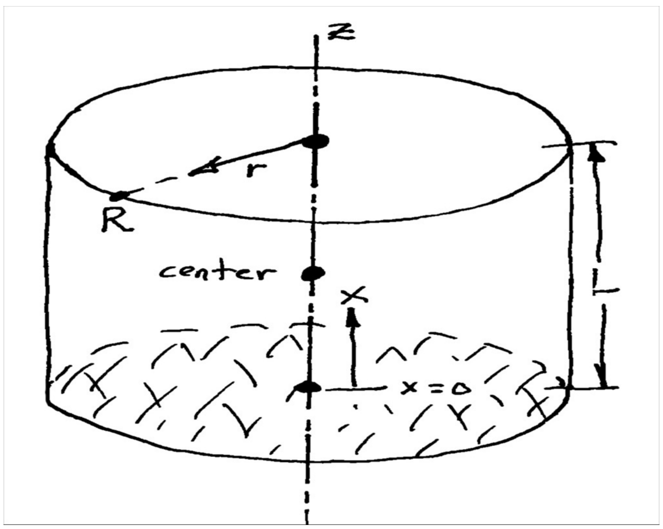
$$t = \frac{1}{\alpha}\left(\frac{\beta k}{\overline{h}}\right)^2 = \begin{cases} 21.5 \text{ sec} & \beta = 0.84 \\ 21.2 \text{ sec} & \beta = 0.8346 \end{cases} \quad \xleftarrow{\text{Answer}}$$

How thick must the wall be for our semi-infinite approximation to apply? Looking at Fig. 5.16, we see that that the temperature remains at T_i ($\Theta = 1$) for $\beta\zeta = \overline{h}x/k \lesssim 3$, or for $x \lesssim 3(0.12)/(80) =$ 4.5 mm. Our approximation is valid for a wall that is at least this thick.

PROBLEM 5.50 Cold butter does not spread as well as warm butter. A small tub of whipped butter bears a label suggesting that, before use, it be allowed to warm up in room air for 30 minutes after being removed from the refrigerator. The tub has a diameter of 9.1 cm with a height of 5.6 cm, and the properties of whipped butter are: $k = 0.125$ W/m·K, $c_p = 2520$ J/kg·K, and $\rho = 620$ kg/m^3. Assume that the tub's plastic walls offer negligible thermal resistance, that $\overline{h} = 10$ W/m^{2}K outside the tub. Ignore heat gained from the countertop below the tub. If the refrigerator temperature was 5°C and the tub has warmed for 30 minutes in a room at 20°C, find: the temperature in the center of the butter tub, the temperature around the edge of the top surface of the butter, and the total energy (in J) absorbed by the butter tub.

SOLUTION

We can model the tub of butter as shown in Fig. 5.27a: the intersection of a cylinder of diameter 9.1 cm with a slab of thickness $2L = 2(5.6) = 11.2$ cm. The slab thickness is twice the height of the tub because the bottom of the tub is presumed to be adiabatic, which means that the bottom acts as the *centerplane* of the slab for superposition purposes. We can use coordinates (x, r) as shown in the figure below.



With eqn. (5.70b):

$$\Theta_{\text{tub}} = \frac{T(r,z,t) - T_\infty}{T_i - T_\infty} = \Theta_{\text{slab}}(\xi, \text{Fo}_s, \text{Bi}_s) \times \Theta_{\text{cyl}}(\rho, \text{Fo}_c, \text{Bi}_c)$$

The thermal diffusivity of the whipped butter is $\alpha = (0.125)/[(620)(2520)] = 8.00 \times 10^{-8}$ m^2/s. After 30 minutes, the Fourier numbers of the slab and cylinder are:

$$\text{Fo}_s = \frac{\alpha t}{L^2} = \frac{8.00 \times 10^{-8}(30)(60)}{(0.056)^2} = 0.04592$$

$$\text{Fo}_c = \frac{\alpha t}{r_o^2} = \frac{8.00 \times 10^{-8}(30)(60)}{(0.0455)^2} = 0.06956$$

The Biot numbers are:

$$\text{Bi}_s = \frac{\overline{h}L}{k} = \frac{(10)(0.056)}{0.125} = 4.48$$

$$\mathrm{Bi}_c = \frac{\overline{h}r_o}{k} = \frac{(10)(0.0455)}{0.125} = 3.64$$

These Fourier numbers are too small for us to use the one-term solutions. We can use the charts, Figs. 5.7 and 5.8. (Note that the charts can only be read to an accuracy of about ±5%, so that different students may come up with slightly different numbers.) The author reads:

$$\Theta_{\text{slab}} \simeq 0.46 \text{ at top, } x/L = 1 \qquad \Theta_{\text{slab}} \simeq 0.95 \text{ in middle, } x/L = 0.5$$
$$\Theta_{\text{cyl}} \simeq 0.40 \text{ at edge, } r/r_o = 1 \qquad \Theta_{\text{cyl}} \simeq 0.97 \text{ at center, } r/r_o = 0$$

so that

$$\Theta_{\text{tub}} = \begin{cases} (0.46)(0.4) = 0.184 & \text{at top outside edge} \\ (0.95)(0.97) = 0.92 & \text{at center} \end{cases}$$

Then, with $T_i - T_\infty = (5 - 20) = -15$°C,

$$T_{\text{tub}} = \begin{cases} (0.184)(-15) + 20 = 17.2\text{°C} & \text{at top outside edge} \\ (0.92)(-15) + 20 = 6.2\text{°C} & \text{at center} \end{cases} \quad \xleftarrow{\text{Answer}}$$

To find the heat gain, we may use eqn. (5.72c):

$$\Phi_{\text{tub}} = \Phi_{\text{slab}} + \Phi_{\text{cyl}}(1 - \Phi_{\text{slab}})$$

The respective values may be read from Fig. 5.10:

$$\Phi_{\text{slab}} \simeq 0.10$$
$$\Phi_{\text{cyl}} \simeq 0.28$$

so

$$\Phi_{\text{tub}} = 0.10 + (0.28)(1 - 0.1) = 0.352$$

The heat *gain* is

$$\begin{aligned} -\int_0^t Q\,dt &= -\rho c_p V(T_i - T_\infty)\Phi \\ &= -(620)(2520)[\pi(0.0455)^2(0.056)](5 - 20)(0.352) \\ &= +3.00 \text{ kJ} \quad \xleftarrow{\text{Answer}} \end{aligned}$$

Problem 5.51 A two-dimensional, 90° annular sector has an adiabatic inner arc, $r = r_i$, and an adiabatic outer arc, $r = r_o$. The flat surface along $\theta = 0$ is isothermal at T_1, and the flat surface along $\theta = \pi/2$ is isothermal at T_2. Show that the shape factor is $S = (2/\pi)\ln(r_o/r_i)$.

Solution Cylindrical coordinates are appropriate for this configuration. The shape factor applies to steady-state heat conduction without heat generation. Further, our problem does not depend on the axial coordinate, z. The heat equation, eqn. (2.11) with eqn. (2.13), can be simplified:

$$\cancelto{0}{\frac{1}{\alpha}\frac{\partial T}{\partial t}} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \cancelto{0}{\frac{\partial^2 T}{\partial z^2}} + \cancelto{0}{\frac{\dot{q}}{k}} \tag{(*)}$$

We may use separation of variables, assuming that $T(r,\theta) = R(r)\Theta(\theta)$:

$$r\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = -\frac{\partial^2 T}{\partial \theta^2}$$

$$\frac{r}{R}\frac{d}{dr}\left(r\frac{dR}{dr}\right) = -\frac{1}{\Theta}\frac{d^2\Theta}{d\theta^2}$$

Since the left-hand side depends on r only and the right-hand side on θ only, the only way for them to be equal is if each take the same constant value. Call that value m^2:

$$\frac{r}{R}\frac{d}{dr}\left(r\frac{dR}{dr}\right) = m^2 = -\frac{1}{\Theta}\frac{d^2\Theta}{d\theta^2}$$

We can separate this into two ordinary differential equations:

$$\frac{d}{dr}\left(r\frac{dR}{dr}\right) - \frac{m^2}{r}R = 0$$

$$\frac{d^2\Theta}{d\theta^2} + m\Theta = 0$$

Our situation looks pretty complicated! For $m \neq 0$, the first equation leads to Bessel functions and the second produces signs and cosines. We'd get a so-called Fourier-Bessel series. But we must also allow for the case $m = 0$. In that case, our equations are:

$$\frac{d}{dr}\left(r\frac{dR}{dr}\right) = 0$$

$$\frac{d^2\Theta}{d\theta^2} = 0$$

We easily find the solutions by integrating each equation twice:

$$R(r) = C_1 \ln r + C_2 = 0$$

$$\Theta(\theta) = C_3\theta + C_4$$

If we can meet the boundary conditions using only the solution for $m = 0$, we can simply omit the solutions for $m \neq 0$.

At $r = r_i$ and $r = r_0$, the boundary is adiabatic, so:

$$\frac{\partial T}{\partial r} = \Theta(\theta)\frac{C_1}{r_i} = \Theta(\theta)\frac{C_1}{r_0} = 0$$

which is only possible for $C_1 = 0$. Thus, $R(r) = C_2$ is just a constant: *this solution is independent of r!* We can take $C_2 = 1$ without loss of generality because C_2 simply multiplies the other unknown constants.

The boundary conditions at $\theta = 0$ and $\theta = \pi/2$ yield:

$$T(r,0) = \Theta(0)\cancelto{1}{C_2} = C_4 = T_1 \qquad T(r,\pi/2) = \Theta(\pi/2)\cancelto{1}{C_2} = C_3\pi/2 + C_4 = T_2$$

so $C_4 = T_1$ and $C_3 = (T_2 - T_1)(2/\pi)$. Collecting all this:

$$T(r,\theta) = T(\theta \text{ only}) = (T_2 - T_1)\frac{\theta}{\pi/2} + T_1 \tag{**}$$

To find the shape factor, we need to calculate the heat flow Q from one isothermal side to the other. The gradient vector in cylindrical coordinates is

$$\nabla T = \frac{\partial T}{\partial r}\mathbf{e}_r + \frac{1}{r}\frac{\partial T}{\partial \theta}\mathbf{e}_\theta + \frac{\partial T}{\partial z}\mathbf{e}_z$$

The heat flux in the θ direction is then

$$q = -k\frac{1}{r}\frac{\partial T}{\partial \theta} = -k(T_2 - T_1)\frac{2}{\pi r}$$

The total heat flow is found by integrating from $r = r_i$ to $r = r_0$ along the line $\theta = 0$:

$$Q = -k(T_2 - T_1)\int_{r_i}^{r_0} \frac{2}{\pi r}\,dr = k(T_1 - T_2)\frac{2}{\pi}\ln\frac{r_0}{r_i}$$

By definition, eqn. (5.66), $Q = S\,k\Delta T$, so the shape factor we seek is:

$$S = \frac{Q}{k\Delta T} = \frac{2}{\pi}\ln\frac{r_0}{r_i} \quad \xleftarrow{\text{Answer}}$$

Comment 1: The astute student may recognize at the outset that the temperature distribution will not depend on r. In that case, the radius derivative in eqn. (*) will also be zero; integration and application of the boundary conditions will lead quickly to eqn. (**).

Comment 2: In the case of a thin sector ($r_o = r_i + t$ for $t \ll r_i$), curvature is unimportant. The problem could be treated as a straight strip of length $(\pi/2)r_i$. A thermal resistance calculation would quickly lead to an expression for S.

In addition, for $t \ll r_i$, $\ln(r_o/r_i) = \ln(1 + t/r_i) \simeq t/r_i$. Then our result becomes $S \simeq 2t/(\pi r_i)$.

Problem 5.52 Suppose that $T_\infty(t)$ is the time-dependent temperature of the environment surrounding a convectively-cooled, lumped object.

a) When T_∞ is not constant, show that eqn. (1.19) leads to

$$\frac{d}{dt}\left(T - T_\infty\right) + \frac{(T - T_\infty)}{\boldsymbol{T}} = -\frac{dT_\infty}{dt}$$

where the time constant $\boldsymbol{T}$ is defined as usual.

b) If the object's initial temperature is T_i, use either an integrating factor or Laplace transforms to show that $T(t)$ is

$$T(t) = T_\infty(t) + \left[T_i - T_\infty(0)\right] e^{-t/\boldsymbol{T}} - e^{-t/\boldsymbol{T}} \int_0^t e^{s/\boldsymbol{T}} \frac{d}{ds} T_\infty(s)\, ds$$

Solution

a) From eqn. (1.19) for constant c, with $T_\infty(t)$ not constant:

$$\begin{aligned} -\overline{h}A(T - T_\infty) &= \frac{d}{dt}\left[\rho c V (T - T_{\text{ref}})\right] = mc\frac{dT}{dt} \\ &= mc\frac{d(T - T_\infty)}{dt} + mc\frac{dT_\infty}{dt} \end{aligned}$$

Setting $\boldsymbol{T} \equiv mc \big/ \overline{h}A$ and rearranging, we obtain the desired result:

$$\boxed{\frac{d}{dt}\left(T - T_\infty\right) + \frac{(T - T_\infty)}{\boldsymbol{T}} = -\frac{dT_\infty}{dt}} \tag{1}$$

b) The integrating factor for this first-order o.d.e. is $e^{t/\boldsymbol{T}}$. Multiplying through and using the product rule, we have

$$\frac{d}{dt}\left[e^{t/\boldsymbol{T}}(T - T_\infty)\right] = -e^{t/\boldsymbol{T}}\,\frac{dT_\infty}{dt}$$

Next integrate from $t = 0$ to t:

$$e^{t/\boldsymbol{T}}(T - T_\infty) - \left[T_i - T_\infty(0)\right] = -\int_0^t e^{s/\boldsymbol{T}}\,\frac{dT_\infty}{ds}\, ds$$

Multiplying through by $e^{-t/\boldsymbol{T}}$ and rearranging gives the stated result:

$$\boxed{T(t) = T_\infty(t) + \left[T_i - T_\infty(0)\right] e^{-t/\boldsymbol{T}} - e^{-t/\boldsymbol{T}} \int_0^t e^{s/\boldsymbol{T}}\,\frac{dT_\infty}{ds}\, ds}$$

Alternate approach: To use Laplace transforms, we first simplify eqn. (1) by defining $y(t) \equiv T - T_\infty$ and $f(t) \equiv -dT_\infty/dt$:

$$\frac{dy}{dt} + \frac{y}{\boldsymbol{T}} = f(t)$$

Next, we apply the Laplace transform $\mathscr{L}\{..\}$, with $\mathscr{L}\{y(t)\} = Y(p)$ and $\mathscr{L}\{f(t)\} = F(p)$:

$$\begin{aligned} \mathscr{L}\left\{\frac{dy}{dt}\right\} + \mathscr{L}\left\{\frac{y}{\boldsymbol{T}}\right\} &= \mathscr{L}\{f(t)\} \\ pY(p) - y(0) + \frac{1}{\boldsymbol{T}}Y(p) &= F(p) \end{aligned}$$

Solving for $Y(p)$:

$$Y(p) = \frac{1}{p + 1/\boldsymbol{T}}\, y(0) + \frac{1}{p + 1/\boldsymbol{T}}\, F(p)$$

Now take the inverse transform, $\mathscr{L}^{-1}\{..\}$:

$$\mathscr{L}^{-1}\{Y(p)\} = \mathscr{L}^{-1}\left\{\frac{1}{p + 1/\boldsymbol{T}}\right\} y(0) + \mathscr{L}^{-1}\left\{\frac{1}{p + 1/\boldsymbol{T}}\, F(p)\right\} \tag{2}$$

With a table of Laplace transforms, we find

$$\mathscr{L}^{-1}\underbrace{\left\{\frac{1}{p + 1/\boldsymbol{T}}\right\}}_{\equiv G(p)} = \underbrace{e^{-t/\boldsymbol{T}}}_{\equiv g(t)}$$

and with $G(p)$ and $g(t)$ defined as shown, the last term is just a convolution integral

$$\mathscr{L}^{-1}\left\{\frac{1}{p + 1/\boldsymbol{T}}\, F(p)\right\} = \mathscr{L}^{-1}\{G(p)F(p)\} = \int_0^t g(t-s) f(t)\, ds$$

Putting all this back into eqn. (2), we find

$$y(t) = e^{-t/\boldsymbol{T}} y(0) + \int_0^t e^{-(t-s)/\boldsymbol{T}} f(t)\, ds$$

and putting back the original variables in place of y and f, we have at length obtained:

$$\boxed{T(t) = T_\infty(t) + \left[T_i - T_\infty(0)\right] e^{-t/\boldsymbol{T}} - e^{-t/\boldsymbol{T}} \int_0^t e^{s/\boldsymbol{T}}\, \frac{dT_\infty}{ds}\, ds}$$

Extra credit. State which approach is more straightforward!

Problem 5.53 Use the equation derived in Problem 5.52b to verify ~~eqn. (5.13)~~ eqn. (5.14).

Solution We have $T_\infty(t) = T_i + bt$. Substituting into the result of Problem 5.52b:

$$
\begin{aligned}
T(t) &= T_\infty(t) + \big[T_i - T_\infty(0)\big]\, e^{-t/\boldsymbol{T}} - e^{-t/\boldsymbol{T}} \int_0^t e^{s/\boldsymbol{T}} \frac{d}{ds}\big[T_i + bs\big]\, ds \\
&= T_i + bt + \big[T_i - T_i\big]\, e^{-t/\boldsymbol{T}} - e^{-t/\boldsymbol{T}} \int_0^t e^{s/\boldsymbol{T}} b\, ds \\
&= T_i + bt - b\boldsymbol{T} e^{-t/\boldsymbol{T}}\big(e^{t/\boldsymbol{T}} - 1\big) \\
&= T_i + bt - b\boldsymbol{T}\big(1 - e^{-t/\boldsymbol{T}}\big)
\end{aligned}
$$

which is eqn. (5.14).

PROBLEM 5.54 Suppose that a thermocouple with an initial temperature T_i is placed into an airflow for which its Bi $\ll 1$ and its time constant is $\boldsymbol{T}$. Suppose also that the temperature of the airflow varies harmonically as $T_\infty(t) = T_i + \Delta T\cos(\omega t)$.

a) Use the equation derived in Problem 5.52b to find the temperature of the thermocouple, $T_{\text{tc}}(t)$, for $t > 0$. (If you wish, note that the real part of $e^{i\omega t}$ is $\mathfrak{Re}\{e^{i\omega t}\} = \cos\omega t$ and use complex variables to do the integration.)
b) Approximate your result for $t \gg \boldsymbol{T}$. Then determine the value of $T_{\text{tc}}(t)$ for $\omega\boldsymbol{T} \ll 1$ and for $\omega\boldsymbol{T} \gg 1$. Explain in physical terms the relevance of these limits to the frequency response of the thermocouple—its ability to follow various frequencies.
c) If the thermocouple has a time constant of $\boldsymbol{T} = 0.1$ sec, estimate the highest frequency temperature variation that it will measure accurately.

SOLUTION

a) The integration can be done in several ways. We'll use complex variables:

$$
\begin{aligned}
T(t) &= T_\infty(t) + \big[T_i - T_\infty(0)\big]e^{-t/\boldsymbol{T}} - e^{-t/\boldsymbol{T}}\int_0^t e^{s/\boldsymbol{T}}\frac{d}{ds}T_\infty(s)\,ds \\
&= T_\infty(t) - \Delta T\,e^{-t/\boldsymbol{T}} - \Delta T e^{-t/\boldsymbol{T}}\int_0^t e^{s/\boldsymbol{T}}\,\mathfrak{Re}\left\{\frac{d}{ds}e^{i\omega s}\right\}ds \\
&= T_\infty(t) - \Delta T\,e^{-t/\boldsymbol{T}} - \Delta T e^{-t/\boldsymbol{T}}\,\mathfrak{Re}\left\{\int_0^t e^{s/\boldsymbol{T}} i\omega e^{i\omega s}\,ds\right\} \\
&= T_\infty(t) - \Delta T\,e^{-t/\boldsymbol{T}} - \Delta T e^{-t/\boldsymbol{T}}\,\mathfrak{Re}\left\{i\omega\left[\frac{e^{(1/\boldsymbol{T}+i\omega)t} - 1}{1/\boldsymbol{T} + i\omega}\right]\right\}
\end{aligned}
$$

…and now a bunch of algebra…

$$
\begin{aligned}
&= T_\infty(t) - \Delta T\,e^{-t/\boldsymbol{T}} - \Delta T\;\mathfrak{Re}\left\{i\omega\left[\left(e^{i\omega t} - e^{-t/\boldsymbol{T}}\right)\left(\frac{1/\boldsymbol{T} - i\omega}{\boldsymbol{T}^{-2} + \omega^2}\right)\right]\right\} \\
&= T_\infty(t) - \Delta T\,e^{-t/\boldsymbol{T}} - \frac{\Delta T\omega}{\boldsymbol{T}^{-2} + \omega^2}\left[-\sin(\omega t)\big/\boldsymbol{T} + \omega\cos(\omega t) - \omega e^{-t/\boldsymbol{T}}\right] \\
&= T_\infty(t) - \Delta T\,e^{-t/\boldsymbol{T}} - \frac{\Delta T}{(\omega\boldsymbol{T})^{-2} + 1}\left[\cos(\omega t) - \sin(\omega t)\big/(\omega\boldsymbol{T}) - e^{-t/\boldsymbol{T}}\right] \\
&= T_\infty(t) + \Delta T\,e^{-t/\boldsymbol{T}}\left[\frac{1}{(\omega\boldsymbol{T})^{-2} + 1} - 1\right] - \frac{\Delta T}{(\omega\boldsymbol{T})^{-2} + 1}\left[\cos(\omega t) - \sin(\omega t)\big/(\omega\boldsymbol{T})\right] \\
&= T_\infty(t) - \Delta T\,e^{-t/\boldsymbol{T}}\left[\frac{1}{1 + (\omega\boldsymbol{T})^2}\right] - \Delta T\,\frac{(\omega\boldsymbol{T})^2}{1 + (\omega\boldsymbol{T})^2}\left[\cos(\omega t) - \sin(\omega t)\big/(\omega\boldsymbol{T})\right]
\end{aligned}
$$

We could stop here…or we can use a trig identity to combine the sine and cosine:

$$A\cos\phi - B\sin\phi = C\cos(\phi + \alpha)$$

for $C^2 = A^2 + B^2$ and $\tan(B/A) = \alpha$. With $A = 1$ and $B = 1/(\omega\boldsymbol{T})$, $\alpha = \tan^{-1}(1/\omega\boldsymbol{T})$ and:

$$T(t) = T_\infty(t)\underbrace{- \Delta T\,e^{-t/\boldsymbol{T}}\left[\frac{1}{1 + (\omega\boldsymbol{T})^2}\right] - \Delta T\sqrt{\frac{(\omega\boldsymbol{T})^2}{1 + (\omega\boldsymbol{T})^2}}\cos(\omega t + \alpha)}_{\text{Measurement error}} \quad \xleftarrow{\text{Answer}} \qquad (*)$$

b) The second term on the right-hand side of eqn. (*) represents the transient response of the thermocouple, which tends to zero for $t \gg \boldsymbol{T}$. We are left with the steady response to the oscillating air temperature:

$$T(t) = T_\infty(t) - \Delta T \sqrt{\frac{(\omega \boldsymbol{T})^2}{1 + (\omega \boldsymbol{T})^2}} \cos(\omega t + \alpha) \tag{**}$$

For low frequencies (with a period much longer than the time constant of the thermocouple), $\omega \boldsymbol{T} \ll 1$ and $\alpha \to \pi/2$. The result reduces to

$$T(t) \simeq T_\infty(t)$$

In this range of frequencies, the thermocouple measures the air temperature accurately.

For high frequencies (with a period much shorter than the time constant of the thermocouple), $\omega \boldsymbol{T} \gg 1$ and $\alpha \to 0$. The result becomes

$$T(t) \to T_\infty(t) - \Delta T \cos(\omega t) = T_i$$

For these very high frequencies, the air temperature fluctuates too rapidly for the thermocouple to follow, and the measured temperature is simply the average air temperature.

c) To measure accurately, we'd like the last term in eqn. (**) to be small. For no more than a 1% error, we would need $\omega \boldsymbol{T} \leqslant 0.01$. Therefore, we need frequencies low enough that $\omega \leqslant 0.01/\boldsymbol{T}$, and with $\omega = 2\pi f$ and $\boldsymbol{T} = 0.1$ sec, this leads to

$$f \leqslant \frac{0.01}{2\pi(0.1)} = 0.0159 \text{ Hz}$$

This frequency corresponds to a maximum period of air temperature change of about 63 sec.

Comment 1: For measurements of fluctuating air temperature in turbulent flow, much higher frequency response is needed, generally in the range of 1 kHz or more. Very short sensor time constants are required, and these can be obtained using micrometer diameter platinum wires. SEE: J. Haugdahl and J.H. Lienhard V, "A low-cost, high-performance DC cold-wire bridge," *J. Phys. E: Sci. Instr.*, Vol. 21, 1988, pp.167-170, doi:10.1088/0022-3735/21/2/008. (PDF file)

Comment 2: The thermocouple is a first-order linear system, and the fluctuating air temperature provides harmonic forcing. In a system dynamics class, these results might be presented using a Bode plot for the amplitude response. To find the phase shift (which is not α), the fluctuating part of $T_\infty(t)$ must be combined with the last term in eqn. (*); this phase shift is 0° at low frequency, increasing to a 90° phase lag at high frequency.

Problem 5.55 A particular tungsten lamp filament has a diameter of 100 μm and sits inside a glass bulb filled with inert gas. The effective heat transfer coefficient for convection and radiation is 750 W/m·K and the electrical current is at 60 Hz. How much does the filament's surface temperature fluctuate if the gas temperature is 200°C and the average wire temperature is 2900°C?

Solution

We may refer to Fig. 5.12 to find the answer once we calculate Bi and ψ. The tungsten wire is at a higher temperature than the data in Table A.1: $k = 114$ W/m·K (at 1000°C) and $\alpha = 6.92\times10^{-5}$ m^2/s (at 20°C). We can do better by using the literature [1]. At 3200 K, $k = 91$ W/m·K. If we take ρc_p to be only weakly temperature dependent, we can estimate α by adjusting the 20°C value $k = 178$ W/m·K to the 3200 K value: $\alpha \approx 6.92 \times 10^{-5}(91/178) = 3.54 \times 10^{-5}$ m^2/s.

$$\text{Bi} = \frac{\overline{h}\delta}{k} = \frac{(750)(50 \times 10^{-6})}{91} = 4.12 \times 10^{-4}$$

$$\psi = \frac{\omega\delta^2}{\alpha} = \frac{2\pi(60)(50 \times 10^{-6})^2}{3.54 \times 10^{-5}} = 0.0266$$

With these values, we can read from Fig. 5.12:

$$\frac{T_{\text{max}} - T_{\text{ave}}}{T_{\text{ave}} - T_\infty} \approx 0.02$$

so that

$$T_{\text{max}} \approx (0.02)(2900 - 200) + 2900 = 2954\ ^\circ\text{C} \qquad \xleftarrow{\text{Answer}}$$

References. [1] R.W. Powell, C.Y. Ho, and P.E. Liley, *Thermal conductivity of selected materials*. Washington, D.C., U.S. Dept. Commerce, National Bureau of Standards, 1966, Figure 13. https://permanent.fdlp.gov/LPS112783/NSRDS-NBS-8.pdf

Problem 5.56 The consider the parameter ψ in eqn. (5.41).

a) If the timescale for heat to diffuse a distance δ is δ^2/α, explain the physical significance of ψ and the consequence of large or small values of ψ.
b) Show that the timescale for the thermal response of a wire of radius δ with Bi $\ll$ 1 is $\rho c_p \delta \big/ \big(2\overline{h}\big)$. Then explain the meaning of the new parameter $\phi = \rho c_p \omega \delta \big/ \big(4\pi \overline{h}\big)$.
c) When Bi $\ll$ 1, is ϕ or ψ a more relevant parameter?

Solution

a) The definition of ψ is $\psi = \omega\delta^2/\alpha$. Physically, ψ is a ratio of timescales:

$$\psi = \frac{\delta^2/\alpha}{1/\omega} = 2\pi \frac{\text{timescale for heat diffusion over distance } \delta}{\text{period of oscillation of heat generation}}$$

with $\omega = 2\pi f$. When $\psi \ll 1$, the heating power oscillates on a timescale much greater than the time required for heat to diffuse over δ, with the result that the surface temperature of the object will not vary much. When $\psi \gg 1$, heat diffuses to the surface much faster than the heat power varies, so that the surface temperature will change as the power fluctuates.

b) For low Biot number, the timescale for an object's temperature change is simply the lumped capacitance time constant. From eqn (1.23)

$$\boldsymbol{T} = mc\left(\frac{1}{\overline{h}A}\right)$$

For a cylindrical wire of radius δ, per unit length, $A = 2\pi\delta$ and $m = \rho\pi\delta^2$. Substituting these values

$$\boldsymbol{T} = \frac{\rho c \delta}{2\overline{h}}$$

The proposed parameter $\phi = \rho c_p \omega \delta \big/ \big(4\pi \overline{h}\big)$, with $f = \omega/2\pi = 1/\text{period}$, has this physical meaning:

$$\phi = \frac{\rho c_p \delta/(2\overline{h})}{2\pi/\omega} = \frac{\text{lumped capacitance time scale}}{\text{period of oscillation}}$$

c) When Bi $\ll 1$, ϕ is clearly the more relevant parameter. In this limit, temperature gradients within the wire are negligible.

From Figs. 5.11 and 5.12, we see that ψ alone predicts the temperature deviation for Bi $\gg 1$. We expect that the deviation would be independent of Bi in the lumped limit, for Bi $\ll 1$, but that behavior does not appear when ψ is used as a parameter. The curves would need to be replotted in terms of ϕ to show independence from Bi at low Bi.

Problem 5.57 Repeat the calculations of Example 5.2 using the one-term solutions. Discuss the differences between your solution and the numbers in the example. Which provides greater accuracy?

Solution

The one-term equations that we need are eqns. (5.42) and (5.43):

$$\Theta = A_1 f_1(r/r_o)\exp\left(-\hat{\lambda}_1^2 \mathrm{Fo}\right) \quad \Phi = 1 - D_1 \exp\left(-\hat{\lambda}_1^2 \mathrm{Fo}\right)$$

From Example. 5.2, $\mathrm{Bi}_{r_o} = 0.498$. Referring to Table 5.2, we read the values for $\mathrm{Bi}_{r_o} = 0.50$:

$$\hat{\lambda}_1 = 1.16556 \quad A_1 = 1.1441 \quad D_1 = 0.9960$$

At the center of the apples, $r/r_o = 0$. For $f_1(0)$ we may refer to Table 5.1, noting that

$$\lim_{x\to 0} \frac{1}{x}\sin x = 1$$

so that $f_1(0) = 1$.

After 1 hr, from Example 5.2, $\mathrm{Fo}_{r_o} = 0.208$, So

$$\Theta = (1.1441)(1)\exp\left[-(1.16556)^2(0.208)\right] = 0.8625$$

which is 2.7% greater than the value found in Example 5.2. From the definition of Θ

$$T_{\text{center}} = (0.8625)(30-5)°\mathrm{C} + 5°\mathrm{C} = 26.6°\mathrm{C} \quad \xleftarrow{\text{Answer}}$$

When the centers reach 10°C, from Example 5.2, $\Theta = 0.20$, so

$$0.20 = (1.1441)(1)\exp\left[-(1.16556)^2 \mathrm{Fo}_{r_o}\right]$$

which can be solved for

$$\mathrm{Fo}_{r_o} = 1.284 \simeq 1.28$$

which is 1.4% less than the value found in Example 5.2, so that

$$t = \frac{1.28(997.6)(4180)(0.0025)}{0.603} = 22{,}129 \text{ s} = 6 \text{ hr } 9 \text{ min} \quad \xleftarrow{\text{Answer}}$$

With $\mathrm{Fo}_{r_o} = 1.284$, we find

$$\Phi = 1 - (0.9960)\exp\left[-(1.16556)^2(1.284)\right] = 0.8259$$

We can use this value of Φ to repeat the calculation in Example 5.2:

$$12\int_0^t Q\,dt = 12(997.6)(4180)\left(\frac{4}{3}\pi(0.05)^3\right)(25)(0.8259) = 541 \text{ kJ} \quad \xleftarrow{\text{Answer}}$$

This value is about 0.7% greater than was calculated in Example 5.2.

Accuracy: The charts can be read to a precision of only about ±5% (although the numerical calculations made when plotting the charts were very precise.) The one-term solutions allow many digits to be computed, but one must also consider the approximation made in reducing the Fourier series solutions to a single term. As noted on page 216 of the next, the one-term solution for a sphere is accurate to better than 0.1% for $\mathrm{Fo} \geqslant 0.28$; that means that the answers for the last two questions are much more accurate with the one-term series than the chart solution. For the first answer (with $\mathrm{Fo} = 0.20$), the one-term solution will be somewhat less accurate (since $\mathrm{Fo} < 0.28$), but likely still much more accurate that reading the chart.

Problem 5.58 The lumped-capacity solution, eqn. (1.22) depends on $t/\boldsymbol{T}$. (a) Write $t/\boldsymbol{T}$ in terms of Bi and Fo for a slab, a cylinder, and a sphere [slab: $t/\boldsymbol{T} = \mathrm{Bi}_L\mathrm{Fo}_l$]. (b) For a sphere with Fo = 1, 2, and 5, plot the lumped-capacity solution as a function of Bi on semilogarithmic coordinates. How do these curves compare to those in Fig. 5.9?

Solution

a) For a very large slab of thickness L and area A (neglecting edges)

$$\frac{t}{\boldsymbol{T}} = \frac{t\overline{h}A}{\rho cLA} = \frac{t\overline{h}}{\rho cL} = \frac{\alpha t}{L^2}\frac{\overline{h}L}{k}\cancelto{1}{\frac{k}{\rho c\alpha}} = \mathrm{Fo}_L\mathrm{Bi}_L$$

For a very long cylinder of radius r_o and length L (neglecting ends)

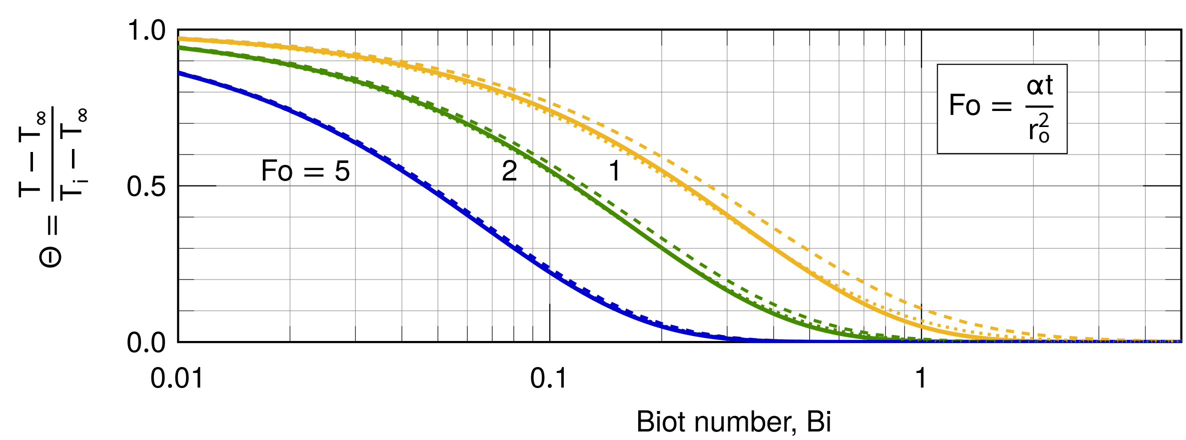
$$\frac{t}{\boldsymbol{T}} = \frac{t\overline{h}(2\pi r_o L)}{\rho c\pi r_o^2 L} = \frac{2t\overline{h}}{\rho c r_o} = 2\frac{\alpha t}{r_o^2}\frac{\overline{h}r_o}{k}\cancelto{1}{\frac{k}{\rho c\alpha}} = 2\,\mathrm{Fo}_{r_o}\mathrm{Bi}_{r_o}$$

For a sphere of radius r_o

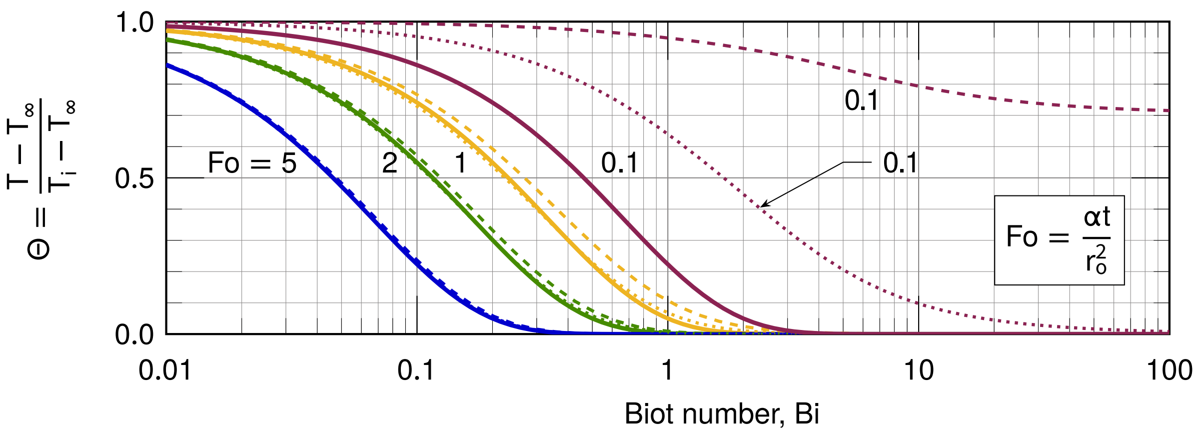
$$\frac{t}{\boldsymbol{T}} = \frac{t\overline{h}(4\pi r_o^2)}{\rho c\pi(4/3)\pi r_o^3} = \frac{3t\overline{h}}{\rho c r_o} = 3\frac{\alpha t}{r_o^2}\frac{\overline{h}r_o}{k}\cancelto{1}{\frac{k}{\rho c\alpha}} = 3\,\mathrm{Fo}_{r_o}\mathrm{Bi}_{r_o}$$

b) The lumped solution is plotted with solid curves in the figure below. The comparison can be made in various ways. In the plot, the curves from Fig. 5.9 are shown in dashed lines for $r/r_o = 0$ and in dotted lines for $r/r_o = 1$. The results from Fig. 5.9 and the lumped solution are similar in all cases, but almost identical for $r/r_o = 1$. The disagreement is greater for higher Bi and lower Fo, as expected, and the most noticeably different case is for for $r/r_o = 0$ (the center) when Fo = 1 and Bi $\gtrsim 0.1$.

The reason that the curves are generally similar is that the lumped solution assumes a uniform temperature through the whole sphere, a condition that best applies for small Biot and large Fourier number.



Comment: If the curves for Fo = 0.1 are plotted, no similarity at all is observed (see figure on next page). For small Fourier numbers, the temperature at the surface and the center are very different unless Bi $\lesssim 0.1$.


1.0
0.5
0.0
$\Theta = \dfrac{T - T_\infty}{T_i - T_\infty}$
Fo = 5
2
1
0.1
0.1
0.1
$\text{Fo} = \dfrac{\alpha t}{r_o^2}$
0.01
0.1
1
10
100
Biot number, Bi

Problem 5.59 Use the lumped-capacity solution to derive an equation for the heat removal, Φ, as a function of t. Then put this equation in terms of Fo and Bi for a cylinder. Plot the result on semilogarithmic coordinates as a function of Bi for Fo = 25, 10, 5, and 2. Compare these curves to Fig. 5.10b.

Solution

By definition, eqn. (5.37),

$$\Phi = \frac{\displaystyle\int_0^t Q\,dt}{\rho c V (T_i - T_\infty)}$$

For a lumped object, with eqn. (1.22),

$$Q = \overline{h}A[T(t) - T_\infty] = \overline{h}A(T_i - T_\infty)\exp(-t/\boldsymbol{T})$$

so that, with $\boldsymbol{T} = \rho c V/\overline{h}A$,

$$\Phi = \frac{\overline{h}A}{\rho c V}\int_0^t \exp(-t/\boldsymbol{T})\,dt = \frac{1}{\boldsymbol{T}}\boldsymbol{T}\big[1 - \exp(-t/\boldsymbol{T})\big] = 1 - \exp(-t/\boldsymbol{T})$$

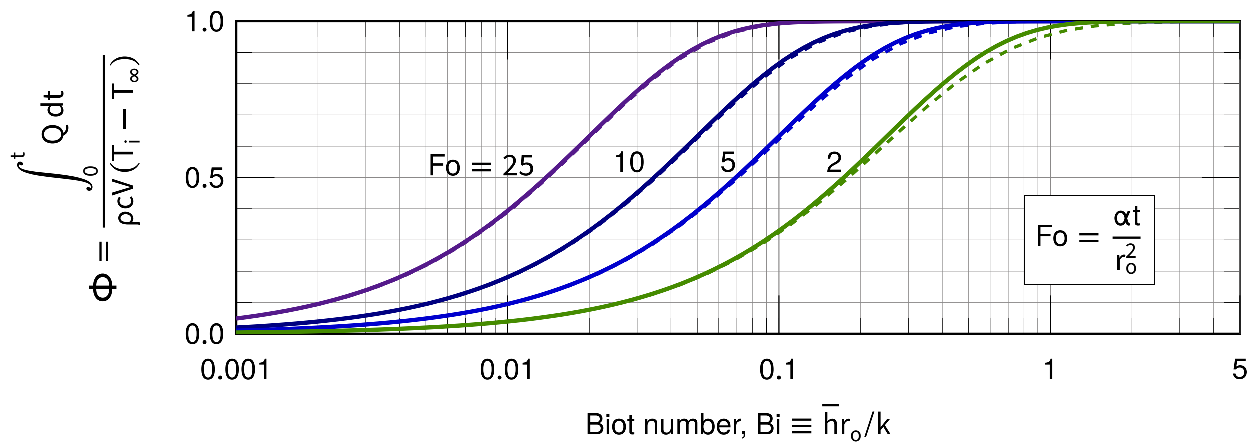
For a cylinder that is very long (so that we neglect the end surfaces),

$$\frac{t}{\boldsymbol{T}} = \frac{t\overline{h}(2\pi r_o L)}{\rho c \pi r_o^2 L} = \frac{2t\overline{h}}{\rho c r_o} = 2\frac{\alpha t}{r_o^2}\,\frac{\overline{h}r_o}{k}\,\frac{k}{\cancelto{1}{\rho c \alpha}} = 2\,\mathrm{Fo}_{r_o}\mathrm{Bi}_{r_o}$$

Thus:

$$\Phi = 1 - \exp(-t/\boldsymbol{T}) = 1 - \exp(-2\,\mathrm{Bi}_{r_o}\mathrm{Fo}_{r_o}) \quad \xleftarrow{\text{Answer}}$$

This equation is plotted below using solid curves. The curves from Fig. 5.10b are shown as dotted curves. The agreement is excellent overall. For smaller Fo and higher Bi, some differences begin to appear. When the Biot number is higher, the internal thermal resistance is no longer negligible (the internal temperature gradients increase), so that the lumped model overestimates the amount of heat removal at a given time (at a given Fo).



Comment: The agreement will be worse for small Fo. As seen in Fig. 5.10b, Fourier numbers less than 1 may never reach $\Phi \simeq 1$ — or even come close to $\Phi = 1$. The lumped solution will *always* reach $\Phi \simeq 1$ if Bi is large enough, but that is not accurate for high Bi.

Overall, the one-term calculations appear to have improved the accuracy by 1–3%. The reader should keep in mind that the *model* used for cooling the apples is far more approximate: apples are not spheres, the heat transfer coefficient may be uncertain by $\pm 20\%$ (or more), and so on.

Problem 5.60 Write down the one-term solutions for Θ for a slab with Bi $= \{0.01, 0.05, 0.1, 0.5, 1\}$. Compare these to the corresponding lumped capacity equation (see Problem 5.58). Ostrogorsky [5.8] has shown that $\hat{\lambda}_1 \simeq \sqrt{m \cdot \text{Bi}}$ for Bi $\leqslant 0.1$, where $m = 1$ for a slab, 2 for a cylinder, and 3 for a sphere. How does that formula compare to your results?

Solution

In Problem 5.58, we found

$$\frac{t}{\boldsymbol{T}} = \text{Bi}_L \text{Fo}_L$$

for a slab, and so the lumped solution may be written

$$\Theta = \exp(-\text{Bi}_L \text{Fo}_L)$$

The one-term equation is eqn. (5.42)

$$\Theta = A_1 f_1(r/r_o) \exp\left(-\hat{\lambda}_1^2 \text{Fo}_L\right)$$

with f_1 from Table 5.1

$$f_1 = \cos\left(\lambda_1 x/L\right)$$

The needed values A_1 and $\hat{\lambda}_1$ may be taken from Table 5.2 as below, where the constants in Θ have been rounded to 3 digits.

Bi_L	$\hat{\lambda}_1$	A_1	Θ	$\sqrt{\text{Bi}_L}$
0.01	0.09983	1.0017	$(1.00)\cos(0.0998x/L)\exp(-0.00997\,\text{Fo}_L)$	0.1000
0.05	0.22176	1.0082	$(1.01)\cos(0.222x/L)\exp(-0.0492\,\text{Fo}_L)$	0.2236
0.10	0.31105	1.0161	$(1.02)\cos(0.311x/L)\exp(-0.0968\,\text{Fo}_L)$	0.3162
0.50	0.65327	1.0701	$(1.07)\cos(0.653x/L)\exp(-0.427\,\text{Fo}_L)$	0.7071
1.00	0.86033	1.1191	$(1.12)\cos(0.860x/L)\exp(-0.740\,\text{Fo}_L)$	1.000

The coefficient of Fo_L in the expression for Θ is very close to Bi_L for $\text{Bi}_L \leqslant 0.1$ (within 3.2% or less). The cosine factor can be studied with a Taylor expansion of $\cos z$:

$$\cos z = 1 - \frac{z^2}{2} + \cdots$$

where $z = \hat{\lambda}_1 x/L$. For $\text{Bi}_L \leqslant 0.1$, $\cos(\lambda_1 x/L) \simeq 1$ with an error of 4.8% or or less. Thus, we conclude that the one-term solution approximates the lumped-capacity result well for $\text{Bi}_L \leqslant 0.1$.

This finding indicates that the one-term solution is acceptable for very low Biot number at any value of the Fourier number.

In every case, Ostrogorsky's formula, $\sqrt{\text{Bi}_L}$, is within 14% or better of $\hat{\lambda}_1$. For $\text{Bi}_L \leqslant 0.1$, Ostrogorsky's formula is within 1.7%.

Problem 5.61 When the one-term solution, eqn. (5.42), is plotted on semilogarithmic coordinates as $\log \Theta$ versus Fo for fixed values of Bi and position, what is the shape of the curve obtained? Make such a plot for a sphere with $\mathrm{Bi} = \{0.5, 1, 2, 5, 10\}$ at $r/r_o = 1$ for $0.2 \leqslant \mathrm{Fo} \leqslant 1.5$.

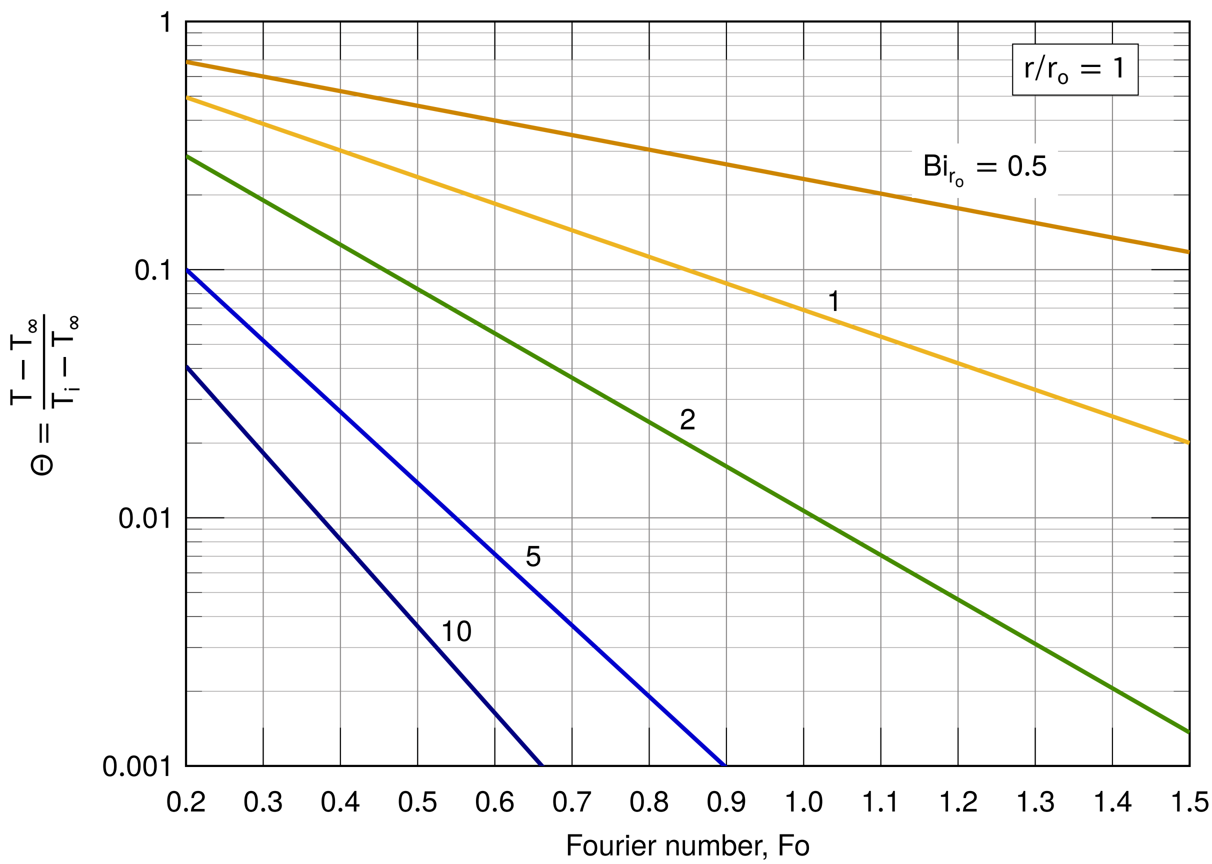
Solution

Note that A_1, f_1, and $\hat{\lambda}_1$ are constant if Bi and the position are fixed. Taking the logarithm of eqn. (5.42), we have

$$\log \Theta = \log(A_1 f_1) - \hat{\lambda}_1^2 \, \mathrm{Fo}$$

which is the equation of a straight line, $y = ax + b$, if $y = \log \Theta$ and $x = \mathrm{Fo}$. $\xleftarrow{\text{Answer}}$

For a sphere with $r/r_o = 1$, we may take f_1 from Table 5.1, giving $f_1(1) = \sin(\hat{\lambda}_1)/\hat{\lambda}_1$. A_1 and $\hat{\lambda}_1$ are functions of Biot number, and their values for $\mathrm{Bi} = \{0.5, 1, 2, 5, 10\}$ are listed in Table 5.2. With those values we may plot Θ vs. Fo_{r_o} for each Bi. Plotting Θ on a logarithmic axis has the same appearance as plotting $\log \Theta$ on a linear axis.



Problem 5.62 The solution for a semi-infinite body with convection, eqn. (5.53), contains a parameter β which is like $\text{Bi}\sqrt{\text{Fo}}$. For cylinders with Bi = 1 and Bi = 10, use eqn. (5.53) to find Θ when Fo = 0.05 for each of the four positions shown in Fig. 5.8, noting that r and x coordinates have different origins. How to these values compare to the values in Fig. 5.8?

Solution

Equation (5.53) is:

$$\Theta = \operatorname{erf}\frac{\zeta}{2} + \exp\left(\beta\zeta + \beta^2\right)\left[\operatorname{erfc}\left(\frac{\zeta}{2} + \beta\right)\right]$$

We can write

$$\beta = \frac{\overline{h}\sqrt{\alpha t}}{k} = \frac{\overline{h}r_o}{k}\sqrt{\frac{\alpha t}{r_o^2}} = \text{Bi}_{r_o}\sqrt{\text{Fo}_{r_o}}$$

To adjust the coordinates to start from the cylinder surface, we set $x = r_o - r$. Then

$$\zeta = \frac{x}{\sqrt{\alpha t}} = \frac{r_o - r}{\sqrt{\alpha t}} = (1 - r/r_o)\left(\frac{r_o}{\sqrt{\alpha t}}\right) = \frac{1 - r/r_o}{\sqrt{\text{Fo}_{r_o}}}$$

For Fo = 0.05:

$$\beta = \text{Bi}_{r_o}\sqrt{\text{Fo}_{r_o}} = 0.2236\,\text{Bi}_{r_o}, \qquad \zeta = \frac{1 - r/r_o}{\sqrt{\text{Fo}_{r_o}}} = 4.472(1 - r/r_o)$$

The calculations take a few steps, to calculate each function with tables or software, but in the end we make a table:

				r/r_o			
Case	Bi_{r_o}	β		0	0.5	0.75	1.0
Semi-infinite			ζ	4.472	2.236	1.118	0
	1	0.2236	Θ	0.9993	0.9863	0.9331	0.7904
	10	2.236	Θ	0.9993	0.9324	0.7104	0.2323
Chart	1	—	Θ	0.99	0.98	0.92	0.77
	10	—	Θ	(0.96)	0.89	0.65	0.20
$\Theta_{\text{semi}}/\Theta_{\text{chart}}$	1			1.009	1.001	1.014	1.026
	10			1.041	1.048	1.093	1.162

Our approximation of the cylinder as a semi-infinite body has the best agreement with the chart for $\text{Bi}_{r_o} = 1$, where the largest difference is less than 3%. For $\text{Bi}_{r_o} = 10$, the greatest difference is 16%.

Problem 5.63 Use eqn. (5.53), for a semi-infinite body, to write an equation for Θ at the surface of a body as a function of Bi and Fo. Plot this function on semilogarithmic axes for Fo = 0.05, 0.02, and 0.01 over the domain $0.01 \leqslant \text{Bi} \leqslant 100$. Compare to Figs. 5.7–5.9. (If you encounter numerical problems for very large values of Bi, note that $e^{x^2}\operatorname{erfc} x \sim 1\big/\sqrt{\pi}x$ as $x \to \infty$.)

Solution

Equation (5.53) is:

$$\Theta = \operatorname{erf}\frac{\zeta}{2} + \exp\left(\beta\zeta + \beta^2\right)\left[\operatorname{erfc}\left(\frac{\zeta}{2} + \beta\right)\right]$$

and at the surface $x = 0$, so that

$$\zeta = \frac{x}{\sqrt{\alpha t}} = 0$$

for $t > 0$. Since $\operatorname{erf}(0) = 0$, eqn (5.53) reduces to

$$\Theta = \exp(\beta^2)\operatorname{erfc}(\beta) \qquad (*)$$

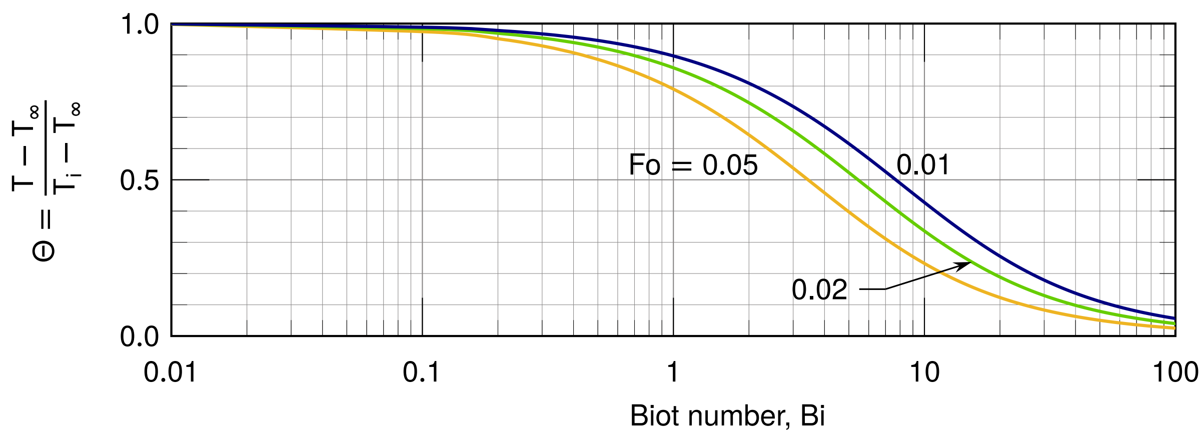
We can write

$$\beta = \frac{\overline{h}\sqrt{\alpha t}}{k} = \frac{\overline{h}L}{k}\sqrt{\frac{\alpha t}{L^2}} = \mathrm{Bi}_L\sqrt{\mathrm{Fo}_L}$$

and we could equally well have put r_o in place of L. So, eqn. (*) may be rewritten as

$$\Theta = \exp\left(\mathrm{Bi}^2\mathrm{Fo}\right)\operatorname{erfc}\left(\mathrm{Bi}\sqrt{\mathrm{Fo}}\right)$$

The evaluation of this function can be done with tables of erfc, but it is best done using software (especially since many values need to be computed to make a chart). The result of such calculations is plotted below.



The curve for $\mathrm{Fo} = 0.05$ can be compared to those for the same Fo and $x/L = 1$ or $r/r_o = 1$ in Figs. 5.7–5.9. This approximation is hardly distinguishable from the slab result at $\mathrm{Fo} = 0.05$. It is roughly 1.5% high for the cylinder, and about 7% for the sphere. Of course, the approximation should become more accurate as Fo is reduced still further.

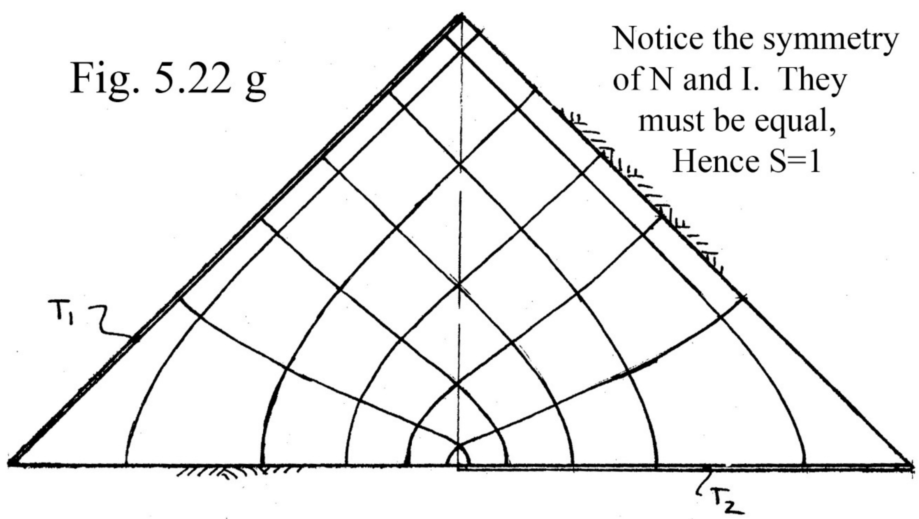
Problem 5.64 Use the method outlined in [5.20] to find the shape factors for Figs. 5.30g and 5.30j.

Solution

Go to Reference [5.20] in the textbook, and click on the link to that short paper. That paper describes how certain symmetries always result in shape factors of $S = 1$. Figures 5.30g and 5.30j are possessed of that symmetry and are, in fact, both used as illustrations of what the author calls "Yin-Yang" symmetry. Therefore, in both cases,

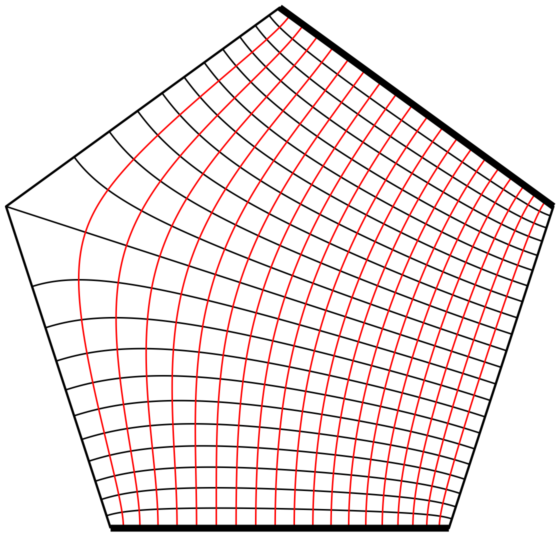
$$S = 1$$

Here is the flux plot for Fig. 5.22g (see Section 5.7):

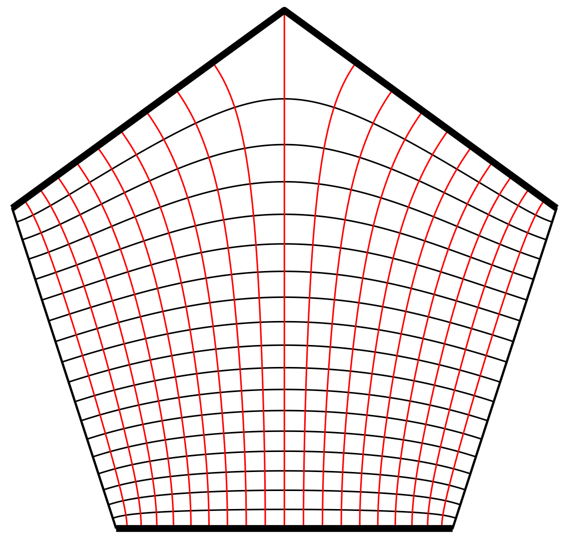


And this is simply a subset of the flux plot for Fig. 5.30j — one quarter of its flux plot.

PROBLEM 5.66 The flux plots Fig. 5.31 are for pentagons with bottom edge at $T = 1$. In (a), the top right edge is at $T = 0$, while in (b), both top edges are at $T = 0$. All other edges are adiabatic. Find the shape factor for each flux plot. What is the product of these two shape factors? Explain why.



(A) Top right edge isothermal



(B) Both top edges isothermal

Flux plots for regular pentagons with isothermal bottom edges and either one or two top edges isothermal.

SOLUTION Let $S^{1\text{-}1}$ be the shape factor for Fig. 5.31a and $S^{2\text{-}1}$ be that for Fig. 5.31b. For Fig. 5.31a, we count $N = 20$ heat flow channels and $I = 22$ temperature increments. For Fig. 5.31b, we count 20 heat flow channels and 18 temperature increments. Therefore, with eqn. (5.67),

$$S^{1\text{-}1} = \frac{N}{I} = \frac{18}{20} = 0.900 \quad \text{and} \quad S^{2\text{-}1} = \frac{20}{18} = 1.111$$

The product $S^{1\text{-}1} \times S^{2\text{-}1} \equiv 1$! The reason is seen by rotating Fig. 5.31a clockwise by one side: the figures are identical except that the isothermal and adiabatic sides have been interchanged, which has the effect of interchanging the heat flow channels and the temperature increments. Thus $N \to I$ and $I \to N$.

***Comment 1*:** The interchange of adiabatic and isothermal will always cause the original shape factor to be replaced by its reciprocal [1].

***Comment 2*:** The flux plots shown here are approximate (the number of temperature increments was numerically forced to be an integer). Finite element method (FEM) solution of these two cases yields $S^{1\text{-}1} = 0.8963$ and $S^{2\text{-}1} = 1.1157$ to an accuracy of about 0.05%.

REFERENCE:

[1] J. Hersch, "On Harmonic Measures, Conformal Moduli and Some Elementary Symmetry Methods," *Journal d'Analyse Mathématique*, **42:**211–228, 1982. doi:10.1007/BF02786880 **NB:** This paper requires a knowledge of conformal mapping.

Problem 6.1 Verify that eqn. (6.13) follows from eqns. (6.11a) and (6.12).

Begin with $\dfrac{\partial u^2}{\partial x} + \dfrac{\partial uv}{\partial y} = -\dfrac{1}{\rho}\dfrac{dp}{dx} + \nu \dfrac{\partial^2 u}{\partial y^2}$

or $u\dfrac{\partial u}{\partial x} + \left(u\dfrac{\partial u}{\partial x} + u\dfrac{\partial v}{\partial y}\right) + v\dfrac{\partial u}{\partial y} = -\dfrac{1}{\rho}\dfrac{dp}{dx} + \nu\dfrac{\partial^2 u}{\partial y^2}$

then multiply the continuity equation by u & get: $u\dfrac{\partial u}{\partial x} + u\dfrac{\partial v}{\partial y} = 0$
and subtract it to get

$$\underline{\underline{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}}}$$

Problem 6.2 Complete the algebra between eqns. (6.16) and (6.20).

Start with $\dfrac{\partial\psi}{\partial y}\dfrac{\partial^2\psi}{\partial x\partial y} - \dfrac{\partial\psi}{\partial x}\dfrac{\partial^2\psi}{\partial y^2} = \nu\dfrac{\partial^3\psi}{\partial y^3}$. Substitute $\psi(x,y) = \sqrt{u_\infty \nu x}\, f(\eta)$ and $\eta \equiv \sqrt{u_\infty/\nu x}\; y$ and get eqn. (6.18).

$$\frac{\partial\psi}{\partial y} = \frac{\partial\psi}{\partial\eta}\frac{\partial\eta}{\partial y} = \left(\sqrt{u_\infty \nu x}\, f'\right)\sqrt{\frac{u_\infty}{\nu x}} = \underline{u_\infty f'} = u$$

$$\frac{\partial\psi}{\partial x} = \frac{\partial\psi}{\partial\eta}\frac{\partial\eta}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial x}{\partial x} = \left(\sqrt{u_\infty \nu x}\, f'\right)\sqrt{\frac{u_\infty}{\nu}}\,\frac{-y}{2x^{3/2}} + \frac{1}{2}\sqrt{\frac{u_\infty \nu}{x}}\, f$$

$$= \underline{-\frac{1}{2}\sqrt{\frac{u_\infty \nu}{x}}\left(f'\eta - f\right)} = v$$

$$\frac{\partial^2\psi}{\partial x\partial y} = u_\infty\frac{\partial f'}{\partial x} = u_\infty\frac{\partial f'}{\partial\eta}\frac{\partial\eta}{\partial x} = -\frac{u_\infty\, y}{2x\sqrt{\frac{\nu x}{u_\infty}}}\, f'' = -\frac{u_\infty}{2x}\,\eta f''$$

$$\frac{\partial^2\psi}{\partial y^2} = u_\infty\frac{\partial f'}{\partial y} = u_\infty\frac{\partial f'}{\partial\eta}\frac{\partial\eta}{\partial y} = \frac{u_\infty}{\sqrt{\frac{\nu x}{u_\infty}}}\, f'' \;;\quad \frac{\partial^3\psi}{\partial y^3} = \frac{u_\infty^2}{\nu x}\, f'''$$

Combine these in the 3rd order mom. eqn. in ψ, and get

$$-u_\infty f'\left(\frac{u_\infty}{2x}\eta f''\right) + \frac{1}{2}\sqrt{\frac{u_\infty \nu}{x}}\left(f'\eta - f\right)\frac{u_\infty}{\sqrt{\frac{\nu x}{u_\infty}}}\, f'' = \nu\,\frac{u_\infty^2}{x}\, f'''$$

or

$$-\frac{u_\infty^2}{2x}\eta f' f'' + \frac{u_\infty^2}{2x}\eta f' f'' - \frac{u_\infty^2}{2x} f f'' = \frac{u_\infty^2}{x} f'''$$

so

$$\underline{\underline{f f'' + 2f''' = 0}}$$

6.3 Solve $ff'' + 2f''' = 0$ subject to the b.c.'s: $f(0) = f'(0) = 0$ and $f'(\infty) = 1$.

We begin by mapping the b.c., $f'(\infty) = 1$ into the origin, thus:

set $f = aF(\xi)$ and $\xi = a\eta$. Then
$$f' = a^2 F'$$
$$f'' = a^3 F''$$
$$f''' = a^4 F'''$$

so $ff'' + 2f''' = 0$ becomes $a^4 FF'' + 2a^4 F''' = 0$ or $\underline{FF'' + 2F''' = 0}$

with the b.c.s
$$f(0) = aF(0) = 0 \quad \text{or} \quad F(0) = 0$$
$$f'(0) = a^2 F'(0) = 0 \qquad F'(0) = 0$$
$$f'(\infty) = a^2 F'(\infty) = 1 \qquad F'(\infty) = 1/a^2$$

Now we would normally have to guess $f''(0) = a^3 F''(0)$. What we shall do is to set $F''(0) = 1$ so that $f''(0) = a^3$, and solve $FF'' + 2F''' = 0$ subject to $F(0) = F'(0) = 0$. This solution will give a certain value of $F'(\infty)$ from which we can calculate $a = [F'(\infty)]^{-1/2}$. Once we know a, we return to our calculated values of F for given values of ξ and correct these back to f and η using $f = aF$ and $\eta = \xi/a$.

There are many ways to solve the system $FF'' + 2F''' = 0$, $F(0) = F'(0) = 0$, $F''(0) = 1$. The simplest is probably to reduce it to three first order d.e.s thus: let $y_1 = F$, $y_2 = F'$, and $y_3 = F''$ so

$$\left.\begin{aligned} \frac{dy_3}{d\xi} &= -\frac{y_1 y_3}{2} \\ \frac{dy_2}{dx} &= y_3 \\ \frac{dy_1}{dx} &= y_2 \end{aligned}\right\} \qquad \begin{aligned} y_1(0) &= 0 \\ y_2(0) &= 0 \\ y_3(0) &= 1 \end{aligned}$$

Runga-Kutta integration schemes are available in computer libraries and can easily be called in to solve such systems of first-order equations.

6.4 Verify that the Blasius solution (given in Table 6.1) satisfies eqn. (7.25). Do this by showing graphically that

$$\frac{d}{dx}\left[\delta\int_0^1 \frac{u}{u_\infty}\left(\frac{u}{u_\infty}-1\right)d\left(\frac{y}{\delta}\right)\right] = -\frac{\nu}{u_\infty\delta}\left.\frac{\partial(u/u_\infty)}{\partial(y/\delta)}\right|_{y=0}$$

is satisfied by the numbers in Table 6.1. We begin by converting the equation with the help of $\delta = 4.92x/\sqrt{Re_x}$:

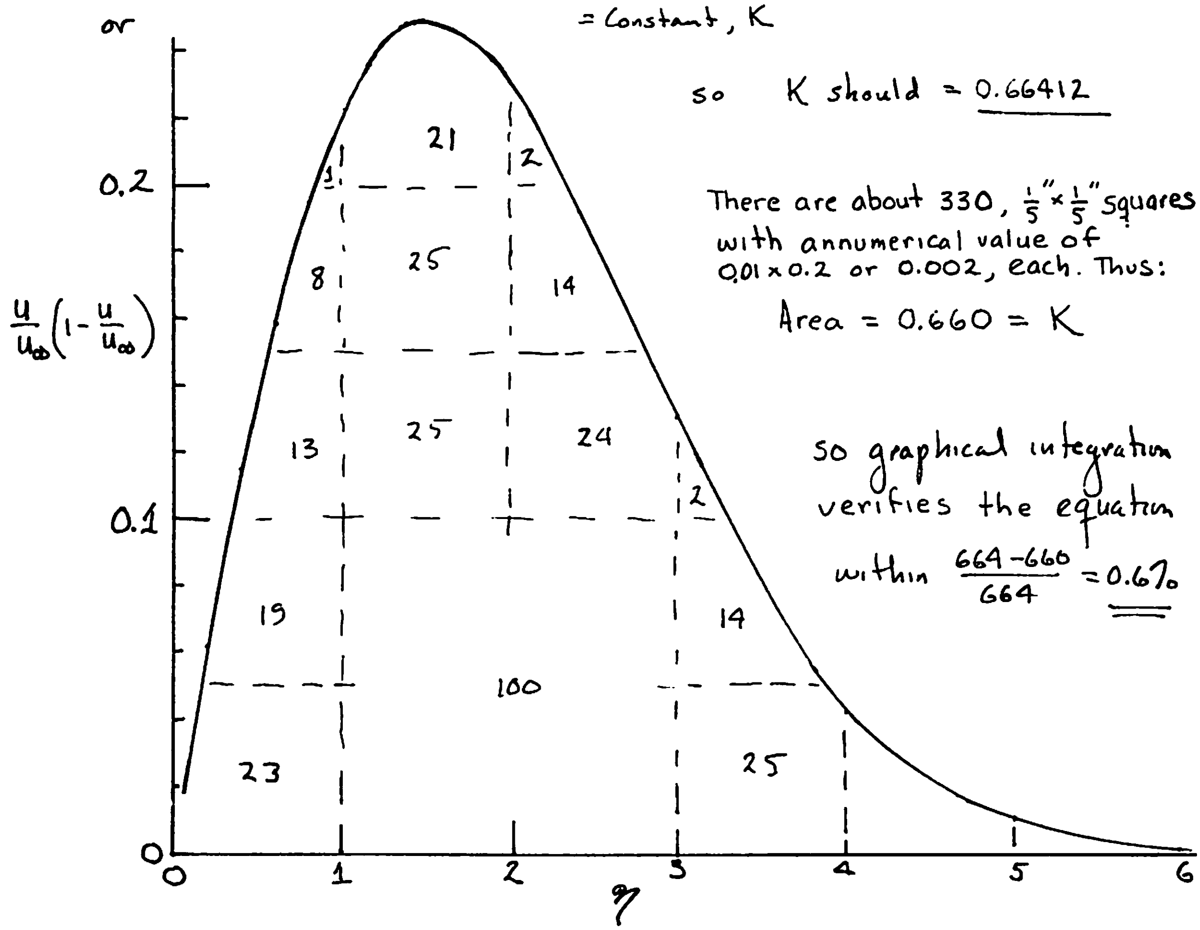
$$\frac{u_\infty}{\nu}\frac{4.92x}{\sqrt{Re_x}}\frac{d}{dx}\left[\frac{4.92x}{\sqrt{Re_x}}\int_0^1\frac{u}{u_\infty}\left(1-\frac{u}{u_\infty}\right)d\underbrace{\left(\frac{y}{4.92\sqrt{\frac{x\nu}{u_\infty}}}\right)}_{\frac{\eta}{4.92}}\right] = +\underbrace{\left.\frac{\partial(u/u_\infty)}{\partial(y/\delta)}\right|_{y=0}}_{=0.33206\,\eta_\delta/\eta\ \text{from Table 6.1}}$$

or

$$\frac{4.92\sqrt{u_\infty x}}{\sqrt{\nu}}\frac{d}{dx}\left[\sqrt{\frac{\nu x}{u_\infty}}\underbrace{\int_0^{4.92}\frac{u}{u_\infty}\left(1-\frac{u}{u_\infty}\right)d\eta}_{=\text{Constant, } K}\right] = 0.33206\times 4.92$$

or

so K should $= \underline{0.66412}$



There are about 330, $\frac{1}{5}'' \times \frac{1}{5}''$ squares with annumerical value of 0.01×0.2 or 0.002, each. Thus:

Area $= 0.660 = K$

so graphical integration verifies the equation within $\frac{664-660}{664} = \underline{\underline{0.6\%}}$

Graphical integration based on Table 6.1

6.5 Verify eqn. (6.30)

Start with $\frac{d}{dx}\left[\delta\int_0^1 \frac{u}{u_\infty}\left(\frac{u}{u_\infty}-1\right)d\left(\frac{y}{\delta}\right)\right] = -\frac{\nu}{u_\infty\delta}\left.\frac{\partial(u/u_\infty)}{\partial(y/\delta)}\right|_{y=0}$

and substitute $\frac{u}{u_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$. Let us call $\xi \equiv y/\delta$. Then:

$$\frac{d}{dx}\Big[\delta\underbrace{\int_0^1\left(\tfrac{3}{2}\xi-\tfrac{1}{2}\xi^3\right)\left(\tfrac{3}{2}\xi-\tfrac{1}{2}\xi^3-1\right)d\xi}_{\cancel{\frac{3}{4}}-\frac{3}{20}-\cancel{\frac{3}{4}}-\frac{3}{20}+\frac{1}{28}+\frac{1}{8}\;=\;-\frac{39}{280}}\Big] = -\frac{\nu}{u_\infty\delta}\left(\frac{3}{2}\right)$$

so:

$$-\frac{39}{280}\frac{d\delta}{dx} = -\frac{3}{2}\frac{\nu}{u_\infty\delta}$$

6.6 Derive τ_w using the momentum integral method.

$$\tau_w = (\mu u_\infty/\delta)\frac{\partial(u/u_\infty)}{\partial(y/\delta)} = \frac{\mu u_\infty}{\delta}\frac{3}{2}$$

substitute eqn. (7.31), $\delta = 4.64\sqrt{\frac{\nu x}{u_\infty}}$.

$$\tau_w = \frac{3/2}{4.64}\sqrt{\frac{u_\infty}{\nu x}}\,\mu u_\infty = 0.3233\frac{\mu u_\infty}{x}Re_x^{1/2}$$

This is only 3% below the exact value, eqn. (6.32)

6.7 Find δ and τ_w using the momentum integral method and assuming $u/u_\infty = y/\delta$ for the velocity profile.

Using eqn. (7.25): $\frac{d}{dx}\Big[\delta\underbrace{\int_0^1 \frac{y}{\delta}\left(\frac{y}{\delta}-1\right)d\frac{y}{\delta}}_{-1/6}\Big] = -\frac{\nu}{u_\infty\delta}\times 1$

so: $\int_0^\delta d\delta^2 = -\int_0^x \frac{12\nu}{u_\infty\delta}dx$ or $\frac{\delta}{x} = \sqrt{12}\frac{1}{\sqrt{Re_x}} = \frac{3.46}{\sqrt{Re_x}}$

And:

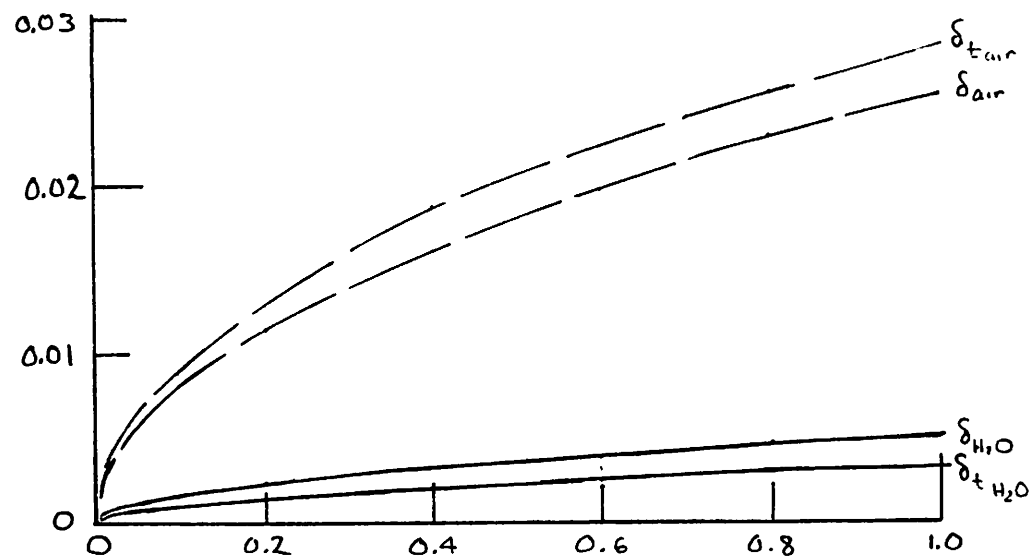
$$\tau_w = \mu\left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{\mu u_\infty}{\delta}\underbrace{\left.\frac{\partial(u/u_\infty)}{\partial(y/\delta)}\right|_{\frac{y}{\delta}=0}}_{=1}$$

so:

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho u_\infty^2} = \frac{2\nu}{\delta u_\infty} = \frac{2}{\sqrt{12}}\frac{\nu}{u_\infty}\sqrt{\frac{u_\infty}{\nu x}} = \frac{0.577}{\sqrt{Re_x}}$$

The use of this extremely crude approximation to u/u_∞ yields a vlaue of x/δ that is low by only 30% and a C_f that is low by only 13%.

6.8 The b.l. thickness for a particular water flow (plotted below) is given by δ m = $0.005\sqrt{x}$ m. Add to the plot: δ_t for the water flow; and δ and δ_t for air at the same temperature and velocity.

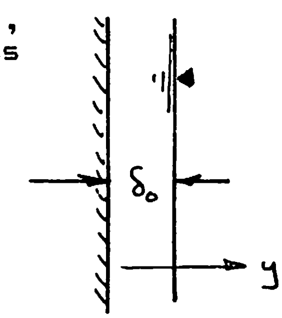


$$\delta_{t_{H_2O}} = \frac{\delta}{Pr_{H_2O}^{1/3}} = \frac{\delta}{4.34^{1/3}} = \underline{0.613\,\delta}\ ; \quad \delta_{air} = \sqrt{\frac{\nu_{air}}{\nu_{H_2O}}}\,\delta_{H_2O} = \sqrt{\frac{17\times10^{-6}}{0.658\times10^{-6}}}\,\delta_{H_2O}$$

$$= \underline{5.083\,\delta_{H_2O}}$$

$$\delta_{t_{air}} = \frac{\delta}{Pr_{air}^{1/3}} = \frac{5.083\,\delta_{H_2O}}{0.704^{1/3}} = \underline{5.714\,\delta_{H_2O}}$$

6.9 A liquid film flows down a plate as shown, at its terminal velocity, with a thickness of δ_o. Derive an expression for u(y).



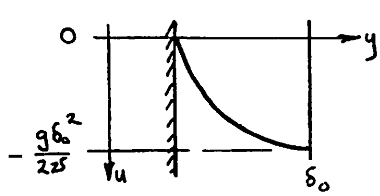
$$\underbrace{u\frac{\partial u}{\partial x}}_{=0} + \underbrace{v\frac{\partial u}{\partial y}}_{=0} = \underbrace{-\frac{1}{\rho}\frac{dp}{dx}}_{=0} - g + \nu\frac{\partial^2 u}{\partial y^2}$$

so $\dfrac{d^2u}{dy^2} = \dfrac{g}{\nu}$; $u = \dfrac{gy^2}{2\nu} + C_1 y + C_2$ w/b.c.'s $u(y=0)=0 \Rightarrow \underline{C_2 = 0}$

$\left.\dfrac{\partial u}{\partial y}\right|_{y=\delta_o} = 0 \Rightarrow \underline{C_1 = -\dfrac{g\delta_o}{\nu}}$

Then: $$\underline{\underline{u = \frac{gy^2}{2\nu} - \frac{g\delta_o^2}{\nu}\frac{y}{\delta_o}}}$$

or:



$$\underline{\underline{\frac{u}{g\delta_o^2/\nu} = \frac{1}{2}\left(\frac{y}{\delta_o}\right)^2 - \frac{y}{\delta_o}}} \quad \left(\text{= negative since it is downward}\right)$$

6.10 Evaluate $\overline{Nu_L}$ for laminar flow over an isothermal flat plate.

We know that $Nu_x = A Re_x^{1/2}$, where $A = \dfrac{0.3387\, Pr^{1/3}}{[1+0.0468/Pr^{2/3}]^{1/4}}$, from eqn. (6.63) so:

$$q(x) = \frac{k\Delta T}{x} Nu_x \quad \text{and} \quad \bar{q} = \frac{kA\Delta T}{L}\int_0^L \sqrt{\frac{u_\infty}{\nu}}\frac{\sqrt{x}}{x}\,dx = 2\frac{kA\Delta T}{L}\sqrt{\frac{u_\infty L}{\nu}}$$

Then:

$$\overline{Nu_L} = \frac{\bar{q}L}{k\Delta T} = A Re_L^{1/2} = \frac{0.6774\, Pr^{1/3} Re_L^{1/2}}{[1+0.0468/Pr^{2/3}]^{1/4}}$$

6.11 Use an integral method to predict Nu_x for a flat plate with q_w = constant, and find ΔT at the leading edge of the plate.

$$\rho c u_\infty \frac{d}{dx}\left[\delta_t \Delta T \underbrace{\int_0^1 \frac{u}{u_\infty}\left(\frac{T-T_\infty}{T_w-T_\infty}\right) d\left(\frac{y}{\delta_t}\right)}_{\frac{3}{20}\phi - \frac{3}{280}\phi^2 \simeq \frac{3}{20}\phi}\right] = q_w$$

$$\rho c u_\infty (\delta_t \Delta T)\left(\frac{3}{20}\frac{\delta_t}{\delta}\right) = q_w x$$

$$\frac{3}{20}\frac{u_\infty}{\alpha}\delta\left(\frac{\delta_t}{\delta}\right)^2 = \frac{q_w x}{\Delta T k} = Nu_x$$

But $Nu_x = \dfrac{3}{2}\dfrac{x}{\delta}\left(\dfrac{\delta}{\delta_t}\right)$ so $\dfrac{3}{20}\dfrac{u_\infty}{\alpha}\left(\dfrac{4.64x}{\sqrt{Re_x}}\right)^2\left(\dfrac{\delta_t}{\delta}\right)^3 = \dfrac{3}{2}x$

$$\frac{\delta}{\delta_t} = \left(\frac{4.64^2}{10}\right)^{1/3} Pr^{1/3} = 1.291\, Pr^{1/3}$$

so

$$Nu_x = \frac{3}{2}\frac{1.291}{4.64} Pr^{1/3} Re_x^{1/2} = 0.417\, Pr^{1/3} Re_x^{1/2}$$

and, since $h \sim Re_x^{1/2}/x \sim 1/\sqrt{x}$

$\Delta T = q_w/h \sim \sqrt{x}$

we conclude that $\Delta T \to 0$ at the leading edge of the heater.

Problem 6.12 (a) Verify that eqn. (6.120) follows from eqn. (6.119). (b) Derive an equation for liquids that is analogous to eqn. (6.119).

Solution

a) Beginning with

$$\begin{aligned}\overline{h} &= \frac{1}{L\Delta T}\int_0^L q_w\,dx \\ &= \frac{1}{L}\left[\int_0^{x_l} h_{\text{laminar}}\,dx + \int_{x_l}^{x_u} h_{\text{trans}}\,dx + \int_{x_u}^{L} h_{\text{turbulent}}\,dx\right] \end{aligned} \tag{6.119}$$

we may evaluate each integral separately. For a uniform temperature surface, the Nusselt numbers are given by these equations:

$$\mathrm{Nu}_{\text{lam}} = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3} \tag{6.58}$$

$$\mathrm{Nu}_{\text{trans}} = \mathrm{Nu}_{\text{lam}}\big(\mathrm{Re}_l,\mathrm{Pr}\big)\left(\frac{\mathrm{Re}_x}{\mathrm{Re}_l}\right)^c \tag{6.114b}$$

$$\mathrm{Nu}_{\text{turb}} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{0.6} \quad \text{for gases} \tag{6.112}$$

The three integrals are thus

$$\frac{1}{L}\int_0^{x_l} h_{\text{lam}}\,dx = \frac{0.332\,k\mathrm{Pr}^{1/3}}{L}\int_0^{x_l}\sqrt{\frac{u_\infty}{\nu x}}\,dx = \frac{0.332\,k\mathrm{Pr}^{1/3}}{L}\,2\sqrt{\frac{u_\infty x_l}{\nu}} = \frac{k}{L}\,0.664\,\mathrm{Re}_l^{1/2}\,\mathrm{Pr}$$

$$\begin{aligned}\frac{1}{L}\int_{x_l}^{x_u} h_{\text{trans}}\,dx &= \frac{k}{L}\frac{\mathrm{Nu}_{\text{lam}}\big(\mathrm{Re}_l,\mathrm{Pr}\big)}{\mathrm{Re}_l^c}\left(\frac{u_\infty}{\nu}\right)^c\int_{x_l}^{x_u} x^{c-1}\,dx = \frac{k}{L}\frac{\mathrm{Nu}_{\text{lam}}\big(\mathrm{Re}_l,\mathrm{Pr}\big)}{\mathrm{Re}_l^c}\left(\frac{u_\infty}{\nu}\right)^c\frac{1}{c}\big(x_u^c - x_l^c\big) \\ &= \frac{k}{L}\frac{\mathrm{Nu}_{\text{lam}}\big(\mathrm{Re}_l,\mathrm{Pr}\big)}{\mathrm{Re}_l^c}\frac{1}{c}\big(\mathrm{Re}_u^c - \mathrm{Re}_l^c\big) = \frac{k}{L}\frac{1}{c}\Big[\mathrm{Nu}_{\text{turb}}\big(\mathrm{Re}_u,\mathrm{Pr}\big) - \mathrm{Nu}_{\text{lam}}\big(\mathrm{Re}_l,\mathrm{Pr}\big)\Big]\end{aligned}$$

where the last step follows because eqn. (6.114b) intersects $\mathrm{Nu}_{\text{turb}}$ at Re_u, and

$$\begin{aligned}\frac{1}{L}\int_{x_u}^{L} h_{\text{turb}}\,dx &= \frac{0.0296\,k\mathrm{Pr}^{0.6}}{L}\left(\frac{u_\infty}{\nu}\right)^{0.8}\int_{x_u}^{L} x^{-0.2}\,dx = \frac{0.0296\,k\mathrm{Pr}^{0.6}}{(0.8)L}\Big(\mathrm{Re}_L^{0.8} - \mathrm{Re}_u^{0.8}\Big) \\ &= \frac{k}{L}0.037\,\mathrm{Pr}^{0.6}\Big(\mathrm{Re}_L^{0.8} - \mathrm{Re}_u^{0.8}\Big)\end{aligned}$$

Collecting these terms, we find:

$$\begin{aligned}\overline{\mathrm{Nu}}_L \equiv \frac{\overline{h}L}{k} &= 0.037\,\mathrm{Pr}^{0.6}\Big(\mathrm{Re}_L^{0.8} - \mathrm{Re}_u^{0.8}\Big) + 0.664\,\mathrm{Re}_l^{1/2}\mathrm{Pr}^{1/3} \\ &\quad + \underbrace{\frac{1}{c}\Big(0.0296\,\mathrm{Re}_u^{0.8}\mathrm{Pr}^{0.6} - 0.332\,\mathrm{Re}_l^{1/2}\mathrm{Pr}^{1/3}\Big)}_{\text{contribution of transition region}} \quad \text{for gases}\end{aligned} \tag{6.120}$$

b) For a liquid flow, the turbulent correlation should be eqn. (6.113):

$$\mathrm{Nu}_{\text{turb}} = 0.032\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{0.43} \quad \text{for nonmetallic liquids} \tag{6.113}$$

and the integral in the turbulent range changes to

$$\frac{1}{L}\int_{x_u}^{L} h_{\text{turb}}\,dx = \frac{0.032\,k\text{Pr}^{0.43}}{L}\left(\frac{u_\infty}{\nu}\right)^{0.8}\int_{x_u}^{L} x^{-0.2}\,dx = \frac{0.032\,k\text{Pr}^{0.43}}{(0.8)L}\left(\text{Re}_L^{0.8} - \text{Re}_u^{0.8}\right)$$
$$= \frac{k}{L}0.040\,\text{Pr}^{0.43}\left(\text{Re}_L^{0.8} - \text{Re}_u^{0.8}\right)$$

Collecting these terms, we find:

$$\overline{\text{Nu}}_L \equiv \frac{\overline{h}L}{k} = 0.040\,\text{Pr}^{0.43}\left(\text{Re}_L^{0.8} - \text{Re}_u^{0.8}\right) + 0.664\,\text{Re}_l^{1/2}\text{Pr}^{1/3}$$
$$+ \underbrace{\frac{1}{c}\left(0.032\,\text{Re}_u^{0.8}\text{Pr}^{0.43} - 0.332\,\text{Re}_l^{1/2}\text{Pr}^{1/3}\right)}_{\text{contribution of transition region}} \quad \text{for nonmetallic liquids}$$

Problem 6.13 Fluid at a uniform speed U flows into a channel between two parallel plates a distance d apart. A laminar boundary layer grows on each plate. (a) At approximately what distance from the inlet will the two boundary layers first touch? (b) If the flow remains laminar, qualitatively sketch the velocity distribution between the plates a long distance after the boundary layers meet, noting that the mass flow rate is constant along the channel. [(a) $x/d \cong 0.01(Ud/\nu)$.]

Solution

a) Initially, a flat-plate boundary layer grows on each wall. The thickness of the b.l. is given by eqn. (6.2):

$$\frac{\delta}{x} = \frac{4.92}{\sqrt{\mathrm{Re}_x}} \tag{6.2}$$

At the location where the boundary layer thickness reaches half the distance between the plates, the boundary layers will meet. Call this position x_e. Then:

$$\frac{d}{2} \cong \delta = \frac{4.92 x_e}{\sqrt{\mathrm{Re}_{x_e}}} = \frac{4.92\sqrt{x_e}}{\sqrt{U/\nu}}$$

or

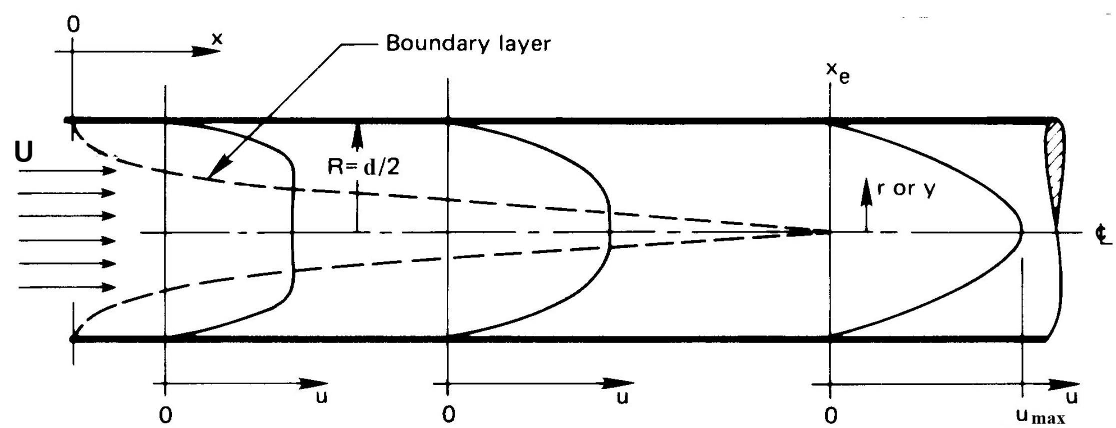
$$\sqrt{x_e} \cong \frac{d\sqrt{U/\nu}}{2(4.92)}$$

Squaring this equation and dividing through by d, we get

$$\frac{x_e}{d} \cong [2(4.92)]^{-2}\,\frac{Ud}{\nu} = 0.01\,\frac{Ud}{\nu} \quad \xleftarrow{\text{Answer}}$$

b) Because mass is conserved, when the fluid in the boundary layers on the walls slows down, the fluid at the center of the channel will have to move at a speed $u_{\max} > U$ so that the total mass rate in any cross-section stays constant. In fact, the same behavior occurs in a tube, as we will see in Chapter 7.

A sketch of the evolving velocity profile is below, for either parallel plates, with coordinates (x, y), or a circular tube, with coordinates (x, r).



Comment: More careful analysis, based upon the solution of the momentum equation, shows that a coefficient several times greater than 0.01 characterizes the distance to achieve the final velocity profile, which is parabolic.

6.14 Do the differentiation in eqn. (6.24)

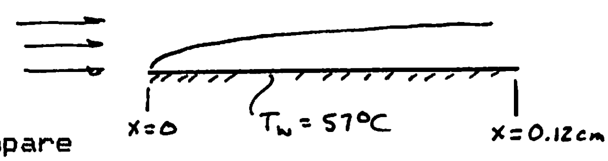
Leibnitz' rule says that: $\dfrac{d}{dx}\displaystyle\int_{a(x)}^{b(x)} f(\alpha,x)\,d\alpha = \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f\,d\alpha + f(b,x)\frac{db}{dx} - f(a,x)\frac{da}{dx}$

Thus:

$$\underline{\underline{\frac{d}{dx}\int_0^{\delta(x)} u(u-u_\infty)\,dy = \int_0^{\delta(x)} \frac{\partial}{\partial x}\left[u(u-u_\infty)\right]dy}} + \underbrace{\underbrace{u(\delta,x)}_{u_\infty}\Big(\underbrace{u(\delta,x)}_{u_\infty}-u_\infty\Big)}_{=0}\frac{d\delta}{dx} - 0$$

and this gives eqn. above eqn. (6.24).

6.15 Glycerin or water flows over a flat plate at 2 m/s. $T_\infty = 23^oC$



Find: $h(x = 0.12)$ and compare the drag forces in each case.

Evaluating properties at $T_{avg} = (57 + 23)/2 = 40^oC$, we get:

water: $k = 0.627\ W/m\text{-}^oC$
$\nu = 0.657\times10^{-6}\ m^2/s$
$\rho = 992\ kg/m^3$
$Pr = 4.36$
$Re_L = 0.12(2)/6.57(10)^{-7} = 365{,}300$
$Nu_L = 0.332\sqrt{365{,}300}\,(4.36)^{1/3} = 328$
$h(x=0.12) = \frac{k}{L}Nu_L = \underline{\underline{1715\ W/m^2\text{-}^oC}}$

glycerin: $k = 0.285\ W/m\text{-}^oC$
$\nu = 0.000227\ m^2/s$
$\rho = 1249\ kg/m^3$
$Pr = 2541$
$Re_L = 0.12(2)/0.000227 = 1057$
$Nu_L = 0.332\sqrt{1057}\,(2451)^{1/3} = 146$
$h(x=0.12) = \frac{k}{L}Nu_L = \underline{\underline{346\ W/m^2\text{-}^oC}}$

Thus the water is a significantly better coolant. Furthermore The ratio of drag forces is:

$$\frac{F_{D\,glyc}}{F_{D\,water}} = \frac{(\tau_w)_{gl}A}{(\tau_w)_{H_2O}A} = \frac{\rho_g\,\overline{C_{f_g}}}{\rho_w\,\overline{C_{f_w}}} = \frac{1249}{992}\sqrt{\frac{Re_{L\,w}}{Re_{L\,g}}} = \underline{\underline{23.4}}$$

So the glycerin exerts 23.4 times as much drag on the plate as the water does.

Problem 6.16 Air at −10 °C flows over a smooth, sharp-edged, almost-flat, aerodynamic surface at 240 km/hr. The surface is at 10 °C. Turbulent transition begins at $\text{Re}_l = 140{,}000$ and ends at $\text{Re}_u = 315{,}000$. Find: (a) the x-coordinates within which laminar-to-turbulent transition occurs; (b) $\overline{h}$ for a 2 m long surface; (c) h at the trailing edge for a 2 m surface; and (d) δ and h at x_l.

Solution

a) We evaluate physical properties at the film temperature, $T_f = (-10+10)/2 = 0\,°\text{C}$: $\nu = 1.332\times10^{-5}$ m^2/s, Pr = 0.711, and $k = 0.244$ W/m·K. Also, $u_\infty = 240(1000)/(3600) = 66.7$ m/s. Then:

$$x_l = \frac{\text{Re}_l \nu}{u_\infty} = \frac{(140000)(1.332\times10^{-5})}{(66.7)} = \underline{0.0280 \text{ m}}$$

$$x_u = \frac{\text{Re}_u \nu}{u_\infty} = \frac{(315000)(1.332\times10^{-5})}{(66.7)} = \underline{0.0629 \text{ m}}$$

Observe that the flow is fully turbulent over 1.937/2.00 = 96.9% of its length.

b) First, we need Re_L:

$$\text{Re}_L = \frac{u_\infty L}{\nu} = \frac{(66.7)(2)}{1.332\times10^{-5}} = 1.00\times10^7$$

Then we get c from eqn. (6.115):

$$c = 0.9922\ \log_{10}(140,000) - 3.013 = 2.09$$

Now we may use eqn. (6.120):

$$\begin{aligned}\overline{\text{Nu}}_L &= 0.037(0.711)^{0.6}\left[(1.00\times10^7)^{0.8} - (3.15\times10^5)^{0.8}\right] \\ &\quad + 0.664\,(1.40\times10^5)^{1/2}(0.711)^{1/3} \\ &\quad + \frac{1}{2.09}\left[0.0296(3.15\times10^5)^{0.8}(0.711)^{0.6} - 0.332\,(1.40\times10^5)^{1/2}(0.711)^{1/3}\right] \\ &= 11248.9 + 221.8 + 236.0 = 1.171\times10^4\end{aligned}$$

Thus

$$\overline{h} = \frac{k}{L}\overline{\text{Nu}}_L = \frac{(0.0244)(1.171\times10^4)}{2} = \underline{143 \text{ W/m}^2\text{K}}$$

c) With eqn. (6.112),

$$\text{Nu}_L = 0.0296\,\text{Re}_L^{0.8}\,\text{Pr}^{0.6} = 0.0296\,(1.00\times10^7)^{0.8}(0.711)^{0.6} = 9603$$

so

$$h(L) = \frac{k}{L}\text{Nu}_L = \frac{(0.0244)(9603)}{2} = \underline{117 \text{ W/m}^2\text{K}}$$

d) The flow is laminar here. From eqn (6.58):

$$\text{Nu}_{x_l} = 0.332\,\text{Re}_l^{1/2}\,\text{Pr}^{1/3} = 0.332\,(1.40\times10^5)^{1/2}(0.711)^{1/3} = 110.9$$

so

$$h(x_l) = \frac{k}{x_l}\text{Nu}_{x_l} = \frac{(0.0244)(110.9)}{0.0280} = \underline{96.6 \text{ W/m}^2\text{K}}$$

With eqn (6.2), we find that the boundary layer here is *very* thin:

$$\delta = \frac{4.92\, x_l}{\sqrt{\mathrm{Re}_{x_l}}} = \frac{4.92(0.0280)}{\sqrt{1.4 \times 10^5}} = 0.000368 \text{ m} = \underline{\underline{0.37 \text{ mm}}}$$

Problem 6.17 Find $\overline{h}$ in Example 6.9 using eqn. (6.120) with $\mathrm{Re}_l = 80,000$. Compare with the value in the example and discuss the implication of your result. *Hint:* See Example 6.10.

Solution Equation (6.120) is

$$\overline{\mathrm{Nu}}_L \equiv \frac{\overline{h}L}{k} = 0.037\,\mathrm{Pr}^{0.6}\left(\mathrm{Re}_L^{0.8} - \mathrm{Re}_u^{0.8}\right) + 0.664\,\mathrm{Re}_l^{1/2}\mathrm{Pr}^{1/3} + \frac{1}{c}\left(0.0296\,\mathrm{Re}_u^{0.8}\mathrm{Pr}^{0.6} - 0.332\,\mathrm{Re}_l^{1/2}\mathrm{Pr}^{1/3}\right) \quad (6.120)$$

From Example 6.9, we have $\mathrm{Re}_L = 1.270 \times 10^6$ and $\mathrm{Pr} = 0.708$. We may find c from eqn. (6.115):

$$c = 0.9922\ \log_{10}(80,000) - 3.013 = 1.85$$

We also need Re_u, which we can find following Example 6.10:

$$\mathrm{Re}_u^{1.85-0.8} = \frac{0.0296(0.708)^{0.6}(80,000)^{1.85}}{0.332(80,000)^{1/2}(0.708)^{1/3}}$$

Solving, $\mathrm{Re}_u = 184,500$. Substituting all this into eqn. (6.120):

$$\overline{\mathrm{Nu}}_L = 0.037(0.708)^{0.6}\left[(1.270\times 10^6)^{0.8} - (1.845\times 10^5)^{0.8}\right] + 0.664\,(8.0\times 10^4)^{1/2}(0.708)^{1/3} + \frac{1}{1.85}\left[0.0296(1.845\times 10^5)^{0.8}(0.708)^{0.6} - 0.332\,(8.0\times 10^4)^{1/2}(0.708)^{1/3}\right]$$

Evaluating, we find the contributions of the turbulent, laminar, and transition regions:

$$\overline{\mathrm{Nu}}_L = \underbrace{1806.6}_{\text{turb.}} + \underbrace{167.4}_{\text{lam.}} + \underbrace{167.1}_{\text{trans.}} = 2,141$$

The transition region contributes 7.8% of the total. The average heat transfer coefficient is

$$\overline{h} = \frac{2141(0.0264)}{2.0} = 28.26\ \mathrm{W/m^2K}$$

and the convective heat loss from the plate is

$$Q = (2.0)(1.0)(28.26)(310 - 290) = \underline{1130\ \mathrm{W}}$$

The earlier transition to turbulence increases the heat removal by $[(1130{+}22)/(756{+}22){-}1]{\times}100 =$ 48%.

Problem 6.18 For system described in Example 6.9, plot the local value of h over the whole length of the plate using eqn. (6.117). On the same graph, plot h from eqn. (6.58) for $\mathrm{Re}_x < 800,000$ and from ~~eqn. (6.115)~~ eqn. (6.112) for $\mathrm{Re}_x > 400,000$. Discuss the results. (Final equation numbers refer to AHTT Version 5.10.)

Solution

The equations that we must work with are as follow (all suitable for air with Pr = 0.708, using the film temperature properties given in Example 6.9):

$$\mathrm{Nu}_x(\mathrm{Re}_x, \mathrm{Pr}) = \left[\mathrm{Nu}_{x,\text{lam}}^5 + \left(\mathrm{Nu}_{x,\text{trans}}^{-10} + \mathrm{Nu}_{x,\text{turb}}^{-10}\right)^{-1/2}\right]^{1/5} \tag{6.117}$$

$$\mathrm{Nu}_{x,\text{lam}} = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3} \tag{6.58}$$

$$\mathrm{Nu}_{x,\text{turb}} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{0.6} \tag{6.112}$$

We need to plot these in terms of the heat transfer coefficient,

$$h(x) = (k/x)\mathrm{Nu}_x = (0.0264/x)\mathrm{Nu}_x \text{ for } x \text{ in meters}$$

Now, we have

$$\mathrm{Re}_x = \frac{u_\infty}{\nu}x = \frac{10\,x}{1.575\times 10^{-5}} = 6.349\times 10^5\,x$$

so that

$$\mathrm{Nu}_{x,\text{lam}} = 0.332\,(6.349\times 10^5\,x)^{1/2}\,(0.708)^{1/3} = 235.8\,x^{1/2}$$

$$\mathrm{Nu}_{x,\text{turb}} = 0.0296\,(6.349\times 10^5\,x)^{0.8}\,(0.708)^{0.6} = 1157\,x^{0.8}$$

Also, we can easily calculate that $\mathrm{Re}_x = 400,000$ at $x = 0.630$ m and $\mathrm{Re}_x = 800,000$ at $x = 1.260$ m. Since the plate is 2 m long, we observe that turbulent flow covers more than half of the plate.

The transitional Reynolds number is given by eqn. (6.114b):

$$\mathrm{Nu}_{\text{trans}} = \mathrm{Nu}_{\text{lam}}(\mathrm{Re}_l, \mathrm{Pr})\left(\frac{\mathrm{Re}_x}{\mathrm{Re}_l}\right)^c \tag{6.114b}$$

The value $c = 2.55$ is the same as in Example 6.9, and with $\mathrm{Re}_l = 400,000$ as in the example,

$$\mathrm{Nu}_{\text{lam}}(400000, 0.708) = 0.332(400000)^{1/2}(0.708)^{1/3} = 187.1$$

so that

$$\mathrm{Nu}_{\text{trans}} = (187.1)\left(\frac{\mathrm{Re}_x}{400,000}\right)^{2.55} = (187.1)\left(\frac{6.349\times 10^5}{4.000\times 10^5}\right)^{2.55} x^{2.55} = 607.7\,x^{2.55}$$

Putting all this into eqn. (6.117), for h in W/m^{2}K and x in meters,

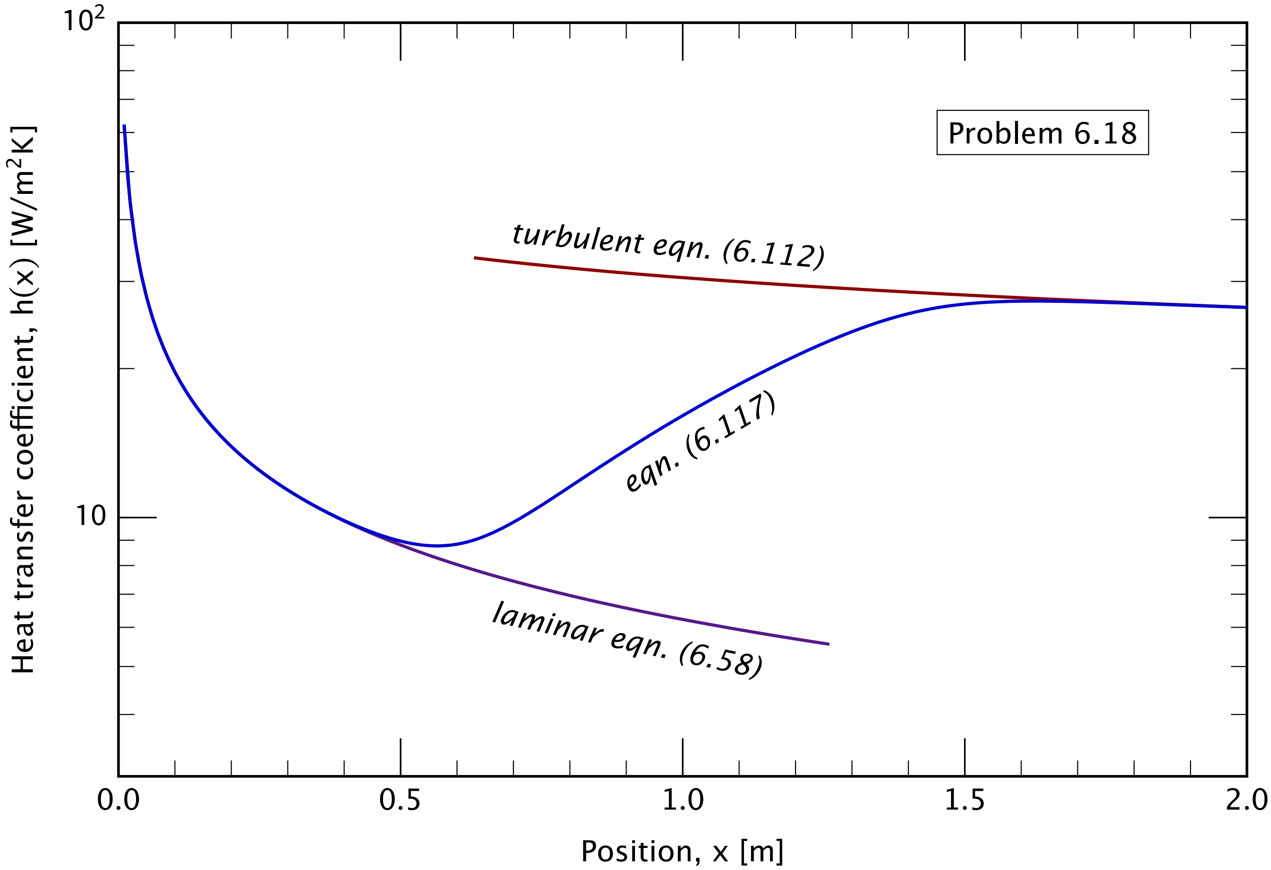
$$h(x) = (0.0264/x)\left[\left(235.8\,x^{1/2}\right)^5 + \left[\left(607.7\,x^{2.55}\right)^{-10} + \left(1157\,x^{0.8}\right)^{-10}\right]^{-1/2}\right]^{1/5}$$

Likewise

$$h(x)_{\text{lam}} = 235.8\,x^{1/2}(0.0264/x) = 6.225\,x^{-0.5}$$

$$h(x)_{\text{turb}} = 1157\,x^{0.8}(0.0264/x) = 30.54\,x^{-0.2}$$

We may now plot all this. Since we are plotting against the physical dimension x, we will not use a logarithmic x-axis. The y-axis may still be plotted on a logarithmic scale.



The chart shows that eqns. (6.58) and (6.117) are identical for $x \lesssim 0.45$ m and that eqns. (6.112) and (6.117) are identical for $x \gtrsim 1.6$ m. The heat transfer coefficient drops sharply with x in the laminar range, as the laminar boundary grows thicker. The heat transfer coefficient rises steeply as turbulent transition begins and develops. In the fully turbulent range, h decreases gradually.

6.19 Mercury flows at 25°C and 0.7 m/s over a 4 cm long plate at 60°C. Find $\bar{h}$, $\overline{\tau_w}$, $h(x = 0.04\text{ m})$, and $\delta(x = 0.04\text{ m})$.

Solution Evaluate properties at $(25+60)/2 = 42.5°\text{C} = 315.5°\text{K}$

$\nu = 1.14(10)^{-7}\,\text{m}^2/\text{s}$, $Pr = 0.0248$, $k = 7.39$, $Re_L = \dfrac{0.7(0.04)}{1.14(10)^{-7}} = 245{,}600$

So:

$$\overline{Nu_L} = 1.13\sqrt{245{,}600(0.0248)} = 88.2\ ; \quad \bar{h} = 88.2\,\frac{7.39}{0.04} = 16{,}293\,\frac{\text{W}}{\text{m}^2\text{-°C}}$$

$$\delta = \frac{4.92(0.04)}{\sqrt{245{,}600}} = \underline{\underline{0.004\text{ m} = 0.4\text{ mm (pretty thin)}}}$$

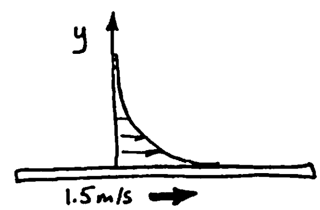
$$\underline{\underline{h_L = \tfrac{1}{2}\bar{h} = 8{,}147\,\frac{\text{W}}{\text{m}^2\text{-°C}}}}$$

$$\tau_w = \tfrac{1}{2}\rho u_\infty C_f = \frac{13{,}573(0.7)^2}{2}\,\frac{1.328}{\sqrt{245{,}600}} = \underline{\underline{8.91\,\frac{\text{N}}{\text{m}^2}}}$$

6.20 A plate is at rest in water at 15°C. It is suddenly translated parallel with itself at 1.5 m/s. Evaluate the liquid velocity, u, 0.015 from the plate at t = 1, 10, and 1000 s.

Pose the problem: $\dfrac{\partial^2 u}{\partial y^2} = \dfrac{1}{\nu}\dfrac{\partial u}{\partial t}$

with b.c.s: $u(y=0) = 1.5\,\text{m/s}$, $t>0$
$u(t=0) = 0$



Compare this with the semi-infinite region solution in Sect 5.6

$$\frac{\partial^2(T-T_i)}{\partial^2 x} = \frac{1}{\alpha}\frac{\partial(T-T_i)}{\partial t} \quad \text{with b.c.'s: } (T-T_i)_{x=0} = T_\infty - T_i\ ;\ (T-T_i)_{t=0} = 0$$

These problems are identical if: $T-T_i \Leftrightarrow u$, $x \Leftrightarrow y$, $\alpha \Leftrightarrow \nu$
and $(T_\infty - T_i) \Leftrightarrow 1.5$

Thus, its solution, $\dfrac{T-T_\infty}{T_i - T_\infty} = \text{erf}\left(\dfrac{x}{2\sqrt{\alpha t}}\right)$ is the solution to our problem once we make these changes. So:

$$\frac{u - 1.5}{-1.5} = \text{erf}\left(\frac{y}{2\sqrt{\nu t}}\right) \qquad \text{where } \nu = 1.184\times10^{-6}\,\frac{\text{m}^2}{\text{s}}$$

Therefore: u at $y = 0.015\text{m}$ is given by

$$u = 1.5\left(1 - \text{erf}\,\frac{6.89}{\sqrt{t}}\right)$$

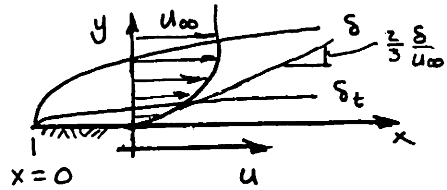
Then at $t = 1$ sec, $\underline{\underline{u \simeq 0\text{ m/s}}}$

$t = 10$ sec, $\underline{\underline{u \simeq 0.003\text{ m/s}}}$

$t = 1000$ sec, $\underline{\underline{u = 1.14\text{ m/s}}}$

6.21 Use the fact that, when Pr is very large, $u/u_\infty = (3/2)(y/\delta)$ inside the thermal b.l., to create an expression for Nu_x during the flow of a High Pr fluid over over a flat isothermal plate.

We begin with the integrated energy equation in the form of eqn. (6.51).



$$u_\infty \Delta T \frac{d}{dx}\left[\delta_t \int_0^1 \underbrace{\frac{u}{u_\infty}}_{\frac{3}{2}\frac{y}{\delta} \,=\, \frac{3}{2}\phi\frac{y}{\delta_t}} \underbrace{\left(\frac{T-T_\infty}{\Delta T}\right)}_{1-\frac{3}{2}\frac{y}{\delta_t}+\frac{1}{2}\left(\frac{y}{\delta_t}\right)^3} d\left(\frac{y}{\delta_t}\right)\right] = -\frac{\alpha \Delta T}{\delta_t} \underbrace{\left.\frac{d\,\frac{T-T_\infty}{\Delta T}}{d(y/\delta_t)}\right|_{\frac{y}{\delta_t}=0}}_{-\frac{3}{2}}$$

or

$$u_\infty \Delta T \frac{d}{dx}\left[\delta_t \underbrace{\int_0^1 \left(\frac{3}{2}\phi x - \frac{9}{4}\phi x^2 + \frac{3}{4}\phi x^4\right) dx}_{\left(\frac{3}{4}-\frac{3}{4}+\frac{3}{20}\right)\phi \,=\, \frac{3}{20}\phi}\right] = \frac{3}{2}\frac{\Delta T}{\delta_t}\alpha$$

or $$\frac{1}{10}\frac{1}{2}\frac{d\delta_t^2}{dx} = \frac{\alpha}{u_\infty \phi} \qquad \text{or} \qquad \delta_t = \sqrt{\frac{20\alpha}{u_\infty \phi}x}$$

Now we introduce $\delta_t = \delta\,\phi$: $$\delta = \sqrt{\frac{20\alpha x}{\phi^3 u_\infty}}$$

Then using eqn. (6.31a) for δ: $$\delta = \sqrt{\frac{280}{13}\frac{\nu x}{u_\infty}}$$

We equate the two expressions for δ and solve for ϕ:

$$\phi = \sqrt[3]{\frac{13}{14}\frac{1}{Pr}}$$

But, in accordance with eqn. (6.57), $h = \frac{3}{2}\frac{k}{\delta_t} = \frac{3k}{2\delta\phi}$

Thus:

$$Nu_x = \frac{3}{2}\,\frac{x}{\frac{4.64x}{\sqrt{Re_x}}\left(\frac{13}{14}\right)^{1/3}\frac{1}{Pr^{1/3}}} = \underline{\underline{0.3314\, Re_x^{1/2}\, Pr^{1/3}}}$$

(The integral approx. in the text gives exactly this for $\phi \Rightarrow 0$)

Problem 6.22 For air flowing above an isothermal plate, plot the ratio of $h(x)_{\text{laminar}}$ to $h(x)_{\text{turbulent}}$ as a function of Re_x in the range of Re_x that might be either laminar or turbulent. What does the plot suggest about designing for effective heat transfer?

Solution For laminar flow of gases or liquids over an isothermal surface, we use eqn. (6.58):

$$\text{Nu}_x = 0.332\,\text{Re}_x^{1/2}\,\text{Pr}^{1/3}$$

For turbulent flow, we can use eqn. (6.111); however, as noted on pgs. 326–327 (and in Fig. 6.22), eqn. (6.112) approximates eqn. (6.111) very closely for gases. Because eqn. (6.112) is less complicated, we will use it:

$$\text{Nu}_x = 0.0296\,\text{Re}_x^{0.8}\,\text{Pr}^{0.6}$$

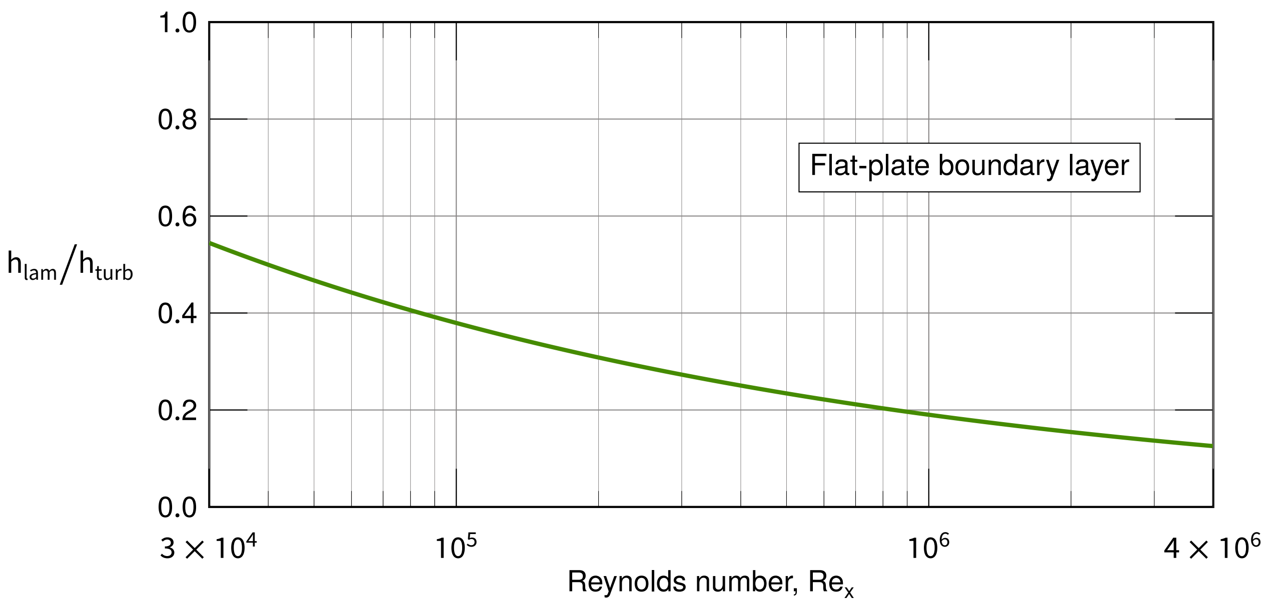
Then, with $\text{Nu}_x = hx/k$, we can simply divide the first equation by the second one:

$$\frac{h(x)_{\text{laminar}}}{h(x)_{\text{turbulent}}} = \frac{0.332\,\text{Re}_x^{1/2}\,\text{Pr}^{1/3}}{0.0296\,\text{Re}_x^{0.8}\,\text{Pr}^{0.6}} = \frac{10.87}{\text{Re}_x^{0.3}\,\text{Pr}^{0.267}}$$

A specific temperature was not stated, but from Table A.6 we see that air has $\text{Pr} \simeq 0.70$ over a very wide range of temperature. So, we may simplify

$$\frac{h(x)_{\text{laminar}}}{h(x)_{\text{turbulent}}} = \frac{10.87}{\text{Re}_x^{0.3}\,0.70^{0.267}} = \frac{12.0}{\text{Re}_x^{0.3}}$$

A boundary layer may be either laminar or turbulent for $3 \times 10^4 \lesssim \text{Re}_x \lesssim 4 \times 10^6$ (see pg. 276 or Fig. 6.4). The corresponding plot is given below.



The figure shows that h_{laminar} is always substantially less than $h_{\text{turbulent}}$. To raise the heat transfer rate from a flat surface, we should aim to cause turbulent transition as early as possible (note, however, that turbulent drag may be increased by an earlier transition).

Comment: As discussed in Sect. 6.9, the transition region between laminar and turbulent flow may be as long as the laminar region itself (see Fig. 6.21). In the transition region, $h(x)$ lies between the lower bound of $h(x)_{\text{laminar}}$ and the upper bound of $h(x)_{\text{turbulent}}$.

6.23 Water at 7°C flows at 0.38 m/s across the top of a 0.207m long, thin copper plate. Methanol at 87°C flows across the bottom of the same plate, at the same speed, but in the opposite direction. Make the obvious first-guess as to the temperature at which to evaluate physical properties. Then plot the plate temperature as a function of position. Do not bother to correct the physical properties in this problem but note Problem 6.24.

We shall first guess that the plate is at the mean temperature of $(7+87)/2 = 47°C$. Evaluate meth. props. at $(47+87)/2 = 67°C$ -- water props. at $(7+47)/2 = 27°C$:

$$Re_{meth_{max.}} = \frac{0.38(0.207)}{0.44(10)^{-6}} = 178{,}773 \;, \quad Re_{H_2O_{max.}} = \frac{0.38(0.207)}{0.826(10)^{-6}} = 95{,}230$$

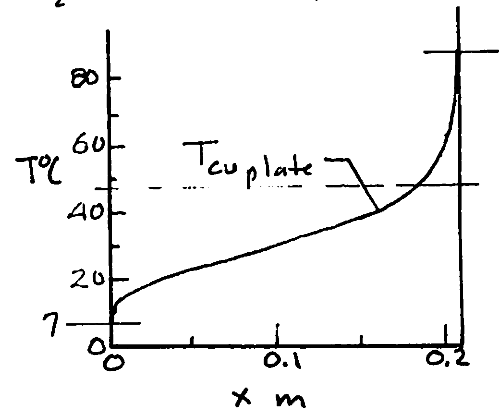
Both are laminar. Use $h = 0.332\,k\,Pr^{1/3}\,Re_x^{1/2}/x$, so

$$h_{meth} = 0.332(0.1908)(4.9)^{1/3}(0.38/0.44(10)^{-6})^{1/2}/\sqrt{x} = \frac{100}{\sqrt{x\,m}}$$

$$h_{H_2O} = 0.332(0.6084)(5.65)^{1/3}(0.38/0.826(10)^{-6})^{1/2}/\sqrt{0.207-x} = \frac{244}{\sqrt{.207-x}}$$

$$U = 1\Big/\left[\frac{1}{h_{meth}} + \frac{1}{h_{H_2O}}\right] \quad \text{and} \quad U(T_{meth} - T_{H_2O}) = h_{meth}(T_{meth} - T_{cu})$$

x m	h_{meth} W/m²-°C	h_{H_2O} W/m²-°C	U W/m²-°C	$T_{cu} = 87 - \frac{U(87-7)}{h_{meth}}$
0	220	∞	220	7
0.05	252	1091	205	22
0.10	305	772	219	30
0.15	419	630	252	39
0.20	1195	546	375	62
0.207	∞	536	536	87



6.24 Work Problem 6.23 taking full account of property variations.

To do this, we regard the previous solution as a first iteration. Now we rework the problem using the plate temperatures above.

x m	T_{plate} °C	$T_{film} = \frac{T_{flow} - T_{cu}}{2}$ meth	H_2O	$\nu\ 10^{-6}\ \frac{m^2}{s}$ meth	H_2O	k W/m-°C meth	H_2O	$Pr^{1/3}$ meth	H_2O	h W/m²-°C meth	H_2O	U $\frac{W}{m^2\text{-}°C}$	T_{cu} °C
0	7	47	7	0.6	1.922	0.1965	0.5818	1.847	2.173	211	∞	211	7
0.05	22	54.5	14.5	0.54	1.20	0.1944	0.5918	1.794	2.043	245	1010	197	22.7
0.10	30	58.5	18.5	0.51	1.08	0.1932	0.5911	1.765	1.967	299	724	212	30.3
0.15	39	63.0	23	0.47	0.95	0.1919	0.6031	1.730	1.873	415	612	247	39.4
0.20	62	74.5	34.5	0.90	0.729	0.1867	0.6190	1.672	1.700	1207	564	384	61.5
0.207	87	87	47	0.36	0.566	0.1851	0.6367	1.626	1.542	∞	587	587	87

No temperature changed more than 0.7°C. We can terminate the calculation. Furthermore, the plot above will stand. No point on it will move by more than a pencil-width.

Better property data have become available since we first worked this problem. Consequently, the numbers will change a bit. But the solution remains essentially correct.

6.25 If in Example 6.6 (with a constant q_w = 420 W/m²) the wall temperature were instead held constant at its average value of 76°C, what would the average wall heat flux be?

$$\overline{Nu_L} = 0.664\, Re_L^{1/2} Pr^{1/3} = 0.664\sqrt{\frac{1.8(0.6)}{1.747(10)^{-5}}}\; 0.712^{1/3} = 147.4$$

where we have evaluated air properties at $(76+15)/2 = 45.5$°C.

Then: $$\overline{h} = \overline{Nu_L}\,\frac{k}{L} = 147.4\,\frac{(0.02482)}{0.6} = 6.10\ \frac{W}{m^2\text{-}°C}$$

$$\overline{q}_w = \overline{h}\,\Delta T = 6.10(76-15) = \underline{\underline{372\ W/m^2}}$$

Problem 6.26 In Sect. 6.4, we noted that the kinetic theory of gases predicts values of Pr ranging from 2/3 for monatomic ideal gases and 1 for complex molecules. Show how this is borne out for gases at 400 K, using Table A.6 in Appendix A.

Solution

Using values given in Table A.6 of Appendix A we find the following.

a) For monatomic gases, He and Ar, $\mathrm{Pr} = 0.663$ and 0.666. These values are in near perfect agreement with the predicted value of 2/3.
b) For diatomic gases: H_2, N_2, O_2 and CO, $\mathrm{Pr} = 0.690, 0.711, 0.734,$ and 0.692. These results are within a few percent of the predicted value: 5/7 or 0.714.
c) For more complex molecules Pr should begin approaching 1. For molecules made up of three atoms, H_2O and CO_2, $\mathrm{Pr} = 0.982$ and 0.738, respectively. Water is close to 1, but carbon dioxide does not depart very far from the value for a diatomic molecule. Ammonia, NH_3, which is made from four atoms, yields $\mathrm{Pr} = 0.858$, which is well on its way to 1.

The simple kinetic theory is thus quite accurate for these monatomic gases, and fairly accurate for these diatomic gases. After that, kinetic theory merely suggests the correct trend toward 1.

Comment: If we moved ahead to Chapter 11, we would find the Eucken equation (11.127), $\mathrm{Pr} = 4\gamma/(9\gamma - 5)$ where γ is the ratio of specific heats. Thus, for N_2 and O_2, $\gamma = 1.4$, and Pr should be 0.737. This value is very close to the actual values above. The ratio γ is 1.31 for ammonia, so Pr would be 0.771, which is low. Eucken's formula, while it is an improvement, is likewise valid for very simple and for very complex molecules [6.7]. But it is only approximate in between. (See Problem 11.20.)

6.27 A two foot square slab of mild steel leaves a forging operation 0.25 in. thick at 1000°C. It is laid flat on an insulating bed and 27°C air is blown over it at 30 m/s. How long will it take to cool to 200°C. Assume the flow is laminar and state your assumptions about property evaluation.

Compute $\overline{h}$ based on air at $\frac{1}{2}\left[\frac{1000+27}{2}+\frac{200+27}{2}\right] = 313.5\,°C$.
(This is pretty coarse. The value of ν drops by a factor of four over the range of film temperature involved.) Then:

$\nu = 4.96\times10^{-5}$, $Pr = 0.698$, $k = 0.0448$

And $\rho c_{steel} = 3.64(10)^6\ J/m^3\text{-}°C$, $k_{steel} = 35$ at 600°C

$$Re_L = \frac{2(0.3048)30}{4.96\times10^{-5}} = 368{,}710$$

$$\overline{Nu}_L = 0.664\,Re_L^{1/2}\,Pr^{1/3} = 357.7\ ;\ \overline{h} = 357.7\,\frac{0.0448}{2(0.3048)} = \underline{26.3\ \frac{W}{m^2\text{-}°C}}$$

Can we use lumped capacity? Check $Bi = \frac{\overline{h}L}{k_s} = \frac{26.3(0.0254\times1/4)}{35} = 0.048 \ll 1$

Lumped capacity is fine. Then: $T = \frac{\rho c\ \text{thickess}}{\overline{h}} = \frac{3.64(10)^6(0.0254/4)}{26.3} = \underline{879\ \text{sec.}}$

Finally:

$$\frac{T-T_\infty}{T_i-T_\infty} = e^{-t/T} = \frac{200-27}{1000-27} = 0.1778 = e^{-t/879}$$

So: $t = 1{,}519$ sec $= \underline{\underline{25.3\ \text{minutes}}}$

This is a long time. Air is an ineffective coolant, 6.28 Do Problem 6.27 numerically, recalculating properties at successive points. If you did Problem 6.27, compare results.

First compute $\bar{h}$ at $T_{plate} = 1000°C, 900°C$, etc.

$$\overline{Nu_L} = 0.664\, Re_L^{1/2} P_r^{1/3}$$

$$\bar{h} = 0.664 \left[\frac{\sqrt{30}}{\sqrt{0.6096}}\right] \frac{P_r^{1/3}}{\nu^{1/2}} k$$

$$\bar{h} = 4.658\, P_r^{1/3} k / \nu^{1/2}$$

T_{plate}	$\frac{1}{2}(T_p+27)+273$	$\nu\ \frac{m^2}{s}$	P_r	$k\ \frac{W}{m°C}$	$\bar{h}\ W/m^2\text{-}°C$
1000°C	786.5°	$8.04(10)^{-5}$	0.704	0.05614	25.95
900	736.5	7.22 "	0.703	0.05334	26.00
800	686.5	6.43 "	0.702	0.05052	26.08
700	636.5	5.68 "	0.700	0.04764	26.15
600	586.5	4.96 "	0.698	0.0448	26.28
500	536.5	4.28 "	0.698	0.04177	26.38
400	486.5	3.63 "	0.699	0.03865	26.52
300	436.5	3.02 "	0.702	0.0359	26.67

Now compute $\mathbf{T} = \rho c$ thickness$/\bar{h} = 23{,}114$ & use $\frac{T-27}{1000-27} = e^{-t/\mathbf{T}}$ to advance 100°C per step. Thus

from previous T to	$\mathbf{T}$ sec	$\frac{T-27}{\text{Prior } T-27}$	$t = -\mathbf{T} \ln \frac{T-27}{T+73}$	t_{total} sec
T = 900°C	891	0.897	96.6 sec	96.6
800	889	0.8855	108.2	204.8
700	886	0.8706	122.8	327.6
600	884	0.8514	142.2	469.8
500	880	0.8255	168.7	638.5
400	876	0.7886	208.1	846.6
300	872	0.7319	272.2	1118.8
200	867	0.6337	395.5	1514.3

We conclude that, since this is within 4.7 sec or 0.31 percent of the approximate result, the averaging that was used in Problem 6.27 is quite good in this case. The high variation of ν is compensated in its influence on $\bar{h}$ by the variation of k. Furthermore the initial under-estimate of $\mathbf{T}$ is compensated by the subsequent over-estimate of $\mathbf{T}$.

Problem 6.29 Plot q_w against x for the situation described in Example 6.9. (If you have already worked Problem 6.18, this calculation will be short.)

Solution

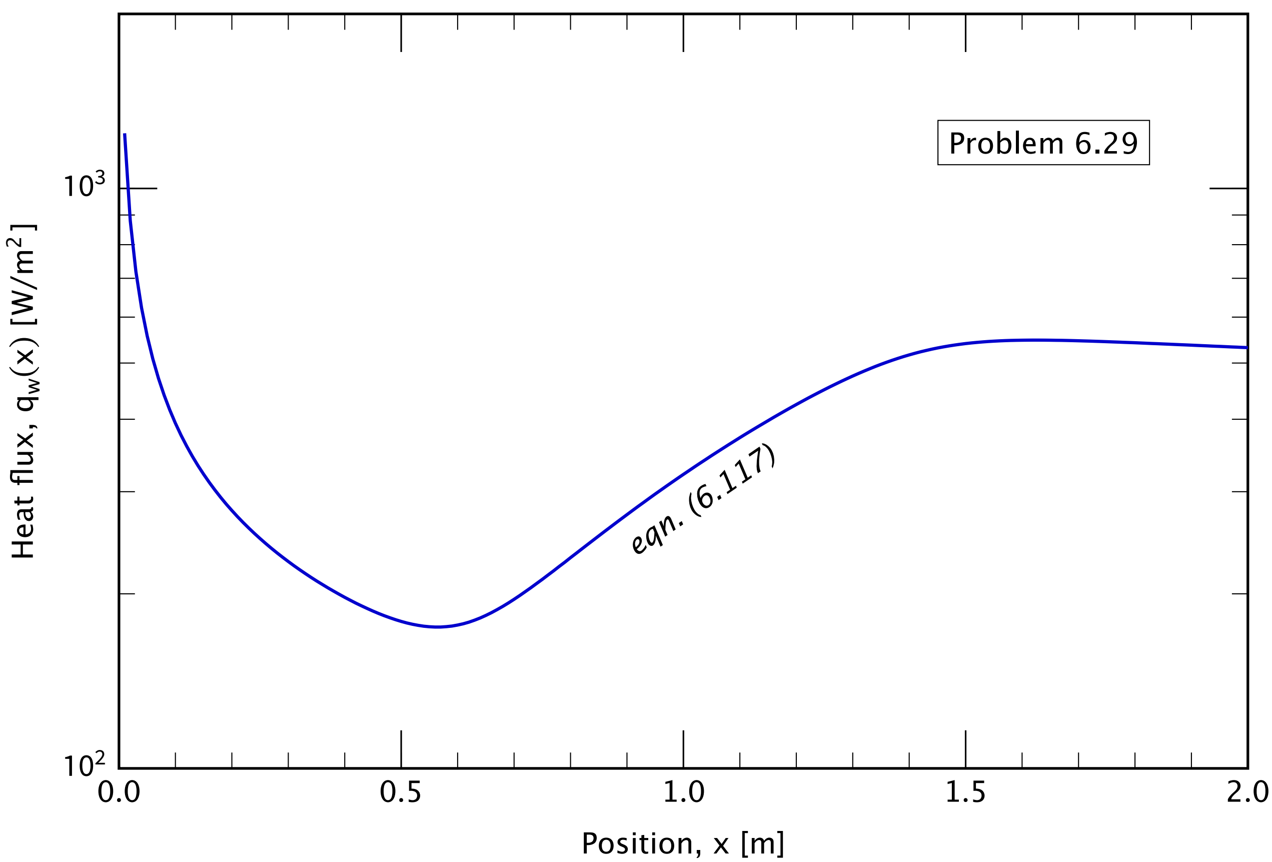
The solution to Problem 6.18 shows that the heat transfer coefficient is given by eqn. (6.117), which may be written in terms of h in W/m^{2}K and x in meters:

$$h(x) = (0.0264/x)\left[\left(235.8\, x^{1/2}\right)^5 + \left[\left(607.7\, x^{2.55}\right)^{-10} + \left(1157\, x^{0.8}\right)^{-10}\right]^{-1/2}\right]^{1/5}$$

The wall heat flux for an isothermal wall is $q_w(x) = h(x)(T_w - T_\infty)$. Here, $(T_w - T_\infty) = (310 - 290) = 20$ K. Hence:

$$q_w(x) = (20)(0.0264/x)\left[\left(235.8\, x^{1/2}\right)^5 + \left[\left(607.7\, x^{2.55}\right)^{-10} + \left(1157\, x^{0.8}\right)^{-10}\right]^{-1/2}\right]^{1/5}$$

The plot follows the considerations in the solution of Problem 6.18.



Problem 6.30 Consider the plate in Example 6.9. Suppose that instead of specifying $T_w = 310$ K, we specified $q_w = 500$ W/m^2. Plot T_w against x for this case.

Solution The solution to Problem 6.18 shows that the heat transfer coefficient is given by eqn. (6.117). In Problem 6.18, the wall was isothermal, with

$$\mathrm{Nu}_{x,\mathrm{lam}} = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3} \tag{6.58}$$

which led to this expression for h in W/m^{2}K and x in meters:

$$h(x)_{\mathrm{isothermal}} = (0.0264/x)\left[\left(235.8\,x^{1/2}\right)^5 + \left[\left(607.7\,x^{2.55}\right)^{-10} + \left(1157\,x^{0.8}\right)^{-10}\right]^{-1/2}\right]^{1/5}$$

The physical properties used in that calculation were based on a film temperature of 300 K, which must check after we have the wall temperature.

In the present problem, the wall is uniform flux with

$$\mathrm{Nu}_{x,\mathrm{lam}} = 0.4587\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3} \tag{6.71}$$

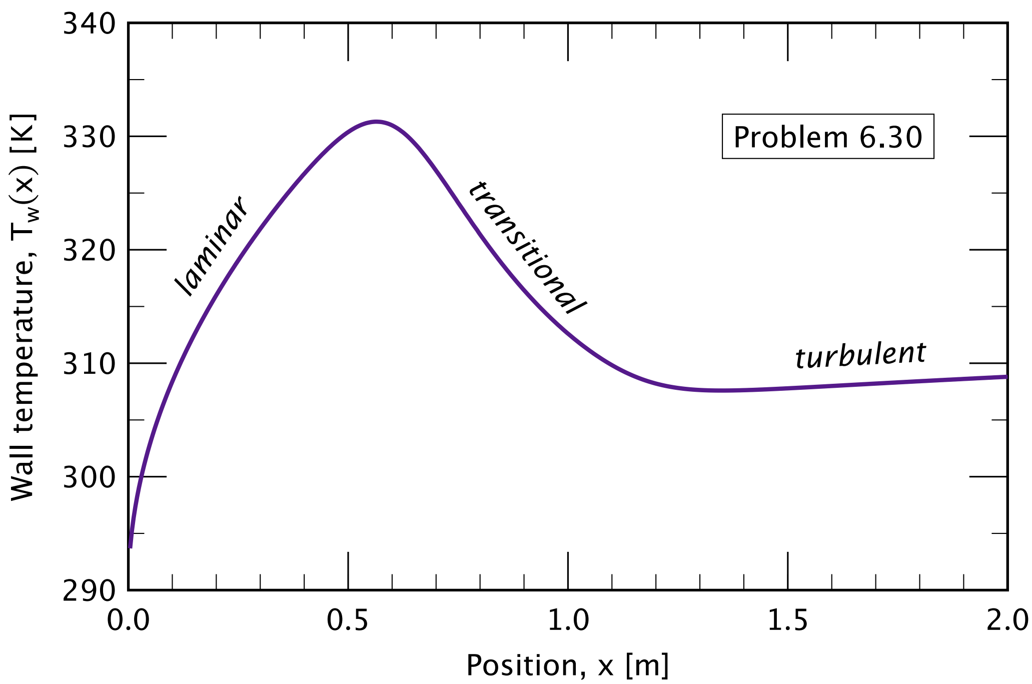
As a result, we must adjust the constants used in the laminar *and transitional* portions of the equation (in red) by the ratio $0.4587/0.332 = 1.38$:

$$h(x)_{\mathrm{uniform\ flux}} = (0.0264/x)\left[\left((1.38)235.8\,x^{1/2}\right)^5 + \left[\left((1.38)607.7\,x^{2.55}\right)^{-10} + \left(1157\,x^{0.8}\right)^{-10}\right]^{-1/2}\right]^{1/5}$$

The heat flux temperature relationship is $q_w = h(x)[T_w(x) - T_\infty]$. Here, $T_\infty = 290$ K and $q_w = 500$ W/m^2. Hence:

$$T_w(x) = T_\infty + q_w\big/h(x)_{\mathrm{uniform\ flux}}$$

The rest is left to software, with the plot below. The plate is hottest where the laminar b.l. is thickest.



The local film temperature is still about 300 K for the right-hand part of the plate. On the left, the local film temperature rises to as much as 310 K; but air's properties change very little from 300 K to 310 K. We conclude that the properties do not need to be updated.

6.31 A thin metal sheet separates air at 44°C, flowing at 48 m/s, from water at 4°C, flowing at 0.2 m/s. Both fluids start at a leading edge and move in the same direction. Plot T_{plate} and q as a function of x up to x = 0.1 m.

Make first calculation evaluating properties on the basis of $T_{plate} = 10°C$ (so $\overline{T}_{air} = 27°C$ and $\overline{T}_{H_2O} = 7°C$) Then:

$\nu_{air} = 1.566(10)^{-5}$ $\quad k_{air} = 0.02614$ $\quad Pr_{air} = 0.711$

$\nu_{H_2O} = 1.422(10)^{-6}$ $\quad k_{H_2O} = 0.6084$ $\quad Pr_{H_2O} = 10.26$

so: $Re_{air} = \frac{u_\infty}{\nu} x = \frac{48\ x}{1.566 \times 10^{-5}} = 3.066 \times 10^6 x \quad ; \quad Re_{H_2O} = \frac{0.2\ x}{1.422 \times 10^{-6}} = 140{,}650x$

Thus the flows should be laminar: $h = \frac{k}{x} 0.322\, Re_x^{1/2}\, Pr^{1/3}$.

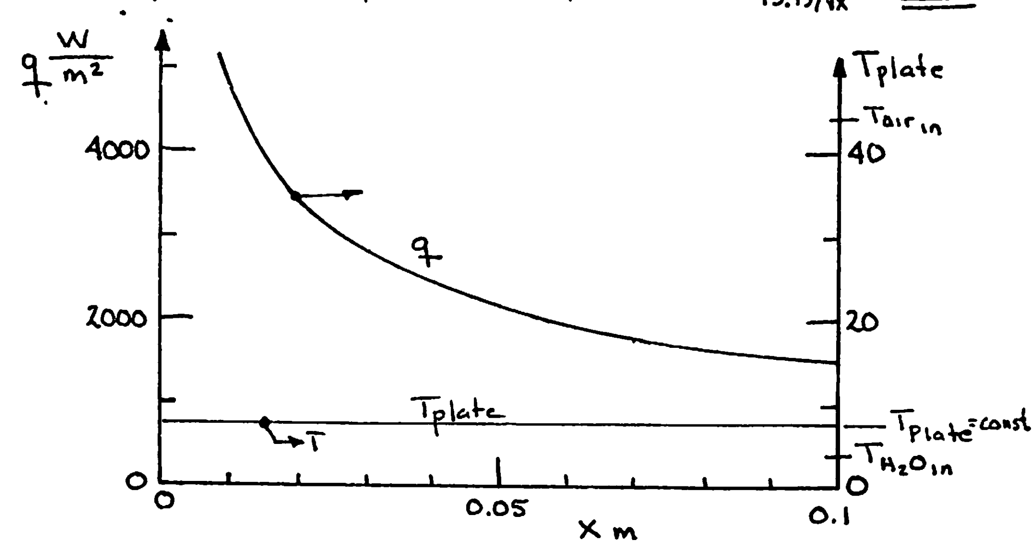
$$h_{air} = \frac{0.02614}{x} 0.322 \left[3.066(10)^6 x\right]^{1/2} 0.711^{1/3} = \underline{13.15/\sqrt{x}}$$

$$h_{H_2O} = \frac{0.6084}{x} 0.322 \left[140{,}650x\right]^{1/2} 10.26^{1/3} = \underline{160.0/\sqrt{x}}$$

Then: $$U = \frac{1}{\frac{1}{h_{air}} + \frac{1}{h_{H_2O}}} = \frac{1}{\frac{1}{13.15} + \frac{1}{160}} \left(\frac{1}{\sqrt{x}}\right) = \underline{12.15/\sqrt{x}}$$

$$q = U\Delta T = \frac{12.15}{\sqrt{x}}(44-4) = \underline{\underline{486/\sqrt{x}}}$$

and since $q = h_{air}(44 - T_{plate})$ so $T_{plate} = 44 - \frac{486/\sqrt{x}}{13.15/\sqrt{x}} = \underline{\underline{7.04°C}}$



6.32 A mixture of 60% glycerin and 40% water flows over a 1m long flat plate. The glycerin is at 20°C and the plate is at 40°. A thermocouple, 1mm above the trailing edge records 35°C. What is u_∞, and what u at the thermocouple?

Using the momentum integral result (eqn. 6.50) we have:

$$\frac{35-20}{40-20} = 1 - \frac{3}{2}\frac{y}{\delta_t} + \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3$$

from which we obtain, by trial and error, $\frac{y}{\delta_t} = 0.1678$

Thus, at $y = 0.001\,m$, $\delta_t = 0.00596\,m$.

And (eqn. 7.55) $\delta = P_r^{1/3}\delta_t = 49.3^{1/3}(0.00596) = 0.02185\,m$

Finally from eqn. (7.2) $\frac{\delta}{x} = \frac{4.92}{Re_x^{1/2}}$; $\delta = \frac{4.92(1)}{\sqrt{\frac{u_\infty}{6.89(10)^{-6}}}} = 0.02185\,m$

so $\underline{\underline{u_\infty = 0.349\ m/s}}$

(Is the flow really laminar? Check it: $Re_{x=L} = \frac{0.349(1)}{6.89(10)^{-6}} = 50{,}702$. Yes)

Then, at the thermocouple: $\frac{y}{\delta} = 0.0458$, so $\frac{u}{u_\infty} = \frac{3}{2}(0.0458) - \frac{1}{2}(0.0458)^3$

$= 0.0686$, $\underline{\underline{u = 0.0239\ m/s}}$

Thus the thermocouple is fairly deeply into the thermal b.l., and very deeply into the flow b.l.. Note that slightly greater accuracy would have resulted from the consistent use of Table 6.1 in place of the integral method approximations.

6.33 What is the maximum $\bar{h}$ that can be achieved in laminar flow over a 5m plate, based on data from Table A.3? What physical circumstances give this result?

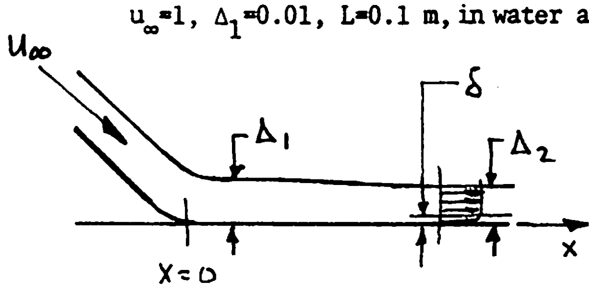
eqn. (6.68) $\bar{h} = 0.664\, k\, Pr^{1/3} \sqrt{\dfrac{u_\infty}{L\nu}}$, but $\dfrac{u_\infty L}{\nu} \leq 3.5(10)^5$ so:

$$\bar{h}_{max} = 0.664\, k\, Pr^{1/3} \sqrt{3.5(10)^5/5^2} = 78.56 k\, Pr^{1/3}$$

The largest $k Pr^{1/3} = 12.3$, for glycerin at 0°C (a viscous oil might will be better if we had k data.) The gives: $\underline{\underline{h_{max} = 966 \text{ W/m}^2\text{-°C}}}$

This corresponds with $u_\infty = 3.5(10)^5(0.0083)/5 = 581$ m/s. This looks high for a real system. It would require a Herculean pump.

6.34 A 17°C sheet of water, Δ_1m thick and moving at a constant speed u_∞m/s, impacts a horizontal plate at 45°, turns, and flows along it. Develop a dimensionless equation for the thickness Δ_2 at a distance L from the point of impact. Assume that $\delta \ll \Delta_2$. Evaluate the result for u_∞=1, Δ_1=0.01, L=0.1 m, in water at 27°C.



Mass balance:

$$\rho u_\infty \Delta_1 = \rho \int_0^\delta u\, dy + \rho \int_\delta^{\Delta_2} u_\infty dy$$

$$1 = \frac{\delta}{\Delta_1} \underbrace{\int_0^1 \left(\frac{3}{2}\xi - \frac{1}{2}\xi^3\right) d\xi}_{\frac{3}{4} - \frac{1}{8} = \frac{5}{8}} + \frac{\Delta_2}{\Delta_1} - \frac{\delta}{\Delta_1}$$

so: $\underline{\dfrac{\Delta_2}{\Delta_1} = 1 + \dfrac{3}{8}\dfrac{\delta}{\Delta_1}}$

The momentum eqn. is already summarized in: $\dfrac{\delta}{L} = \dfrac{4.92}{\sqrt{Re_L}}$

$$\therefore \quad \underline{\underline{\frac{\Delta_2}{\Delta_1} = 1 + \frac{3}{8} \frac{4.92}{\sqrt{Re_L}} \frac{L}{\Delta_1} = 1 + \frac{1.845}{\sqrt{Re_L}} \frac{L}{\Delta_1}}}$$

In the case in point: $Re_L = \dfrac{0.1 \times 1}{0.826(10)^{-6}} = 121{,}065$ so

$$\frac{\Delta_2}{\Delta_1} = 1 + \frac{1.845}{\sqrt{121{,}065}} 10 = \underline{\underline{1.053}}$$

(at this point $\delta = 0.001414$ m which is $\ll \Delta_2$) Thus, contrary to the sketch above, the sheet swells to accommodate the reduced speed near the wall.)

6.35 A good approximation to the temperature dependence of μ in gases is given by the Sutherland formula: $\mu/\mu_{ref} = \left(\frac{T}{T_{ref}}\right)^{1.5} \frac{T_{ref}+S}{T+S}$, where the reference state can be chosen anywhere. Use data for air at two points to evaluate S for air. Use this value to predict a third point. (T and T_{ref} are expressed in °K.)

Students might use any points from Table A-6. Let us do the problem for air using a value of T_{ref} = 300°K and a temperature, T, of interest equal to 500°K.

First we calculate S based on the known values of

$$\mu(T = T_{ref} = 300^{\circ}K) = 1.857(10)^{-5} \text{ kg/m-s}$$

$$\mu(T = 400^{\circ}K) = 2.310(10)^{-5} \text{ kg/m-s}$$

Using these values in Sutherland's formula, we get

$$\underline{\underline{S = 120.7^{\circ}K}}$$

Then, using this S and $\mu_{ref}(T_{ref})$ in Sutherland's formula, we obtain:

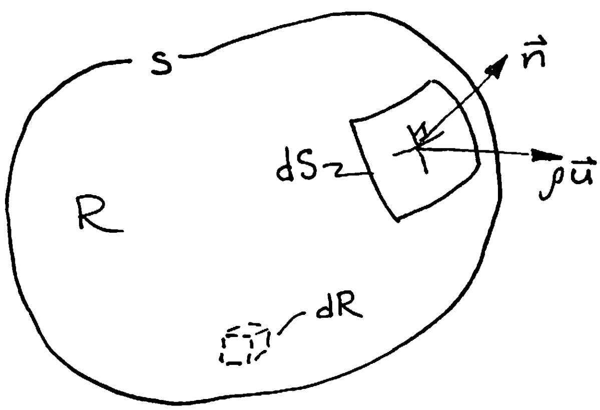
$$\mu(T = 500^{\circ}K) = \underline{\underline{2.71\,(10)^{-5} \text{ kg/m-s}}}$$

which is exactly the tabled value to three decimal places.

6.36 We have derived a steady-state continuity equation in Section 6.3. Derive the time-dependent three-dimensional version of the equation:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0$$

To do this, paraphrase the development of equation (2.14) requiring that mass be conserved instead of energy.



the mass into the control volume

$$= \frac{d}{dt}(\text{rate of mass storage})$$

or

$$-\int_S (\rho\vec{u})\cdot\vec{n}\,dS = \frac{d}{dt}\int_R \rho\,dR$$

Now we use Gauss' theorem, $\int_S (\rho\vec{u})\cdot\vec{n}\,dS = \int_R \vec{\nabla}\cdot\rho\vec{u}\,dR$ to get:

$$\int_R \left(\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\rho\vec{u}\right)dR = 0$$

(where we have used Leibnitz' rule as we did in the context of eqn. (6.24.)

Finally, since the integral must vanish identically, we obtain:

$$\underline{\underline{\frac{\partial}{\partial t}\rho + \vec{\nabla}\cdot\rho\vec{u} = 0}}$$

Notice that, since $\vec{\nabla}\cdot\rho\vec{u} = \rho\vec{\nabla}\cdot\vec{u} + \vec{u}\cdot\vec{\nabla}\rho$, this can be rewritten as:

$$\underline{\frac{D\rho}{Dt} + \rho\vec{\nabla}\cdot u = 0}$$

where $\frac{D}{Dt}$ is the substantial derivative, $\left(\frac{\partial \rho}{\partial t} + \vec{u}\cdot\vec{\nabla}\rho\right)$.

6.37 Various considerations show that the smallest scale motions in a turbulent flow have no prefered spatial orientation at large enough values of Re. Moreover, these small eddies are responsible for most of the viscous dissipation of kinetic energy. The dissipation rate, ϵ (W/kg), may be regarded as given information about the small scale motion, since it is set by the larger scale motion. Both ϵ and ν are governing parameters of the small scale motion.

a.) Find the characteristic length and velocity scales of the small scale motion. These are called the <u>Kolmogorov scales</u> of the flow.

b.) Compute Re for the <u>small scale</u> motion, and interpret the result.

c.) The Kolmogorov length scale characterizes the smallest motions found in a turbulent flow. If ϵ is 10 W/kg and the mean free path is $7(10)^{-6}$ m, for air at standard conditions, show that turbulent motion is a continuum phenomenon and it is thus properly governed by the equations of this chapter.

a.) Length & velocity scales, η & v, can be formed from $\epsilon \ \frac{W}{kg} = \epsilon \ \frac{m^2}{s^3}$ and $\nu \ \frac{m^2}{s}$. We get:

$$\text{length scale, } \eta = \left(\frac{\nu^3}{\epsilon}\right)^{1/4}$$

$$\text{velocity scale, } v = (\nu\epsilon)^{1/4}$$

b.) $Re = \dfrac{v\eta}{\nu} = \dfrac{1}{\nu}\left(\dfrac{\nu^3}{\epsilon}\right)^{1/4}(\nu\epsilon)^{1/4} = \underline{\underline{1}}$ since viscosity balances inertia, the small scales are extremely viscous.

c.) For air at 300°K, $\nu = 1.566(10)^{-5}$ m²/s. we get

$$\eta = \left(1.566\times10^{-5}\right)^{3/4}/10^{1/4} = 0.00014 \text{ m}$$

This is far larger than the mean free path. Therefore turbulent motion is a continuum phenomenon

Problem 6.38 The temperature outside is 35°F, but with the wind chill it's ~~≥15°F~~ 24°F. And you forgot your hat. If you go outdoors for long, are you in danger of freezing your ears? Why or why not?

Solution

The air temperature outside is 35°F. In accordance with the Second Law of Thermodynamics, heat cannot flow from your face to any temperature less than that. Hence:

The answer to the question is No, one's ears could never freeze. $\xleftarrow{\text{Answer}}$

What then does 24°F mean? The heat transfer from our face to the surroundings increases greatly when wind (forced convection) increases the heat transfer coefficients around our face relative to the heat transfer coefficients is still air (natural convection and radiation). In a sufficient wind, our face cools at the same rate it would in still air at 24°F, and so our face feels far colder than it would in still air. For that reason, the news announcers often report the wind chill temperature as the "feels like" temperature.

As a matter of interest, here is the National Weather Service's *Wind Chill Chart*:

www.weather.gov/safety/cold-wind-chill-chart

From this chart, we find that a wind of 19 mph would cause a wind chill of 24°F at an air temperature of 35°F.

PROBLEM 6.39 To heat the airflow in a wind tunnel, an experimenter uses an array of electrically heated, Nichrome V strips. Each strip is 20 cm by 2.5 cm and very thin. They are stretched across the flow with the thin edge facing into the wind. The air flows along both ~~2.54 cm~~ sides. The strips are spaced vertically, each 1 cm above the next. Air at 1 atm and 20°C ~~passes over them~~ enters the array of strips at 10 m/s.

a) How much power must each strip deliver to raise the mean temperature of the airstream to 30°C?
b) What is the heat flux if the electrical dissipation in the strips is uniform?
c) What are the average and maximum temperatures of the strips?

SOLUTION

a) We can consider the airflow around one strip. At 20°C (293 K), the density of air is $\rho = 1.206$ kg/m^3 and the heat capacity is $c_p = 1006$ J/kg-K from Table A.6.

Each strip heats half of the channels between it and the next strip above and the next strip below, in other words 0.5 cm above it and 0.5 cm below it. An energy balance on the cross-sectional area A_c, accounting for both sides of the strip, gives

$$\text{Electrical power dissipation in each strip, } P = \dot{m} c_p \Delta T$$

The mass flow rate for air heated by one strip, $\dot{m}$

$$\dot{m} = \rho u_\infty A_c = (1.206)(10)(0.2)(0.01) = 0.0241 \text{ kg/m}^3$$

and, in steady flow, this rate is constant as the air passes over the strip, even if its density drops (the speed increases in proportion; see Sect. 7.2). Then

$$\begin{aligned} P &= \dot{m} c_p \Delta T \\ &= (0.0241)(1006)(30-20) \\ &= 242.6 \text{ W} \quad \xleftarrow{\text{Answer}} \end{aligned}$$

b) The power is provided by heat leaving both sides of the strip. The area of one side is 20 cm by 2.5 cm, so

$$2(0.20)(0.025)q = P = 242.6 \text{ W}$$

where q is the heat flux on one side of the strips. Then

$$q = \frac{242.6}{2(0.2)(0.025)} = 24.26 \text{ kW/m}^2 \quad \xleftarrow{\text{Answer}}$$

c) To find the average and maximum temperatures, we need the average heat transfer coefficient and the minimum heat transfer coefficient. The latter should occur at the end of the plate, where the boundary layer has grown thickest. For both of these calculations, we must determine Re_L.

We need the physical properties of air. Since we don't know the temperature of the strips, it's hard to precisely estimate the film temperature. Let's guess a value that we can read from Table A.6 without interpolation, and we can adjust once we have a better idea of the strip temperature, if necessary. Take $T_f = 310$ K. Then

$$\nu = 1.670 \times 10^{-5} \text{ m}^2\text{/s}, \quad k = 0.0271 \text{ W/m·K}, \quad \text{Pr} = 0.706$$

and the Reynolds number is

$$\mathrm{Re}_L = \frac{u_\infty L}{\nu} = \frac{(10)(0.025)}{1.670\times10^{-5}} = 1.50\times10^4$$

The Reynolds number is quite low and the flow is laminar over the whole strip. We can use eqns. (6.71) and (6.72), both with the Reynolds number at the end of the plate, Re_L:

$$\mathrm{Nu}_L = 0.4587\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3} = 0.4587(1.50\times10^4)^{1/2}(0.706)^{1/3} = 50.0 \tag{6.71}$$

$$\overline{\mathrm{Nu}}_L = 0.688\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3} = 0.688(1.50\times10^4)^{1/2}(0.706)^{1/3} = 75.0 \tag{6.72}$$

Then:

$$h(L) = \frac{k}{L}\mathrm{Nu}_L = \frac{0.0271}{0.025}(50.0) = 54.2\ \mathrm{W/m^2K}$$

$$\overline{h} = \frac{k}{L}\overline{\mathrm{Nu}}_L = \frac{0.0271}{0.025}(75.0) = 81.3\ \mathrm{W/m^2K}$$

The temperatures are:

$$T(L) = T_\infty + \frac{q}{h(L)} = 20 + \frac{24.26\times10^3}{54.2} = 468\ ^\circ\mathrm{C}$$

$$\overline{T} = T_\infty + \frac{q}{\overline{h}} = 20 + \frac{24.26\times10^3}{81.3} = 318\ ^\circ\mathrm{C}$$

The film temperature we guessed was too low. Let's use the average temperature just computed to find an updated, average film temperature:

$$T_f = \frac{\overline{T}+T_\infty}{2} = \frac{318+20}{2} = 169\ ^\circ\mathrm{C} = 442\ \mathrm{K}$$

and the properties are

$$\nu = 3.109\times10^{-5}\ \mathrm{m^2/s},\quad k = 0.0363\ \mathrm{W/m{\cdot}K},\quad \mathrm{Pr} = 0.698$$

Notice that the kinematic viscosity is much higher, but so is k. These changes affect h oppositely. Recalculating the Reynolds number, we get $\mathrm{Re}_L = 8057$. The Nusselt numbers are

$$\mathrm{Nu}_L = 0.4587(8057)^{1/2}(0.698)^{1/3} = 36.5$$
$$\overline{\mathrm{Nu}}_L = 0.688(8057)^{1/2}(0.698)^{1/3} = 54.8$$

and the heat transfer coefficients are

$$h(L) = \frac{k}{L}\mathrm{Nu}_L = \frac{0.0363}{0.025}(36.5) = 53.0\ \mathrm{W/m^2K}$$

$$\overline{h} = \frac{k}{L}\overline{\mathrm{Nu}}_L = \frac{0.0363}{0.025}(54.8) = 79.6\ \mathrm{W/m^2K}$$

These values are about 2% lower than the previous ones. The revised temperatures are:

$$T(L) = T_\infty + \frac{q}{h(L)} = 20 + \frac{24.26\times10^3}{53.0} = 478\ ^\circ\mathrm{C} \quad \xleftarrow{\text{Answer}}$$

$$\overline{T} = T_\infty + \frac{q}{\overline{h}} = 20 + \frac{24.26\times10^3}{79.6} = 325\ ^\circ\mathrm{C} \quad \xleftarrow{\text{Answer}}$$

Further iteration would shift the values only slightly.

Problem 6.40 An airflow sensor consists of a 5 cm long, heated copper slug that is smoothly embedded 10 cm from the leading edge of a flat plate. The overall length of the plate is 15 cm, and the width of the plate and the slug are both 10 cm. The slug is electrically heated by an internal heating element; but, owing to its high thermal conductivity, the slug has a nearly uniform temperature along its airside surface. The heater's controller adjusts its power to keep the slug surface at a fixed temperature. The air velocity is calculated from measurements of the slug temperature, the air temperature, and the heating power.

a) If the air is at 280 K, the slug is at 300 K, and the heater power is 5.0 W find the airspeed assuming the flow is laminar. *Hint:* For $x_1/x_0 = 1.5$, integration shows that

$$\int_{x_0}^{x_1} x^{-1/2}\left[1-(x_0/x)^{3/4}\right]^{-1/3} dx = 1.0035\,\sqrt{x_0}$$

b) Suppose that a disturbance trips the boundary layer near the leading edge, causing it to become turbulent over the whole plate. The air speed, air temperature, and the slug's set-point temperature remain the same. Make a very rough estimate of the heater power that the controller now delivers, without doing a lot of analysis.

Solution

a) This configuration has an unheated starting length, with $x_0 = 10$ cm. Based on the given information, the slug is isothermal at a temperature T_w that is measured. The electrical power dissipated in the slug, P, is also known and equals the heat transfer rate from the slug, Q. The flow speed (and Reynolds number) are unknown.

We may apply eqn. (6.64):

$$\mathrm{Nu}_x = \frac{0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1-(x_0/x)^{3/4}\right]^{1/3}} \tag{6.64}$$

The local heat flux, for $\Delta T = T_w - T_\infty$, is

$$q_w = \frac{k\Delta T}{x}\,\frac{0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1-(x_0/x)^{3/4}\right]^{1/3}} = \frac{0.332\,k\Delta T\,\mathrm{Pr}^{1/3}\sqrt{u_\infty/\nu}}{\sqrt{x}\left[1-(x_0/x)^{3/4}\right]^{1/3}}$$

The electrical power dissipated in the slug is the total transfer rate from the slug, which can be found by integration. Let $x_1 = 15$ cm be the position at the end of the slug, and let $w = 15$ cm be the width of the plate:

$$\begin{aligned} P = Q &= w\int_{x_0}^{x_1} q_w\,dx \\ &= 0.332\,wk\Delta T\,\mathrm{Pr}^{1/3}\sqrt{u_\infty/\nu}\;\underbrace{\int_{x_0}^{x_1} x^{-1/2}\left[1-(x_0/x)^{3/4}\right]^{-1/3} dx}_{1.0035\sqrt{x_0}} \\ &= 0.333\,wk\Delta T\,\mathrm{Pr}^{1/3}\sqrt{u_\infty x_0/\nu} \qquad (*) \end{aligned}$$

where the integral was given in the problem statement (or see ***Comment*** below).

For the case given, we may take a film temperature $T_f = (280 + 300)/2 = 290$ K, and with Table A.6 the properties of air are

$$\nu = 1.482 \times 10^{-5}\ \mathrm{m^2/s}, \quad k = 0.0256\ \mathrm{W/m{\cdot}K}, \quad \mathrm{Pr} = 0.707$$

From eqn. (*):

$$\begin{aligned} u_\infty &= \frac{\nu P^2}{x_0\big(0.333\, wk\Delta T\,\mathrm{Pr}^{1/3}\big)^2} \\ &= \frac{(1.482 \times 10^{-5})(25)}{(0.10)(0.15)^2(0.333)^2(0.0256)^2(20)^2(0.707)^{2/3}} \\ &= 7.11\ \mathrm{m/s} \quad \xleftarrow{\text{Answer}} \end{aligned}$$

We can that for laminar flow by looking at the Reynolds number at the plate's trailing edge:

$$\mathrm{Re}_{x_1} = \frac{(7.11)(0.15)}{(1.482 \times 10^{-5})} = 7.2 \times 10^4$$

which normally corresponds to laminar flow.

b) To make a rough estimate, we ignore the unheated starting length. This approach is more acceptable for a turbulent boundary layer than a laminar boundary layer because turbulent mixing tends to erase the upstream history of the temperature distribution (recall from Sects. 6.7 and 6.8 that a turbulent boundary layer is not very sensitive to upstream conditions).

For turbulent flow on an isothermal plate, we have the local heat transfer coefficient from either eqn. (6.111) or (because this fluid is air) eqn. (6.112):

$$\mathrm{Nu}_x = 0.0296\ \mathrm{Re}_x^{0.8}\mathrm{Pr}^{0.6}$$

We may find h with eqn. (6.112) and integrate it from x_0 to x_1 as before:

$$\begin{aligned} P = Q &= w\int_{x_0}^{x_1} h(x)\Delta T\, dx \\ &= 0.0296\ wk\Delta T\,\mathrm{Pr}^{0.6}(u_\infty/\nu)^{0.8}\int_{x_0}^{x_1} x^{-0.2}\, dx \\ &= \underbrace{\frac{0.0296}{0.8}}_{=0.037}\ wk\Delta T\,\mathrm{Pr}^{0.6}(u_\infty/\nu)^{0.8}\big(x_1^{0.8} - x_0^{0.8}\big) \\ &= (0.037)(0.15)(0.0256)(20)(16/(1.482 \times 10^{-5})^{0.8}\big[(0.15)^{0.8} - (0.1)^{0.8}\big] \\ &= 11.6\ \mathrm{W} \quad \xleftarrow{\text{Answer}} \end{aligned}$$

Comment: The integral in Part (a) can evaluated by putting $s = x/x_0$. Call the integral I:

$$I = \sqrt{x_o}\int_1^{x_1/x_0} s^{-1/2}\big(1 - s^{-3/4}\big)^{-1/3}\, ds = \sqrt{x_o}\int_1^{x_1/x_0} s^{-1/4}\big(s^{3/4} - 1\big)^{-1/3}\, ds$$

We can use the substitution $u = (s^{3/4} - 1)$ to simplify further and integrate directly. We leave it to the reader to check that

$$I = \sqrt{x_o}\left[2\sqrt{s}\big(1 - s^{-3/4}\big)^{2/3}\right]\Bigg|_1^{1.5} = 2\sqrt{1.5}\big(1 - (1.5)^{-3/4}\big)^{2/3}\sqrt{x_o} \doteq 1.003485\sqrt{x_o}$$

Problem 6.41 Equation (6.64) gives Nu_x for a flat plate with an unheated starting length. This equation may be derived using the integral energy equation (6.47), the velocity and temperature profiles from eqns. (6.29) and (6.50), and $\delta(x)$ from eqn. (6.31a). Equation (6.52) is again obtained; however, in this case, $\phi = \delta_t/\delta$ is a function of x for $x > x_0$. Derive eqn. (6.64) by starting with eqn. (6.52), neglecting the term $3\phi^3/280$, and replacing δ_t by $\phi\delta$. After some manipulation, you will obtain

$$x\frac{4}{3}\frac{d}{dx}\phi^3 + \phi^3 = \frac{13}{14\,\mathrm{Pr}}$$

Show that the solution of this o.d.e. is

$$\phi^3 = Cx^{-3/4} + \frac{13}{14\,\mathrm{Pr}}$$

for an unknown constant C. Then apply an appropriate initial condition and the definition of q_w and Nu_x to obtain eqn. (6.64).

Solution

We start with eqn. (6.52):

$$\delta_t\,\frac{d}{dx}\left[\delta_t\,\underbrace{\int_0^1\left(\frac{3}{2}\eta\phi - \frac{1}{2}\eta^3\phi^3\right)\left(1 - \frac{3}{2}\eta + \frac{1}{2}\eta^3\right)d\eta}_{=\frac{3}{20}\phi - \frac{3}{280}\phi^3 \cong \frac{3}{20}\phi}\right] = \frac{3\alpha}{2u_\infty} \tag{6.52}$$

which, when approximated as shown in the underbrace, simplifies to

$$\delta_t\,\frac{d}{dx}\left[\delta_t\frac{3}{20}\phi\right] = \frac{3\alpha}{2u_\infty}$$

Now put $\delta_t = \phi\delta$:

$$\delta\phi\,\frac{d}{dx}(\delta\phi^2) = \frac{10\alpha}{u_\infty} \tag{*}$$

Equation (6.31a) gives

$$\delta^2 = \frac{280}{13}\frac{\nu x}{u_\infty} \tag{6.31a}$$

so

$$\delta = \sqrt{\frac{280}{13}}\sqrt{\frac{\nu x}{u_\infty}}$$

We can substitute this into eqn. (*) above and get

$$\frac{280}{13}\frac{\nu}{u_\infty}\sqrt{x}\phi\frac{d}{dx}\left(\phi^2\sqrt{x}\right) = \frac{10\alpha}{u_\infty}$$

or

$$\sqrt{x}\phi\frac{d}{dx}\left(\phi^2\sqrt{x}\right) = \frac{13}{28}\frac{1}{\mathrm{Pr}}$$

Now expand the derivative on the left-hand side

$$\frac{1}{2}\phi^3 + 2\phi^2 x\frac{d\phi}{dx} = \frac{13}{28}\frac{1}{\mathrm{Pr}}$$

or

$$\frac{2}{3}x\frac{d}{dx}\phi^3 + \frac{1}{2}\phi^3 = \frac{13}{28}\frac{1}{\mathrm{Pr}}$$

$$\frac{4}{3}x\frac{d}{dx}\phi^3 + \phi^3 = \frac{13}{14}\frac{1}{\mathrm{Pr}} \quad \longleftarrow \text{Answer} \tag{**}$$

To solve, we notice that eqn. (**) is just a first-order, linear o.d.e. for ϕ^3. We need the particular and homogeneous solutions. By inspection, a particular solution is

$$\phi_p^3 = \frac{13}{14}\frac{1}{\mathrm{Pr}}$$

The homogenous equation can be integrated without difficulty:

$$\frac{4}{3}x\frac{d}{dx}\phi_h^3 + \phi_h^3 = 0$$
$$\frac{4}{3}x\frac{d\phi_h^3}{\phi_h^3} = -\frac{3}{4}\frac{dx}{x}$$
$$\ln\phi_h^3 = -\frac{3}{4}\ln x + \text{constant}$$
$$\phi_h^3 = Cx^{-3/4}$$

for a constant C. Adding the two solutions, we get

$$\phi^3 = Cx^{-3/4} + \frac{13}{14}\frac{1}{\mathrm{Pr}} \qquad \xleftarrow{\text{Answer}}$$

The initial condition may be applied at $x = x_0$, where heating starts and $\delta_t = 0$: $\phi = 0$ at $x = x_0$. This condition is met when

$$C = -\frac{13}{14}\frac{1}{\mathrm{Pr}}x_0^{3/4}$$

Then

$$\phi = \left\{\frac{13}{14}\frac{1}{\mathrm{Pr}}\left[1-\left(\frac{x_0}{x}\right)^{3/4}\right]\right\}^{1/3}$$

The heat flux and the heat transfer coefficient may be calculated from the temperature profile and boundary layer thickness (as in Sect. 6.5), and, as before, that calculation results in eqn. (6.57):

$$h = \frac{3}{2}\frac{k}{\delta}\frac{\delta}{\delta_t} = \frac{3}{2}k\frac{1}{\delta\phi} \tag{6.57}$$

Then

$$\mathrm{Nu}_x = \frac{hx}{k} = \frac{3}{2}\frac{x}{\delta\phi}$$

We again can substitute the square root of eqn. (6.31a), and algebra produces the final result:

$$\mathrm{Nu}_x = \frac{3}{2}\sqrt{\frac{13}{280}}\left(\frac{14}{13}\right)^{1/3}\sqrt{\frac{u_\infty x}{\nu}}\mathrm{Pr}^{1/3}\left[1-\left(\frac{x_0}{x}\right)^{3/4}\right]^{-1/3}$$

$$\mathrm{Nu}_x = \frac{0.3313\,\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3}}{\left[1-(x_0/x)^{3/4}\right]^{1/3}} \quad \text{for } x > x_0 \qquad \xleftarrow{\text{Answer}}$$

This result is given in the text as eqn. (6.64).

Problem 6.42 Make a spreadsheet to compare eqn. (6.111) to eqn. (6.112) and eqn. (6.113) for Prandtl numbers of 0.7, 6, 50, and 80 over the range $2 \times 10^5 \leqslant \text{Re}_x \leqslant 10^7$, keeping in mind the ranges of validity of the various equations. What conclusions do you draw?

Solution

The equations of interest are as follow. Equation (6.111) applies for any $\text{Pr} \geqslant 0.5$:

$$\text{St} = \frac{\text{Nu}_x}{\text{Re}_x \text{Pr}} = \frac{C_f/2}{1 + 12.7(\text{Pr}^{2/3} - 1)\sqrt{C_f/2}} \qquad \text{Pr} \geqslant 0.5 \tag{6.111}$$

with eqn. (6.102) for C_f:

$$C_f(x) = \frac{0.455}{\left[\ln(0.06\,\text{Re}_x)\right]^2} \tag{6.102}$$

Equation (6.112) applies for gases (in our case, for $\text{Pr} = 0.7$):

$$\text{Nu}_x = 0.0296\,\text{Re}_x^{0.8}\,\text{Pr}^{0.6} \quad \text{for gases} \tag{6.112}$$

And eqn. (6.113) applies for non-metallic liquids (which in our case would cover $\text{Pr} = 6, 50, 80$):

$$\text{Nu}_x = 0.032\,\text{Re}_x^{0.8}\,\text{Pr}^{0.43} \quad \text{for nonmetallic liquids} \tag{6.113}$$

These equations can be coded into a spreadsheet range requested, with the results below:

	A	B	C	D	E	F	G	H
1								
2	Pr	Re	C_f	6.111	6.112	1-6.112/6.111	6.113	1-6.113/6.111
3								
4	0.7	200000	0.005157	418	416	0.48%		
5	0.7	300000	0.004739	573	576	-0.52%		
6	0.7	500000	0.004281	856	866	-1.21%		
7	0.7	700000	0.004015	1118	1134	-1.36%		
8	0.7	1000000	0.003759	1489	1508	-1.26%		
9								
10	6	200000	0.005157	1245			1204	3.34%
11	6	300000	0.004739	1760			1665	5.41%
12	6	500000	0.004281	2730			2506	8.21%
13	6	700000	0.004015	3650			3280	10.15%
14	6	1000000	0.003759	4973			4363	12.28%
15								
16	50	200000	0.005157	2831			2996	-5.82%
17	50	300000	0.004739	4052			4144	-2.27%
18	50	500000	0.004281	6381			6236	2.27%
19	50	700000	0.004015	8617			8162	5.28%
20	50	1000000	0.003759	11862			10857	8.48%
21								
22	80	200000	0.005157	3347			3667	-9.57%
23	80	300000	0.004739	4795			5072	-5.77%
24	80	500000	0.004281	7563			7632	-0.92%
25	80	700000	0.004015	10224			9990	2.29%
26	80	1000000	0.003759	14089			13289	5.68%

The calculations show that eqn. (6.112) is within 1.4% of eqn. (6.111) over the range considered. This difference is within the accuracy of either equation (see pp. 625–626), so we can use the simpler result, eqn. (6.112), when convenient.

Equation (6.113) is within 12.3% of eqn. (6.111) over the range considered, with the largest disagreements occurring at a different Reynolds number for different values of Prandtl number. Equation (6.113) fit the liquid data of Žukauskas and coworkers to about ±15% (see pg. 327), whereas eqn (6.111) is likely to be more accurate (however, liquid data for flat-plate boundary layers are scarce; see Lienhard [6.6]).

Problem 6.43 Liquid metal flows past a flat plate. Axial heat conduction is negligible, and the momentum b.l. has negligible thickness. (a) If the plate is isothermal, use eqn. (5.54) to derive eqn. (6.62). (b) Derive the corresponding expression for the local Nusselt number if the plate has a constant wall heat flux. (c) Find the average Nusselt number in both cases.

Solution

a) Before starting the analysis, consider the problem stated. A liquid metal (Pr $\ll$ 1) has a very thin momentum boundary layer relative to its thermal boundary layer (see Sect. 6.4 and Fig. 6.14). As result, we can neglect the momentum boundary layer and think of a slice of the liquid metal as a solid vertical slab that flows past the plate at speed u_∞. Because axial conduction is negligible in this case, all the heat flow is perpendicular to the flat plate. When the slice of liquid metal reaches the plate, at position $x = 0$ and time $t = 0$, its bottom temperature changes from T_i to T_∞. The time needed to reach any downstream position is $t = x/u_\infty$. Therefore, the slice of liquid metal experiences the same heat transfer process as a semi-infinite body if we replace the time t by x/u_∞.

Equation (5.54) provides the heat flux to a semi-infinite body whose surface temperature changes from T_i to T_∞ at $t = 0$:

$$q(t) = \frac{k(T_\infty - T_i)}{\sqrt{\pi \alpha t}}$$

Converting to the coordinate x, this expression is

$$q(x) = \frac{k(T_\infty - T_i)}{\sqrt{\pi \alpha x \,/\, u_\infty}}$$

The Péclet number is $\mathrm{Pe}_x = u_\infty x \,/\, \alpha$, and we may rearrange to get

$$\frac{qx}{k(T_\infty - T_i)} = \sqrt{\frac{\mathrm{Pe}_x}{\pi}}$$

$$\frac{hx}{k} = \sqrt{\frac{\mathrm{Pe}_x}{\pi}}$$

$$\mathrm{Nu}_x = 0.564\ \mathrm{Pe}_x^{1/2} \quad \xleftarrow{\text{Answer}}$$

b) In this case, we need the semi-infinite body solution for a step change in wall heat flux at $t = 0$ (corresponding to $x = 0$). That's given by eqn. (5.56):

$$T_w(t) - T_i = \frac{2q_w}{k}\sqrt{\frac{\alpha t}{\pi}}$$

As before, $t = x/u_\infty$. We can rearrange the equation as follows:

$$\frac{q_w x}{k(T_w - T_i)} = \frac{x}{2}\sqrt{\frac{\pi u_\infty}{\alpha x}}$$

$$\frac{hx}{k} = \frac{\sqrt{\pi}}{2}\sqrt{\mathrm{Pe}_x}$$

$$\mathrm{Nu}_x = 0.886\ \mathrm{Pe}_x^{1/2} \quad \xleftarrow{\text{Answer}}$$

c) In the uniform wall temperature case, the average heat transfer coefficient is given by eqn (6.65):

$$\overline{h} = \frac{1}{L}\int_0^L h(x)\,dx = \frac{1}{L}\int_0^L \frac{k}{x}\sqrt{\frac{u_\infty x}{\pi\alpha}}\,dx = \frac{2k}{L}\sqrt{\frac{u_\infty L}{\pi\alpha}} = \frac{2k}{L}\sqrt{\frac{\mathrm{Pe}_L}{\pi}}$$

so that

$$\frac{\overline{h}L}{k} = 2\sqrt{\frac{\mathrm{Pe}_L}{\pi}}$$

$$\overline{\mathrm{Nu}}_L = 1.13\ \mathrm{Pe}_L^{1/2} \quad \xleftarrow{\text{Answer}}$$

which is the same as eqn. (6.69).

In the uniform heat flux case, we use eqn. (6.66)

$$\overline{h} = \frac{q_w}{\dfrac{1}{L}\displaystyle\int_0^L (T_w - T_\infty)\,dx} = \frac{q_w}{\dfrac{1}{L}\displaystyle\int_0^L \frac{2q_w}{k}\sqrt{\frac{\alpha x}{\pi u_\infty}}\,dx} = \frac{1}{\dfrac{4}{3k}\sqrt{\dfrac{\alpha L}{\pi u_\infty}}}$$

or

$$\overline{\mathrm{Nu}}_L = \frac{\overline{h}L}{k} = \frac{3\sqrt{\pi}}{4}\sqrt{\frac{u_\infty L}{\alpha}} = 1.33\ \mathrm{Pe}_L^{1/2} \quad \xleftarrow{\text{Answer}}$$

PROBLEM 6.44 Beginning with eqn. (6.73) show that $\overline{\text{Nu}}_L$ is given over the entire range of Pr for a laminar b.l. on a flat, constant flux surface by:

$$\overline{\text{Nu}}_L = \frac{0.696\,\text{Re}_L^{1/2}\,\text{Pr}^{1/3}}{\left[1 + (0.0205/\text{Pr})^{2/3}\right]^{1/4}} \tag{6.124}$$

SOLUTION

Equation (6.73), for laminar flow over a constant flux surface, is

$$\text{Nu}_x = \frac{0.464\,\text{Re}_x^{1/2}\,\text{Pr}^{1/3}}{\left[1 + (0.0205/\text{Pr})^{2/3}\right]^{1/4}} \quad \text{for Pe}_x > 100 \tag{6.73}$$

The equation applies over the full range of Pr if $\text{Pe}_x = \text{Re}_x\text{Pr} > 100$.

To get the average heat transfer coefficient, $\overline{h} = q_w \big/ \overline{T_w - T_\infty}$, we need to find the average temperature difference with $\text{Nu}_x = h(x)x/k$:

$$\begin{aligned}
\overline{T_w - T_\infty} &= \frac{1}{L}\int_0^L (T_w - T_\infty)\,dx = \frac{1}{L}\int_0^L \frac{q_w}{h(x)}\,dx \\
&= \frac{1}{L}\int_0^L \frac{q_w x\left[1 + (0.0205/\text{Pr})^{2/3}\right]^{1/4}}{k\left(0.464\sqrt{u_\infty/\nu}\,\text{Pr}^{1/3}\right)}\frac{dx}{\sqrt{x}} \\
&= \frac{q_w\left[1 + (0.0205/\text{Pr})^{2/3}\right]^{1/4}}{k\left(0.464\sqrt{u_\infty/\nu}\,\text{Pr}^{1/3}\right)}\frac{2L^{3/2}}{3L}
\end{aligned}$$

Then we may rearrange, using the definition

$$\overline{\text{Nu}}_L = \frac{q_w L}{k\left(\overline{T_w - T_\infty}\right)}$$

and finding that

$$\overline{\text{Nu}}_L = \frac{(3/2)\left(0.464\sqrt{u_\infty L/\nu}\,\text{Pr}^{1/3}\right)}{\left[1 + (0.0205/\text{Pr})^{2/3}\right]^{1/4}} = \frac{0.696\,\text{Re}_L^{1/2}\,\text{Pr}^{1/3}}{\left[1 + (0.0205/\text{Pr})^{2/3}\right]^{1/4}} \quad \longleftarrow \text{Answer}$$

Problem 6.45 For laminar flow over a flat plate flow with $\mathrm{Pr} > 0.6$, how does $\overline{h}$ for T_w constant compare to $\overline{h}$ for q_w constant? At what location on a plate with q_w constant is the local plate temperature the same as the average plate temperature? At what location on a plate with T_w constant is the local heat flux the same as the average heat flux?

Solution

For the isothermal plate with $\mathrm{Pr} > 0.6$, we use eqns. (6.58) and (6.68):

$$\mathrm{Nu}_x = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3} \tag{6.58}$$

$$\overline{\mathrm{Nu}}_L = 0.664\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3} \tag{6.68}$$

For the uniform flux plate with $\mathrm{Pr} > 0.6$, we use eqns. (6.71) and (6.72):

$$\mathrm{Nu}_x = 0.4587\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3} \tag{6.71}$$

$$\overline{\mathrm{Nu}}_L = 0.688\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3} \tag{6.72}$$

Then, we have the following for the average heat transfer coefficients:

$$\overline{h} = \begin{cases} 0.664\,(k/L)\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3} & \text{for } T_w \text{ constant} \\ 0.688\,(k/L)\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3} & \text{for } q_w \text{ constant} \end{cases}$$

The ratio is just $0.664/0.688 = 0.965$, so the average heat transfer coefficients differ by $\underline{< 4\%}$.

For the fixed flux plate, the local and average temperatures are:

$$T(x) - T_\infty = \frac{q_w}{h(x)} = \frac{q_w x}{k}\left(0.4587\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}\right)^{-1}$$

$$\overline{T} - T_\infty = \frac{q_w}{\overline{h}} = \frac{q_w L}{k}\left(0.688\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3}\right)^{-1}$$

Setting these equal, we get

$$\frac{q_w x}{k}\left(0.4587\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}\right)^{-1} = \frac{q_w L}{k}\left(0.688\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3}\right)^{-1}$$

or

$$x\left(0.4587\,x^{1/2}\right)^{-1} = L\left(0.688\,L^{1/2}\right)^{-1}$$

Rearranging shows that these temperatures are the same when

$$\frac{x}{L} = \left(\frac{0.4587}{0.688}\right)^2 = 0.445 \quad \xleftarrow{\text{Answer}}$$

For the fixed temperature plate, with $\Delta T = T_w - T_\infty$, the local and average heat fluxes are:

$$q_w = h(x)\Delta T = 0.332\,\frac{k}{x}\Delta T\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$$

$$\overline{q}_w = \overline{h}\Delta T = 0.664\,\frac{k}{L}\Delta T\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3}$$

Setting these equal and simplifying gives the solution:

$$0.332\,\frac{k}{x}\Delta T\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3} = 0.664\,\frac{k}{L}\Delta T\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3}$$

$$\frac{x^{1/2}}{x} = \frac{2L^{1/2}}{L}$$

$$\frac{x}{L} = \left(\frac{1}{2}\right)^2 = 0.25 \quad \xleftarrow{\text{Answer}}$$

Comment: Observe that for uniform flux ΔT increases as $\sqrt{x}$, whereas for uniform temperature q_w decreases as $1/\sqrt{x}$.

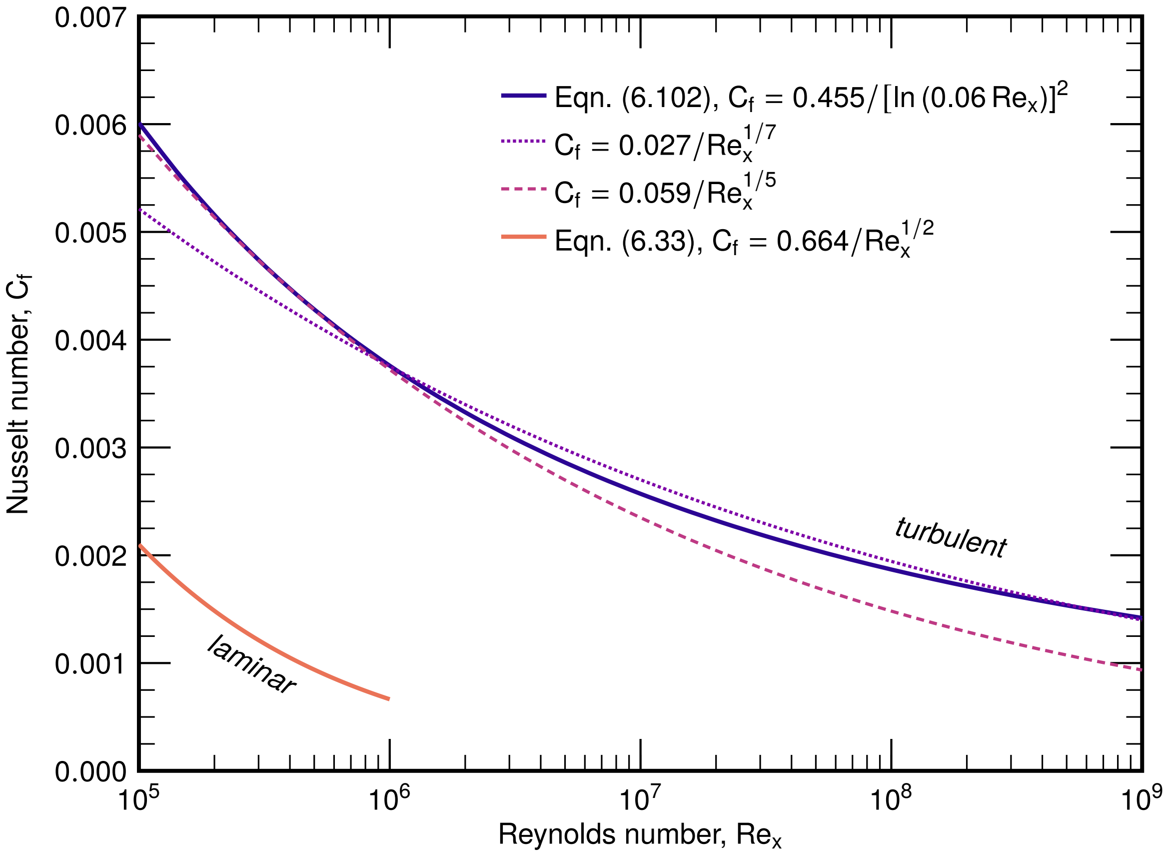
Problem 6.46 Two power laws are available for the skin friction coefficient in turbulent flow: $C_f(x) = 0.027\,\mathrm{Re}_x^{-1/7}$ and $C_f(x) = 0.059\,\mathrm{Re}_x^{-1/5}$. The former is due to White and the latter to Prandtl [6.4]. Equation (6.102) is more accurate and wide ranging than either. Plot all three expressions on semi-log coordinates for $10^5 \leqslant \mathrm{Re}_x \leqslant 10^9$. Over what range are the power laws in reasonable agreement with eqn. (6.102)? Also plot the laminar equation (6.33) on same graph for $\mathrm{Re}_x \leqslant 10^6$. Comment on all your results.

Solution The figure shows the two power laws and the mentioned turbulent and laminar expressions:

$$C_f = \frac{0.455}{\left[\ln(0.06\,\mathrm{Re}_x)\right]^2} \tag{6.102}$$

$$C_f = \frac{0.664}{\sqrt{\mathrm{Re}_x}} \tag{6.33}$$

The 1/7 power law is within 5% of eqn. (6.102) for $3.5 \times 10^5 \leqslant \mathrm{Re}_x \leqslant 10^9$, while the 1/5 power law is within 5% for $10^5 \leqslant \mathrm{Re}_x \leqslant 5 \times 10^7$. We also observe that skin friction in laminar flow is far less than in turbulent flow.



Problem 6.47 Reynolds et al. [6.27] provide the following measurements for air flowing over a flat plate at 127 ft/s with T_∞ = 86 °F and T_w = 63 °F. Plot these data on log-log coordinates as Nu_x vs. Re_x, and fit a power law to them. How does your fit compare to eqn. (6.112)?

$\mathrm{Re}_x \times 10^{-6}$	$\mathrm{St} \times 10^3$	$\mathrm{Re}_x \times 10^{-6}$	$\mathrm{St} \times 10^3$	$\mathrm{Re}_x \times 10^{-6}$	$\mathrm{St} \times 10^3$
0.255	2.73	1.353	2.01	2.44	1.74
0.423	2.41	1.507	1.85	2.60	1.75
0.580	2.13	1.661	1.79	2.75	1.72
0.736	2.11	1.823	1.84	2.90	1.68
0.889	2.06	1.970	1.78	3.05	1.73
1.045	2.02	2.13	1.79	3.18	1.67
1.196	1.97	2.28	1.73	3.36	1.54

Solution The film temperature is $T_f = (63 + 86)/2 = 74.5$ °F = 23.6 °C = 296.8 K. At this temperature, Table A.6 gives Pr = 0.707. We can convert the given data to $\mathrm{Nu}_x = \mathrm{St}\,\mathrm{Re}_x \mathrm{Pr}$ using a spreadsheet.

To make a fit, we must recognize that Pr does not vary. We have no basis for fitting a Pr exponent. So, we can fit to

$$\mathrm{Nu}_x = A\ \mathrm{Re}_x^b$$

This fit may be done by linear regression if we first take the logarithm:

$$\ln \mathrm{Nu}_x = \ln A + b \ln \mathrm{Re}_x$$

Using a spreadsheet, we can calculate the logarithms and perform the linear regression to find $A = 0.0187$ and $b = 0.814$ ($r^2 = 0.9978$), or

$$\boxed{\mathrm{Nu}_x = 0.0187\ \mathrm{Re}_x^{0.814}}$$

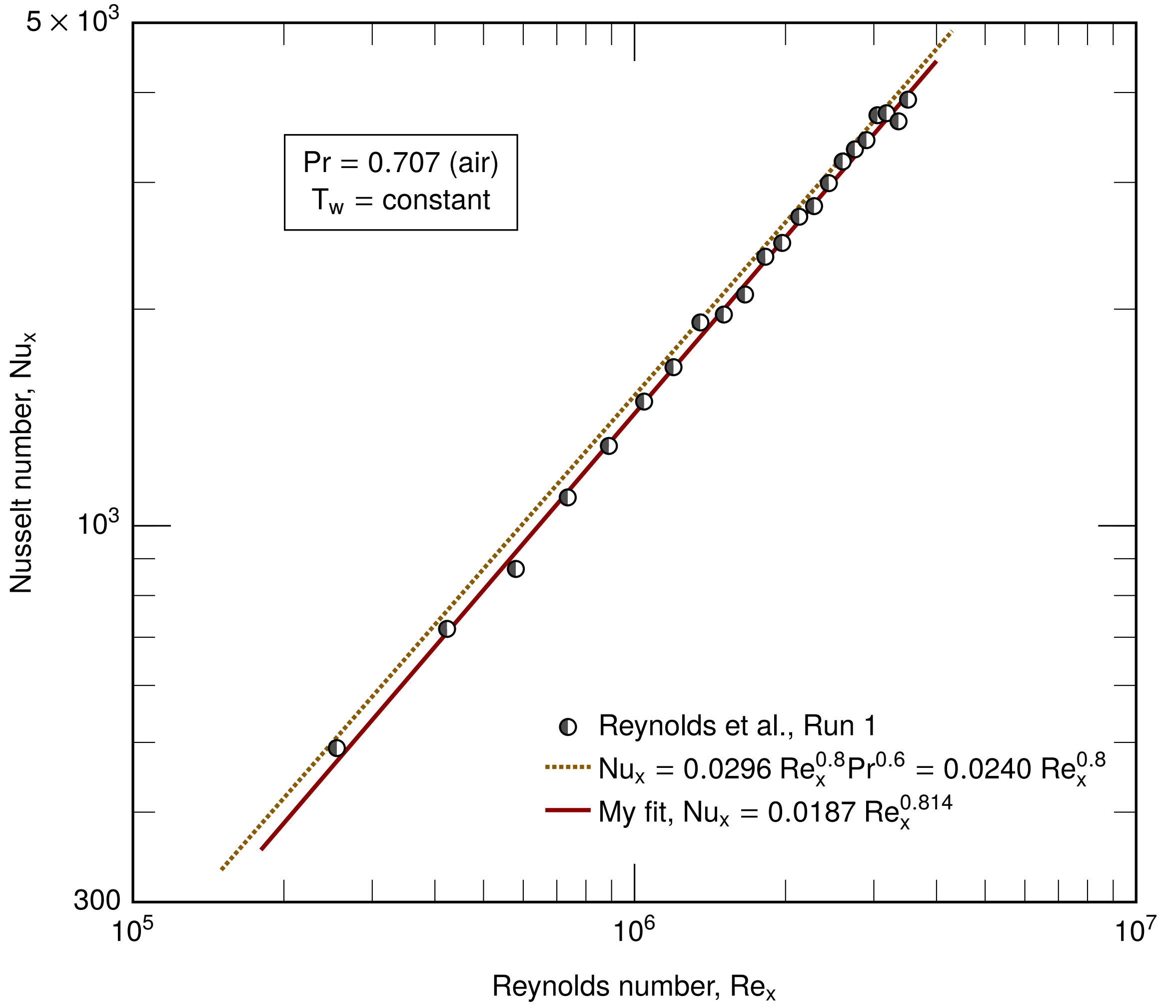
The fit is plotted with the equation, and the agreement is excellent.

With some additional effort, we may use the spreadsheet to find that the standard deviation of the data with respect to the fit is $s_x = 2.81\%$, which provides a 95% confidence interval (two-sided t-statistic for 21 points, $\pm 2.08 s_x$) of $\pm 5.8\%$.

Equation (6.112) for Pr = 0.712,

$$\mathrm{Nu}_x = 0.0296\ \mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{0.6} = 0.0240\ \mathrm{Re}_x^{0.8} \tag{6.112}$$

is also plotted in the figure, but it is systematically higher than this data set and our fit. (Reynolds et al. had 7 other data sets and reported an overall $s_x = 4.5\%$ for a $\pm 9\%$ uncertainty at 95% confidence.)



5 × 10^3
10^3
300
Nusselt number, Nu_x
10^5
10^6
10^7
Reynolds number, Re_x
Pr = 0.707 (air)
T_w = constant
Reynolds et al., Run 1
Nu_x = 0.0296 Re_x^0.8 Pr^0.6 = 0.0240 Re_x^0.8
My fit, Nu_x = 0.0187 Re_x^0.814

PROBLEM 6.48 Blair and Werle [6.36] reported the b.l. data below. Their experiment had a uniform wall heat flux with a 4.29 cm unheated starting length, u_∞ = 30.2 m/s, and T_∞ = 20.5°C.

a) Plot these data as Nu_x versus Re_x on log-log coordinates. Identify the regions likely to be laminar, transitional, and turbulent flow.
b) Plot the appropriate theoretical equation for Nu_x in laminar flow on this graph. Does the equation agree with the data?
c) Plot eqn. (6.112) for Nu_x in turbulent flow on this graph. How well do the data and the equation agree?
d) At what Re_x does transition begin? Find values of c and Re_l that fit eqn. (6.116b) to these data, and plot the fit on this graph.
e) Plot eqn. (6.117) through the entire range of Re_x.

$\mathrm{Re}_x \times 10^{-6}$	$\mathrm{St} \times 10^3$	$\mathrm{Re}_x \times 10^{-6}$	$\mathrm{St} \times 10^3$	$\mathrm{Re}_x \times 10^{-6}$	$\mathrm{St} \times 10^3$
0.112	2.94	0.362	1.07	1.27	2.09
0.137	2.23	0.411	1.05	1.46	2.02
0.162	1.96	0.460	1.01	1.67	1.96
0.183	1.68	0.505	1.05	2.06	1.84
0.212	1.56	0.561	1.07	2.32	1.86
0.237	1.45	0.665	1.34	2.97	1.74
0.262	1.33	0.767	1.74	3.54	1.66
0.289	1.23	0.865	1.99	4.23	1.65
0.312	1.17	0.961	2.15	4.60	1.62
0.338	1.14	1.06	2.24	4.83	1.62

SOLUTION

a) Calculate the Nusselt number from the values of Stanton number using $\mathrm{Nu}_x = \mathrm{St}\,\mathrm{Pr}\,\mathrm{Re}_x$. This is easily done with software (or by hand if you are patient) using Pr = 0.71. The results are plotted on the next page. The regions can be identified from the changes in slope and curvature (part b makes the laminar regime more obvious).
b) The appropriate formula is eqn. (6.116) for a laminar b.l. with an unheated starting length:

$$\mathrm{Nu}_{\mathrm{lam}} = \frac{0.4587\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1-(x_0/x)^{3/4}\right]^{1/3}} \tag{6.116}$$

We have only Re_x, not x. However,

$$\frac{x_0}{x} = \frac{\mathrm{Re}_{x_0}}{\mathrm{Re}_x} \quad \text{and} \quad \mathrm{Re}_{x_0} = \frac{u_\infty x_0}{\nu} = \frac{(30.2)(0.0429)}{1.516\times10^{-5}} = 8.546\times10^4$$

With this, the expression can be plotted. The agreement is pretty good. (Equation (6.71) is shown for comparison.)
c) The equation,

$$\mathrm{Nu}_{\mathrm{turb}} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{0.6} \tag{6.112}$$

is plotted in the figure, with excellent agreement.
d) To use eqn. (6.114b), we can start by visualizing a straight line through the transitional data on the log-log plot to determine the slope, c. This slope can be determined iteratively if using

software, or by drawing the line if working by hand. The slope is well fit by $c = 2.5$. Once the slope is found, we find the point at which this line intersects the laminar, unheated starting length curve. That point is well represented by $\mathrm{Re}_l = 500{,}000$ and $\mathrm{Nu}_{\mathrm{lam}}(\mathrm{Re}_l, \mathrm{Pr}) = 321$. Hence,

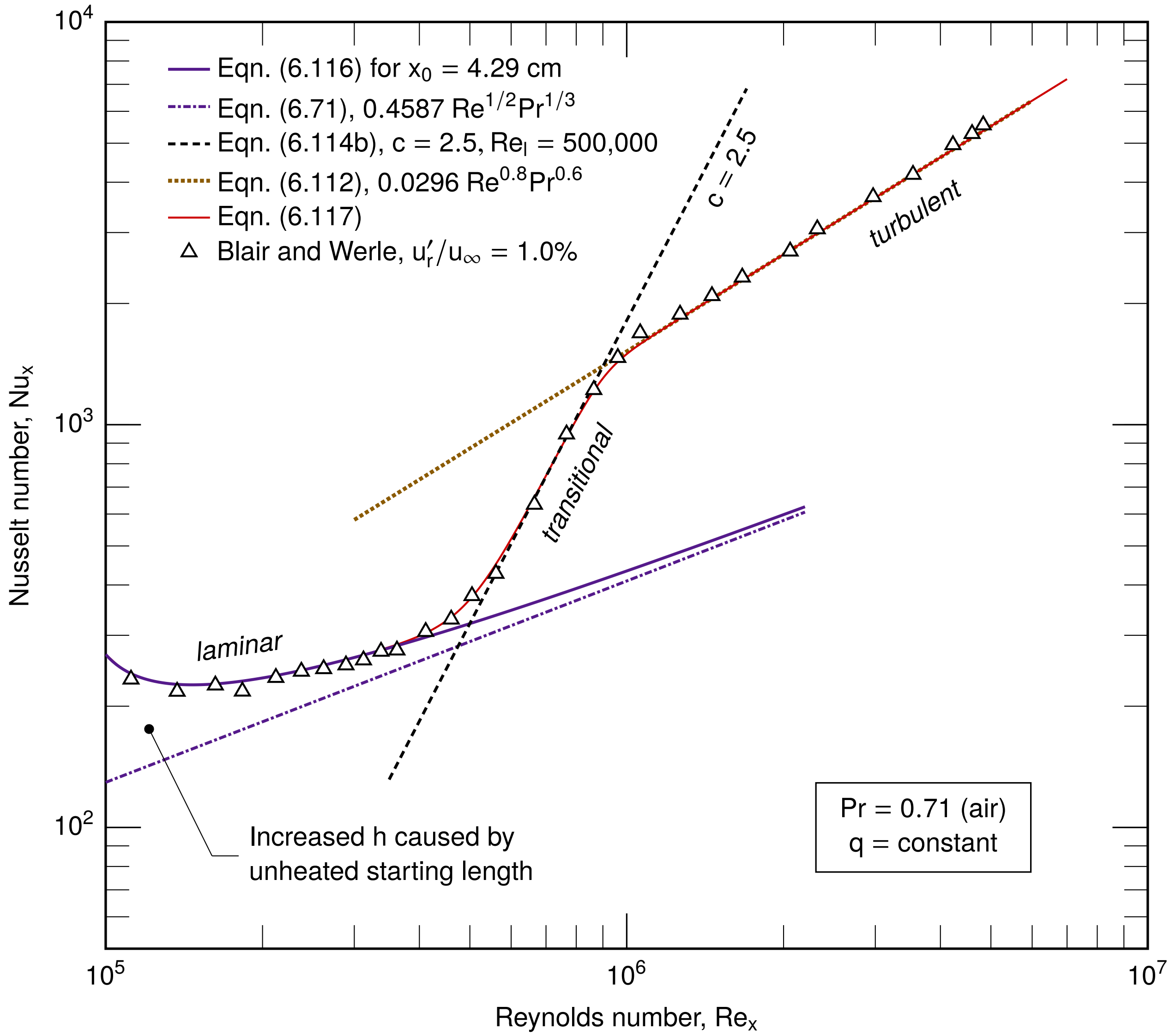
$$\mathrm{Nu}_{\mathrm{trans}} = \mathrm{Nu}_{\mathrm{lam}}(\mathrm{Re}_l, \mathrm{Pr}) \left(\frac{\mathrm{Re}_x}{\mathrm{Re}_l} \right)^c = 321 \left(\frac{\mathrm{Re}_x}{500{,}000} \right)^{2.5} \tag{6.114b}$$

This equation is plotted in the figure, with very good agreement. *Note that slightly different values of Re_l and Nu_{lam} may produce a good fit, if they lie on the same line. The best approach is to find Re_l and then* calculate *Nu_{lam} from eqn. (6.116).*

e) Equation (6.117) uses the laminar, transitional, and turbulent Nusselt numbers from parts (b), (c), and (d):

$$\mathrm{Nu}_x(\mathrm{Re}_x, \mathrm{Pr}) = \left[\mathrm{Nu}_{x,\mathrm{lam}}^5 + \left(\mathrm{Nu}_{x,\mathrm{trans}}^{-10} + \mathrm{Nu}_{x,\mathrm{turb}}^{-10} \right)^{-1/2} \right]^{1/5} \tag{6.117}$$

This equation is plotted in the figure as well, with very good agreement.



Problem 6.49 Figure 6.21 shows a fit to the following air data from Kestin et al. [6.29] using eqn. (6.117). The plate temperature was 100 °C (over its entire length) and the free-stream temperature varied between 20 and 30 °C. Follow the steps used in Problem 6.48 to reproduce that fit and plot it with these data.

$\mathrm{Re}_x \times 10^{-3}$	Nu_x	$\mathrm{Re}_x \times 10^{-3}$	Nu_x	$\mathrm{Re}_x \times 10^{-3}$	Nu_x
60.4	42.9	445.3	208.0	336.5	153.0
76.6	66.3	580.7	289.0	403.2	203.0
133.4	85.3	105.2	71.1	509.4	256.0
187.8	105.0	154.2	95.1	907.5	522.0
284.5	134.0	242.9	123.0		

Solution

a) The results are plotted on the next page. The regions can be identified from the changes in slope.

b) The appropriate formula is eqn. (6.58) for a laminar b.l. on a uniform temperature plate:

$$\mathrm{Nu}_{\mathrm{lam}} = 0.332\, \mathrm{Re}_x^{1/2}\, \mathrm{Pr}^{1/3} \tag{6.58}$$

The film temperature is between 60 and 65 °C, so Pr = 0.703. This equation is plotted on the figure. Only two data points touch the line, but they are in excellent agreement.

c) The appropriate equation,

$$\mathrm{Nu}_{\mathrm{turb}} = 0.0296\, \mathrm{Re}_x^{0.8}\, \mathrm{Pr}^{0.6} \tag{6.112}$$

is plotted in the figure, with very good agreement.

d) To use eqn. (6.114b), we can start by visualizing a straight line through the transitional data on the log-log plot to determine the slope, c. The slope is well fit by $c = 1.7$. Once the slope is found, we find the point at which this line intersects the laminar, unheated starting length curve. That point is well represented by $\mathrm{Re}_l = 60{,}000$ and $\mathrm{Nu}_{\mathrm{lam}}(\mathrm{Re}_l, \mathrm{Pr}) = 72.3$. Hence,

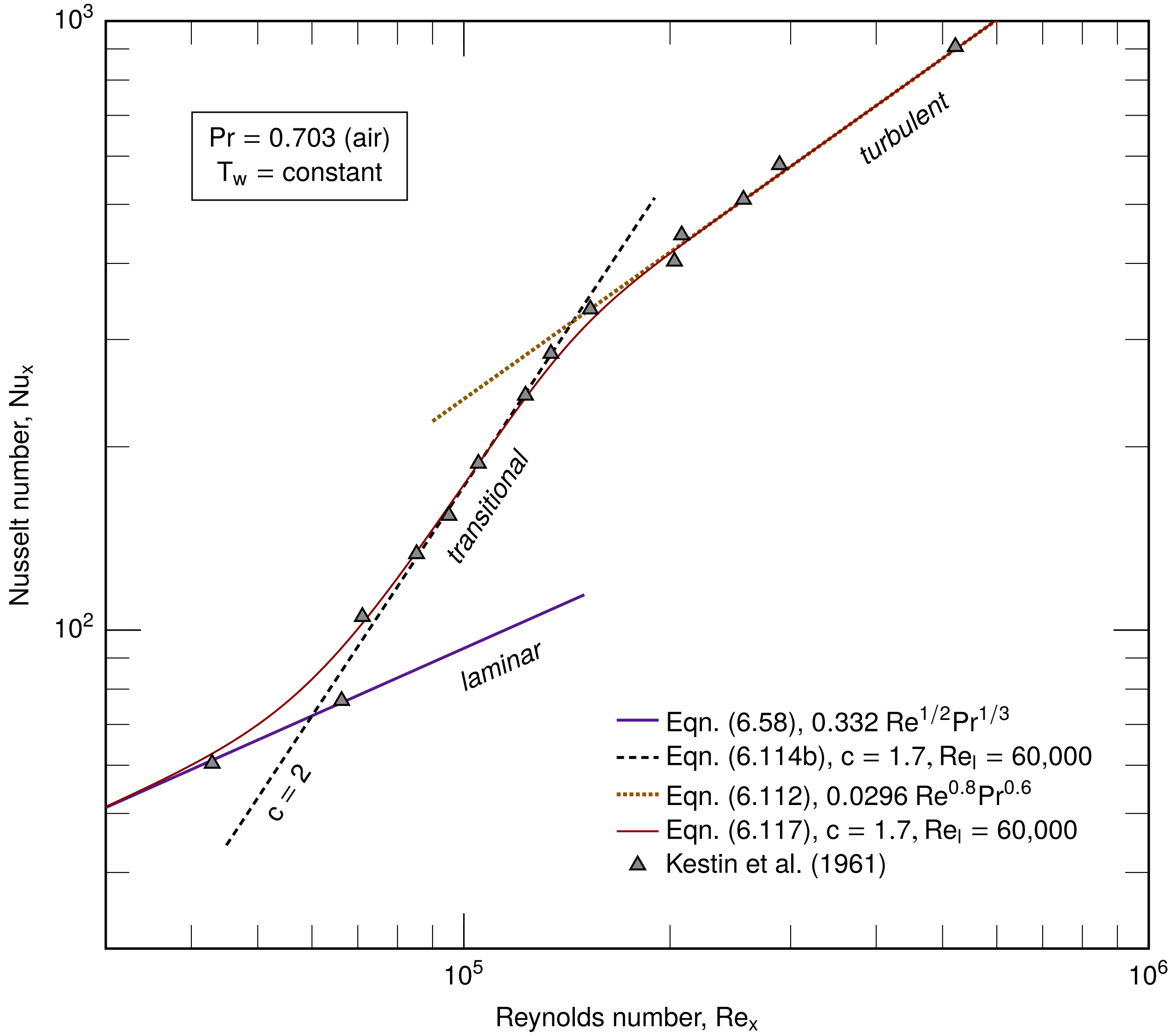
$$\mathrm{Nu}_{\mathrm{trans}} = \mathrm{Nu}_{\mathrm{lam}}\big(\mathrm{Re}_l, \mathrm{Pr}\big)\left(\frac{\mathrm{Re}_x}{\mathrm{Re}_l}\right)^c = 72.3\left(\frac{\mathrm{Re}_x}{60000}\right)^{1.7} \tag{6.114b}$$

This equation is plotted in the figure, with good agreement. Note that the most consistent approach is to find Re_l and then *calculate* $\mathrm{Nu}_{\mathrm{lam}}$ from eqn. (6.58).

e) Equation (6.117) uses the laminar, transitional, and turbulent Nusselt numbers from parts (b), (c), and (d):

$$\mathrm{Nu}_x(\mathrm{Re}_x, \mathrm{Pr}) = \left[\mathrm{Nu}_{x,\mathrm{lam}}^5 + \left(\mathrm{Nu}_{x,\mathrm{trans}}^{-10} + \mathrm{Nu}_{x,\mathrm{turb}}^{-10}\right)^{-1/2}\right]^{1/5} \tag{6.117}$$

This equation is plotted in the figure as well, with very good agreement in the turbulent and transitional ranges. The laminar fit looks good with one data point, but not the other one. The data themselves make a sharp leap between Re_x of 66,300 and 85,300. (Kestin et al. varied the Reynolds number between these data by increasing the air speed, u_∞—these data are not from spatially sequential points (unlike the data of Blair in Problem 6.48). The onset of turbulence is an instability, and the change in flow conditions may well have affected the transition.)



Pr = 0.703 (air)
T_w = constant
turbulent
transitional
laminar
c = 2
Eqn. (6.58), 0.332 $Re^{1/2}Pr^{1/3}$
Eqn. (6.114b), c = 1.7, Re_l = 60,000
Eqn. (6.112), 0.0296 $Re^{0.8}Pr^{0.6}$
Eqn. (6.117), c = 1.7, Re_l = 60,000
Kestin et al. (1961)
Nusselt number, Nu_x
Reynolds number, Re_x
10^3
10^2
10^5
10^6

Problem 6.50 A study of the kinetic theory of gases shows that the mean free path of a molecule in air at one atmosphere and 20 °C is 67 nm and that its mean speed is 467 m/s. Use eqns. (6.45) obtain C_1 and C_2 from the known physical properties of air. We have asserted that these constants should be on the order of 1. Are they?

Solution We had found that

$$\mu = C_1\left(\rho\overline{C}\ell\right) \tag{6.45c}$$

and

$$k = C_2\left(\rho c_v\overline{C}\ell\right) \tag{6.45d}$$

We may interpolate the physical properties of air from Table A.6: $\mu = 1.82 \times 10^{-5}$ kg/m·s, $k = 0.0259$ W/m·K, $\rho = 1.21$ kg/m^3, and $c_p = 1006$ J/kg·K. In addition, the specific heat capacity ratio for air is $\gamma = c_p/c_v = 1.4$.

Rearranging:

$$C_1 = \frac{\mu}{\rho\overline{C}\ell} = \frac{1.82 \times 10^{-5}}{(1.21)(467)(67 \times 10^{-9})} = 0.481$$

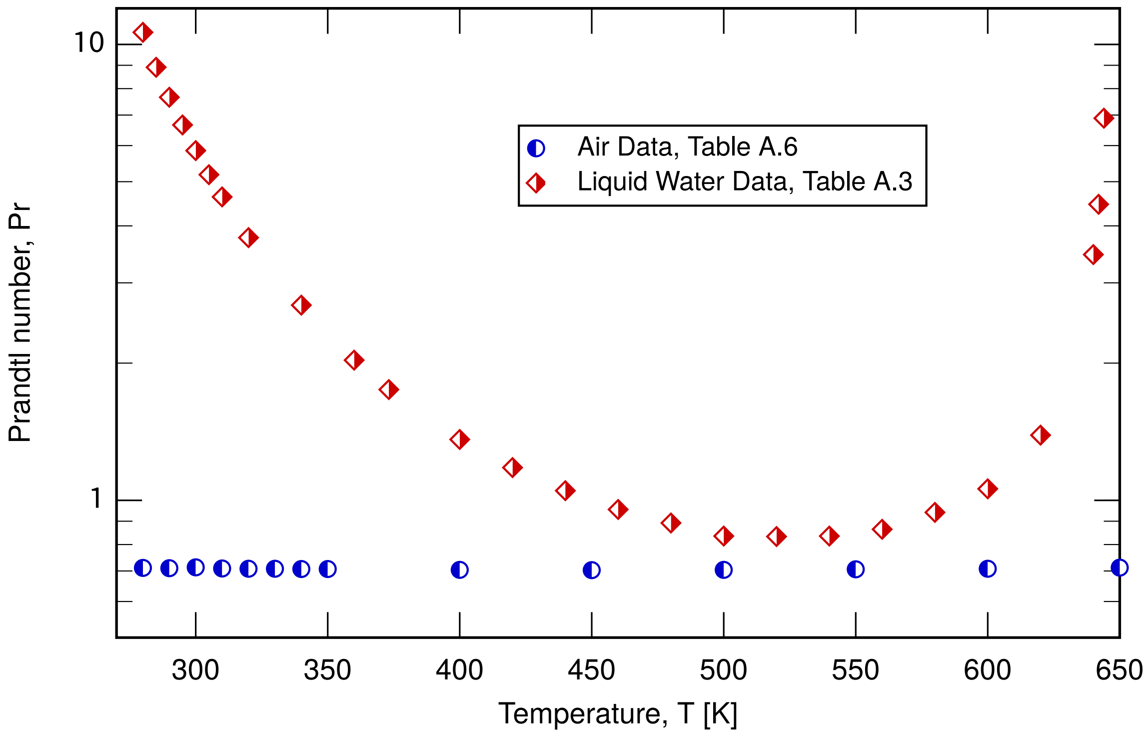
and

$$C_2 = \frac{k\gamma}{\rho c_p\overline{C}\ell} = \frac{(0.0259)(1.4)}{(1.21)(1006)(467)(67 \times 10^{-9})} = 0.952$$

The constants are indeed $\mathcal{O}(1)$.

Problem 6.51 The two most important fluids for thermal engineering are air and water. Using data from Appendix A, plot the Prandtl number of air and of saturated liquid water from 280 K to 650 K (for water, stop plotting at 644 K, which is very close to the critical point temperature of 647.1 K). Comment on the trends in this chart.

Solution The data are plotted below.



This chart shows that Pr for air is essentially independent of temperature with an approximate value 5/7, in accord with Section 6.4 (note that the kinetic theory predicting a value of 5/7 applies only to gases).

For liquid water, the Prandtl number drops steeply with rising temperature in the range up to 400 K. That decrease is mainly caused by the rapid decrease of water's viscosity with rising temperature. For temperatures from 400 K to 620 K, water has a Prandtl number on the order of 1, only about $1/10^{\text{th}}$ the value for cold water. The Prandtl number rises very abruptly near the critical point temperature, in the interval 640 K to 647 K; the reason is that $\mathrm{Pr} = \nu/\alpha = \mu c_p/k$ and $c_p \to \infty$ at the critical point.

7.1 Relate u_{avg} to dp/dx in laminar pipe flow.

$$\rho u_{avg} \frac{\pi}{4} D^2 = \rho \int_0^{D/2} u(r)\, 2\pi r\, dr = \frac{\rho\left(\frac{D}{2}\right)^2}{4\mu}\left(-\frac{dp}{dx}\right) 2\pi \underbrace{\int_0^{D/2}\left[1-\left(\frac{2r}{D}\right)^2\right] r\,dr}_{\underbrace{\frac{1}{2}\left(\frac{D}{2}\right)^2-\left(\frac{2}{D}\right)^2\frac{1}{4}\left(\frac{D}{2}\right)^4}_{+\frac{1}{4}\left(\frac{D}{2}\right)^2}}$$

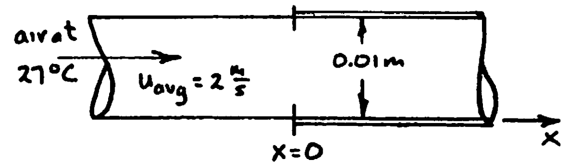
so

$$\underline{\underline{u_{avg} = -\frac{dp/dx}{32\mu} D^2 = -\frac{dp/dx}{8\mu} D^2}} \longleftarrow$$

7.2 Consider the air flow shown:
After x = 0 either:

(a) T_w = 68.4°C

or (b) q_w = 378 W/m^2



Plot T_w, q_w, and T_b vs. x in each case.

first evaluate $$\underline{\frac{2}{Gz} = \frac{2x}{u_{avg} D^2/\alpha} = \frac{2x}{2(2)(0.01)^2/2.205(10)^{-5}} = 0.2205\, x}$$

And $$\underline{Nu_D = \frac{q_w D}{(T_w - T_b)k} = \frac{0.01}{0.02614}\,\frac{q_w}{\Delta T} = 0.383\,\frac{q_w}{\Delta T}}$$

From eqn. (7.4) $$q_w = \frac{\rho c\, u_{avg} D}{4}\,\frac{dT_b}{dx} = \frac{1.193(1003)(2)(0.01)}{4}\,\frac{dT_b}{dx}$$

or $$\underline{q_w = 5.93\,\frac{dT_b}{dx}}$$

a) $T_w = 68.9°C$ = constant

$$\frac{dT_b}{dx} \simeq \frac{T_{b_{i+1}} - T_{b_i}}{\Delta x} = 0.169\, q_w = 0.169\, Nu_D\,\frac{T_w - T_{b_{i+1}}}{0.383}$$

so $$T_{b_{i+1}} = T_{b_i} + 0.441\,\Delta x\, Nu_D\,(68.9 - T_{b_{i+1}})$$

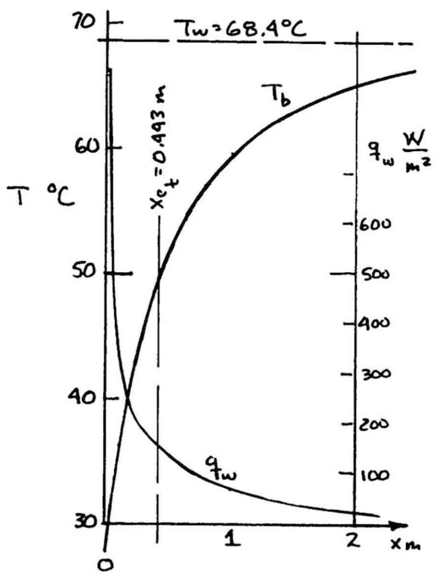
or $$\underline{T_{b_{i+1}} = \frac{T_{b_i} + 30.16\,\Delta x\, Nu_D}{1 + 0.441\,\Delta x\, Nu_D}}$$

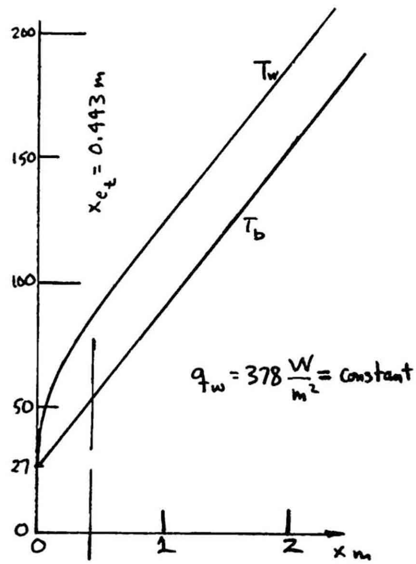
Now we pick an x = 0 where Nu_x will equal ∞. This gives $T_{b_{i+1}}$ = 27.0. Then we advance by Δx = 0.02, where we can get a new Nu_D from Fig. 8.4. Then we march forward as shown in the table below.

(b) $\frac{dT_b}{dx} = 0.169(378) = 63.88$ so $\underline{T_b = 27 + 63.88x}$

and $T_w = \frac{0.383}{Nu_D} 378 + T_b = \frac{145}{Nu_D} + 27 + 63.88x$ These are also tabled below.

Δx (m)	x (m)	$2/Gz = 0.2205x$	Nu_D, Fig. 7.4, T_w = const.	Nu_D, Fig. 7.4, q_w = const	$T_{b_{i+1}} = \frac{T_{b_i} + 30.16 Nu_D \Delta x}{1 + 0.441 Nu_D \Delta x}$	$q_w = 2.61 Nu_D \times (68.4 - T_{b_{i+1}})$	$T_w = \frac{145}{Nu_D} + 27 + 63.88x$ for q_w = const.
0.02	0	0	∞	∞	27 (given)	∞	27.0
↓	0.02	0.00441	7.7	9.5	29.63	779	43.5
↓	0.04	0.00882	6.15	7.7	31.63	590	48.4
↓	0.06	0.01323	5.55	6.8	33.34	420	52.2
↓	0.08	0.01762	5.0	6.5	34.82	363	54.4
0.1	0.10	0.02205	4.75	5.9	36.17	331	58.0
0.1	0.2	0.0441	4.1	5.05	41.11	242	68.5
0.1	0.3	0.0662	3.85	4.7	45.07	194	77.0
0.2	0.4	0.0882	3.75	4.55	48.38	162	84.4
0.2	0.6	0.1323	3.66	4.364	53.26	120	98.6
0.2	0.8	0.1762	↓	↓	56.96	90.9	111
1	1.0	0.2205	↓	↓	59.75	63.4	124
1	2	0.441	↓	↓	65.08	26.2	188
7	3	0.662	↓	↓	67.12	10.1	252
—	10	2.205	↓	↓	68.3	0.8	699





Problem 7.2: Added Note

Equation (7.30) expresses Nu_D as a function of the local Graetz Number, for a constant wall heat flux. We could thus use it, with the help of a spreadsheet, rather than reading from Fig. 7.4.

For a constant wall temperature, we can use equation (7.57) to find T_b. And equation (7.29) gives the overall heat transfer coefficient. We can then let L be the local value of x, and use these two equations to calculate T_b and the overall heat transfer coefficient at points along the tube. Once again, a spreadsheet would allow us to carry out the calculations and to plot the graph.

7.3 Prove that $C_f = 16/Re_D$ in laminar pipe flow.

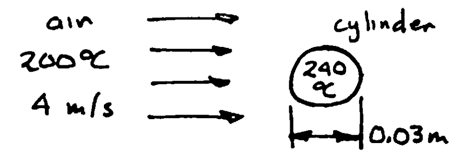
$$C_f = \frac{\tau_w}{\rho \bar{u}^2/2}, \text{ but } \tau_w = \left|\mu \frac{\partial u}{\partial r}\Big|_{r=R}\right| = \mu \left| 2\bar{u}\left(0 - 2\frac{r}{R}\right)_{r=R}\right| = \left| -4\mu\bar{u}/R \right| \quad \text{(eqn. (8.8))}$$

so: $$C_f = \frac{2}{\rho \bar{u}^2}\left(\frac{4\mu\bar{u}}{R}\right) = \frac{8\nu}{\bar{u}R} = \frac{16}{\bar{u}D/\nu} = \underline{\underline{\frac{16}{Re_D}}}$$

$$\left[\& \ f = 4C_f = 64/Re_D\right]$$

7.4 (a) Find $\bar{h}$ for the flow shown:

(b) If the flow were H_2O at 200°C, what velocity would give the same $\bar{h}$, the same $\overline{Nu_D}$, and same Re_D?



Evaluate properties at 220°C

(c) Would it be possible to model this heat transfer situation using water at some other temperature. Discuss.

a) $Re_D = \dfrac{u_\infty D}{\nu} = \dfrac{4(0.03)}{3.106\times10^{-5}} = \underline{3863}$ so from Fig. 7.14 $\dfrac{\overline{Nu_D}-0.3}{Pr^{1/3}}\left[1+\left(\dfrac{0.4}{Pr}\right)^{2/3}\right]^{1/4} = 39$

but $Pr = 0.698$ so we calculate $\overline{Nu_D} = 30.64$

or: $\bar{h} = \dfrac{k\overline{Nu_D}}{D} = \dfrac{0.03595(30.64)}{0.03} = \underline{\underline{36.7\ \dfrac{W}{m^2\text{°C}}}}$

b) For H_2O, $Re_D = \dfrac{u_\infty(0.03)}{1.444\times10^{-7}} = 3863$, so $u_\infty = $ only $\underline{\underline{0.0186\ \dfrac{m}{s}}}$ for same Re_D

At $\overline{Nu_D} = 30.64$, $\dfrac{30.34}{(0.871)^{1/3}}\left[1+\left(\dfrac{0.4}{0.871}\right)^{2/3}\right]^{1/4} = 35.7$

where we read from Fig. 7.14 $Re_D \simeq 3250 = \dfrac{u_\infty(0.03)}{1.444\times10^{-7}}$

so $\underline{\underline{u_\infty = 0.0156\ \dfrac{m}{s}}}$ for same $\overline{Nu_D}$

If $\bar{h} = 36.7$: $\dfrac{\dfrac{36.7(0.03)}{0.691}-0.3}{(0.871)^{1/3}}\left[1+\left(\dfrac{0.4}{.871}\right)^{2/3}\right]^{1/4} = 1.67$

so we use eqn. (7.66) to calculate $Re = (1.67/0.62)^2 = 7.255$.

This gives $7.255 = \dfrac{u_\infty(0.07)}{1.444(10)^{-7}}$ so $\underline{\underline{u_\infty = 0.000035\ \dfrac{m}{s}}}$

which is absurdly slow!

c) $\overline{Nu_D} = fn(Re_D, Pr)$

In hot water, $Pr \Rightarrow Pr_{air}$ closely enough to make a good approximation.

This can be made equal to Re_D for an airflow, if u_∞ is kept very low.

Thus we can model the air flow approximately in water.

Problem 7.5 Compare the h value computed in Example 7.3 with values predicted by the Dittus-Boelter, Colburn, McAdams, and Sieder-Tate equations. Comment on this comparison.

Solution: Taking values of components from Example 7.3, we get:

$$h_{DB} = (k/D)(0.0243)(Pr)^{0.4}(Re_D)^{0.8}$$

$$= (0.661/0.12)(0.0243)(3.61)^{0.4}(412{,}300)^{0.8} = \underline{\underline{6747\ W/m^2\text{-}K}}$$

$$h_{Colburn} = (k/D)(0.023)(Pr)^{1/3}(Re_D)^{0.8}$$

$$= (0.661/0.12)(0.023)(3.61)^{1/3}(412{,}300)^{0.8} = \underline{\underline{6193\ W/m^2\text{-}K}}$$

$$h_{Mcdams} = (k/D)(0.0225)(Pr)^{0.4}(Re_D)^{0.8} = (0.0225/0.0243)h_{DB}$$

$$= \underline{\underline{6247\ W/m^2\text{-}K}}$$

$$h_{ST} = h_{Colburn}(\mu_b/\mu_w)^{0.14} = 6193(1.75)^{0.14} = 6193(1.081)$$

$$= \underline{\underline{6698\ W/m^2\text{-}K}}$$

The more accurate Gnielinski equation gives h = 8400 W/m^2-K. Therefore, these old equations are low by roughly 20%, 26%, 26%, and 25%, respectively.

Why such consistently large deviations? It is because the old correlations represent much more limited data sets than Gnielinski's correlation. In this case, Re_D = 412,000 was a good deal higher than the Re_D values used to build the old correlations.

7.8 If u_∞ and T_w vary in Example 7.4, but all other conditions remain the same, plot u_∞ against T_w.

With reference to the Example, we write:

$$u_\infty = \frac{\nu}{D}Re_D = \frac{\nu}{D}\left[\frac{\frac{Q}{\pi D(T_w - T_\infty)}\frac{D}{k} - 0.3}{0.62(Pr)^{1/3}}\left[1+\left(\frac{0.4}{Pr}\right)^{2/3}\right]^{1/4}\right]^2$$

$$u_\infty = \frac{1.596\times 10^{-5}}{10^{-4}}\left[\frac{\frac{17.8}{\pi(T_w-20)(0.0299)} - 0.3}{0.62\,(0.71)^{1/3}}\left(1+\left(\frac{0.4}{0.71}\right)^{2/3}\right)^{1/4}\right]^2$$

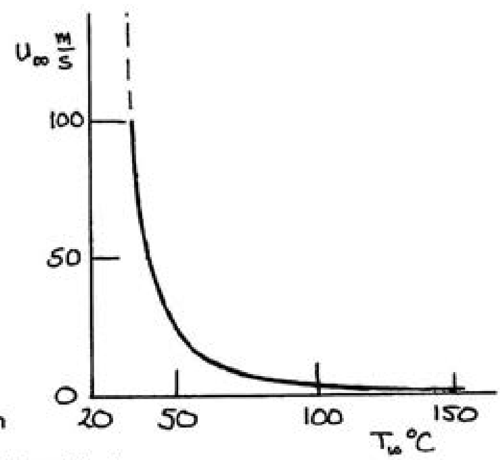
so

$$u_\infty = 0.677\left(\frac{189.4}{T_w - 20} - 0.3\right)^2$$

The speed is too high up to here

T_w °C	u_∞ $\frac{m}{s}$
20	∞
25	956
30	235
45	35.8
50	24.5

T_w °C	u_∞ $\frac{m}{s}$
60	13.3
80	5.52
100	2.89
150	0.91
200	0.38



(When $u_\infty \tilde{>} 100$ m/s, the incompressible assumption breaks down and the prediction of q is more difficult.)

7.10 NAK flows full-developed at 8 m/s and 395°C in a 0.05 m I.D. tube. What is h if T_w is 403°C?

$k = 26.7\ \mathrm{W/m\text{-}^\circ C}$, $Pr = 0.0068$, $\nu = 2.67 \times 10^{-7}\ \mathrm{m^2/s}$ at $\bar{T} = 399°C$.

$$Re_D = \frac{8(0.05)}{2.67(10)^{-7}} = 1.498 \times 10^6 \quad \text{so the flow is } \underline{\underline{\text{turbulent}}}.$$

Then eqn. (8.32) gives: $Nu_D = 0.625(1.498 \times 10^6 \times 0.0068)^{0.4} = 25.07$

$$\therefore\ h = \frac{26.7}{0.05}\,25.07 = \underline{\underline{13{,}385\ \frac{W}{m^2\text{-}^\circ C}}}$$

7.11 Water enters a 0.07 m diam., 73°C, pipe at 5°C. u_{avg} = 0.86 m/s. Plot T_b vs. x, neglecting entry conditions. Assume the pipe is smooth. Thus:

$$Nu_D = \frac{(f/8)\,Re_D\,Pr}{1.07 + 12.7\sqrt{\frac{f}{8}}\,(Pr^{2/3} - 1)}\left(\frac{\mu_b}{\mu_w}\right)^{0.11} \quad ; \quad f = \frac{1}{(1.82\log_{10} Re_D - 1.64)^2}$$

If we move down the pipes in increments of δx,

$$T_{b_{x+\delta x}} = T_{b_x} + \frac{\bar{h}\,\pi D\,\delta x\,(T_w - T_{b_x})}{\rho_f c_p\,\pi\left(\frac{D}{2}\right)^2 u_{avg}} = T_{b_x} + \frac{4\,\bar{h}\,\delta x\,(T_w - T_{b_x})}{(\rho_f c_p)\,D\,u_{avg}}$$

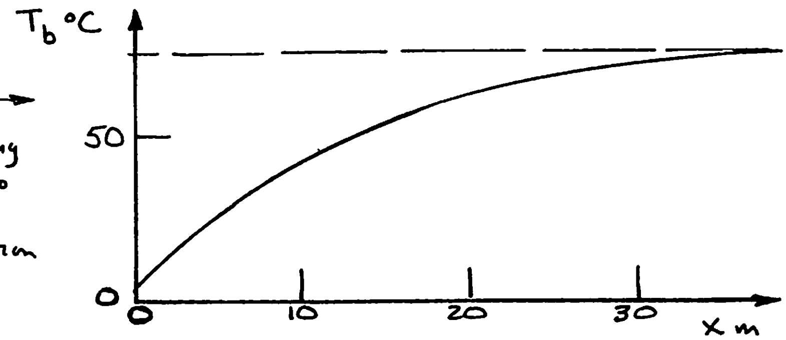
x(m)	T_{b_x} °C	$\frac{T_{b_x}+T_w}{2}$ °K	$\nu \times 10^6$ k $\rho c \times 10^{-6}$ Pr	$Re_D = \frac{0.0602}{\nu}$	$(\mu_b/\mu_w)^{0.11} \simeq \left(\frac{\nu(T_{b_x})}{0.393 \times 10^{-6}}\right)^{0.11}$	$\frac{f}{8}$	Nu_D	$h = \frac{k}{D} Nu_D$	$T_{b_{x+\delta x}}$ °C
0 (δx=2 m)	5	312	0.67 0.6253 4.14 4.48	85,851	1.157	0.0023	413	3689	13.0
2 (δx=2)	13	316	0.618 0.631 4.14 4.07	97,411	1.135	0.0023	453	4083	20.9
4 (δx=3)	20.9	320	0.566 0.6367 4.13 3.67	106,360	1.112	0.00222	508	4618	32.5
7 (δx=4)	32.5	326	0.532 0.6433 4.12 3.35	113,158	1.074	0.00219	494	4536	44.3
11 (δx=5)	44.3	331.6	0.482 0.6493 4.11 3.06	124,896	1.048	0.00214	498	4620	55.0
16 (δx=8)	55.0	337	0.440 0.6183 4.10 2.78	136,818	1.03	0.00210	512	4431	65.3
24 (δx=12)	65.3	342	0.411 0.6571 4.10 2.55	146,472	1.01	0.00207	497	4667	72.3

(continued ...)

7.11 (continued)

Plot the results ⟶

This might be converging too quickly owing to the large steps taken later in the calculation (cf Problem 7.13)



7.14 This problem can have hundreds of solutions. It has the value of putting the student in an active (attack) mode.

7.15 Water at 24°C flows at u_{avg} = 0.8 m/s in a 0.015 m smooth tube which is kept at 30°C. The flow could be either laminar or turbulent. Calculate $h_{turb.}/h_{laminar}$ if the flow is fully developed.

The properties at $T = \frac{24+30}{2} = 27°C$ or 300°K are: $\nu = 0.826 \times 10^{-6}$, $k = 0.6084$, $\rho c = 4.161 \times 10^{6}$, $Pr = 5.65$

$$Re_D = \frac{u_{avg} D}{\nu} = \underline{14,528}$$

for laminar flow: $Nu_D = 3.658$, $h = \frac{0.6084}{0.015} 3.658 = \underline{\underline{148.4 \frac{W}{m^2\text{-}°C}}}$

for turbulent flow:

$$\frac{f}{8} = \frac{1}{8(1.82 \log_{10} 14,528 - 1.64)^2} = \underline{0.00355}$$

7.15 (continued)

$$Nu_D = \frac{\frac{f}{8} Re_D Pr}{1.07 + 12.7\sqrt{\frac{f}{8}}(Pr^{2/3} - 1)} \left(\frac{\mu_b}{\mu_w}\right)^{0.11}$$

but $(\mu_b/\mu_w)^{0.11} \simeq (\nu_b/\nu_w)^{0.11} = \left(\frac{0.915}{0.787}\right)^{0.11} = 1.017$, so:

$$Nu_D = 109.2, \quad h = \frac{0.6084}{0.015} 109.2 = \underline{\underline{4429 \frac{W}{m^2 °C}}}$$

Thus:

$$h_{turbulent} / h_{laminar} = \frac{4429}{148.4} = \underline{\underline{29.8}}$$

Turbulent flow (in this case) gives 30 times the heat transfer in laminar flow.

7.16 Laboratory observations of heat transfer during the forced flow of air at 27°C over a bluff body, 12 cm wide, kept at 77°C, yield $q = 646\ W/m^2$ when the air moves 2 m/s and 3590 W/m^2 when it moves 18 m/s. In another test, everything else is the same, but now 17°C water flowing 0.4 m/s yields 131,000 W/m^2. The correlations in Chapt. 7 suggest that, with such limited data, we can probably create a fairly good correlation in the form: $\overline{Nu_L} = CRe^a Pr^b$. Estimate the constants C, a, and b, by cross-plotting the data on log-log paper.

for the air case: $Re_L = \frac{(2 \text{ or } 18)(0.12)}{1.809(10)^{-5}} = 13,267 \text{ or } 119,400$

and $Pr = 0.709$.

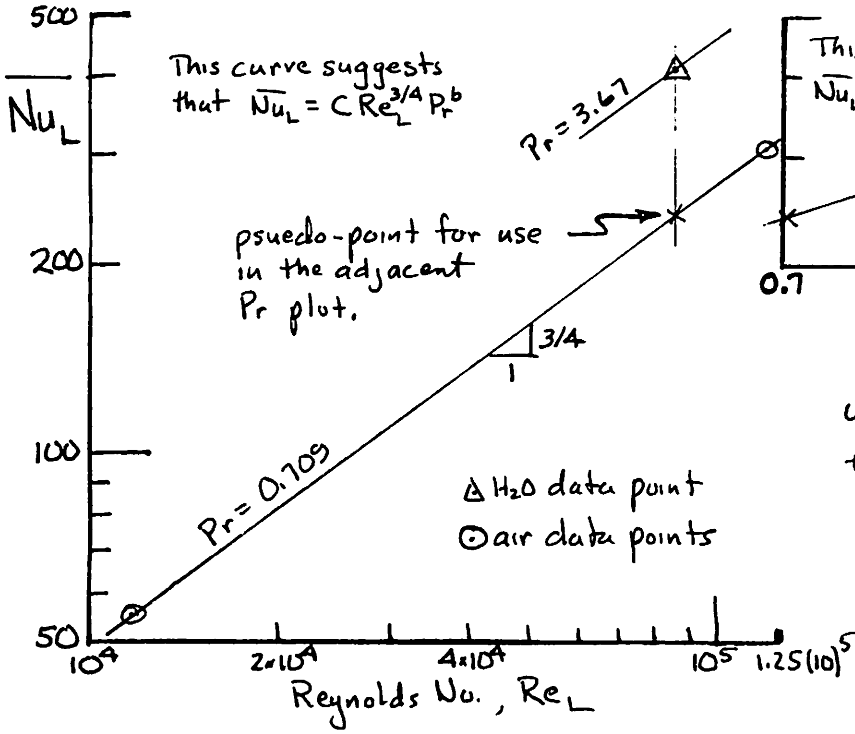
$$Nu_L = \frac{qL}{\Delta T\, k} = \frac{(646 \text{ or } 3590)(0.12)}{(77-27)(0.02792)} = \underline{55.5} \text{ or } \underline{308.6}$$

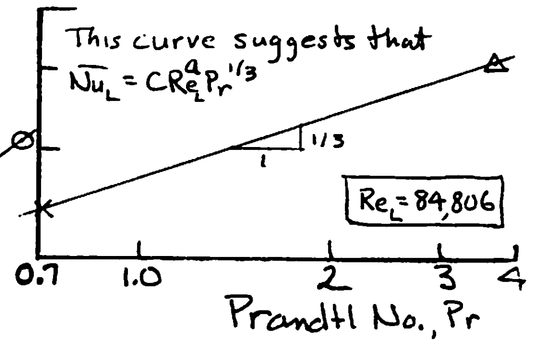
for water: $Re_L = \frac{0.4(0.12)}{0.566(10)^{-6}} = 84,806, \quad Pr = 3.67,$

and $Nu_L = \frac{131,000\,(0.12)}{(77-17)(0.6367)} = \underline{411.5}$

(over)

7.16 (continued)





use $411.5 = C(84806)^{3/4}(3.67)^{1/3}$

to get $C = 0.0537$

Then:

$$\overline{Nu}_L = 0.0537\,Re_L^{3/4}\,Pr^{1/3}$$

PROBLEM 7.17 Air at 1.38 MPa (200 psia) flows at 12 m/s in an 11 cm I.D. duct. At one location, the bulk temperature is 40 °C and the pipe wall is at 268 °C. Evaluate h if $\varepsilon/D = 0.002$.

SOLUTION We evaluate the bulk properties at 40°C = 313.15 K. Since the pressure is elevated, we must use the ideal gas law to find the density of air with the universal gas constant, R°, and the molar mass of air, M:

$$\rho = \frac{pM}{R^\circ T} = \frac{(1.38 \times 10^6)(28.97)}{(8314.5)(313.15)} = 15.36 \text{ kg/m}^3$$

The dynamic viscosity, conductivity, and Prandtl number of a gas depend primarily upon temperature. At 313 K, $\mu = 1.917 \times 10^{-5}$ kg/m·s, $k = 0.0274$ W/m·K, and Pr = 0.706. Hence,

$$\mathrm{Re}_D = \frac{\rho u_{\mathrm{av}} D}{\mu} = \frac{(15.36)(12)(0.11)}{1.917 \times 10^{-5}} = 1.058 \times 10^6$$

The friction factor may be calculated with Haaland's equation, (7.50):

$$f = \left\{ 1.8 \, \log_{10} \left[\frac{6.9}{1.058 \times 10^6} + \left(\frac{0.002}{3.7} \right)^{1.11} \right] \right\}^{-2} = 0.02362$$

We can see from Fig. 7.6 that this condition lies in the fully rough regime, as confirmed by eqns. (7.48):

$$\mathrm{Re}_\varepsilon \equiv \frac{u^* \varepsilon}{\nu} = \mathrm{Re}_D \, \frac{\varepsilon}{D} \sqrt{\frac{f}{8}} = (1.058 \times 10^6)(0.002)\sqrt{\frac{0.02362}{8}} = 114.9 > 70$$

Next, we may compute the Nusselt number from eqn. (7.49):

$$\begin{aligned}
\mathrm{Nu}_D &= \frac{(f/8)\, \mathrm{Re}_D \, \mathrm{Pr}}{1 + \sqrt{f/8}\left(4.5\, \mathrm{Re}_\varepsilon^{0.2} \mathrm{Pr}^{0.5} - 8.48\right)} \\
&= \frac{(0.02362/8)\,(1.058 \times 10^6)(0.706)}{1 + \sqrt{0.02362/8}\left(4.5(114.9)^{0.2}(0.706)^{0.5} - 8.48\right)} \\
&= 2061
\end{aligned}$$

The temperature difference is quite large, so we should correct for variable properties using eqn. (7.45):

$$\mathrm{Nu}_D = \mathrm{Nu}_D\Big|_{T_b} \left(\frac{T_b}{T_w} \right)^{0.47} = (2061)\left(\frac{313.15}{541.15} \right)^{0.47} = 1594$$

Finally,

$$h = \frac{k}{D}\, \mathrm{Nu}_D = \frac{0.0274}{0.11}\,(1594) = \underline{397 \text{ W/m}^2\text{K}}$$

7.18 How does $\bar{h}$ vary with the heater diameter during crossflow over a cylindrical heater when Re_D is very large.

From eqn. (8.34)

$$\lim_{\text{large } Re_D} \overline{Nu}_D = \frac{0.62\, Re_D^{1/2}\, Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}} \left(\frac{Re_D}{282000}\right)^{\frac{5}{8}\cdot\frac{4}{5}} = fn(Pr)\, Re_D$$

Therefore:

$$\bar{h} = k\, fn(Pr)\, \frac{u_\infty}{\nu}$$ which is independent of D

We encounter this size-independence again in natural convection when the size is large. See Problem 8.31.

7.20 Write Re_D in terms of $\dot{m}$ in pipe flow and explain why this repesentation could be particularly useful in dealing with compressible pipe flows.

$$Re_D = \frac{\rho \bar{u} D}{\mu} = \frac{\rho \bar{u} A}{\mu} \frac{4D}{\pi D^2} = \frac{4\dot{m}}{\pi \mu D}$$

$\dot{m}$ must remain constant in a compressible pipe flow while both ρ and u vary. In an isothermal gas flow with a pressure drop, we see that Re_D actually stays constant -- a fact that is not clear in the conventional form.

7.21 NAK at 394°C flows at 0.57 m/s across a 1.82 m length of 0.036 m O.D. tube. The tube is kept at 404°C. Find $\bar{h}$ and the heat removal rate from the tube.

Evaluate the properties a (94 + 404)/2 = 399°C:

$$Re_D = \frac{uD}{\nu} = \frac{0.57(0.036)}{2.67\times10^{-7}} = 78{,}854, \quad Pr = 0.0068, \quad Pe_b = Re_b Pr = 523$$

So we use eqn. (8.35)

$$\overline{Nu_D} = 0.3 + \frac{0.62(78{,}854)^{1/2}(0.0068)^{1/3}}{[1+(0.4/0.0068)^{2/3}]^{1/4}} = 16.55; \quad \bar{h} = \overline{Nu_D}\frac{k}{D} = 16.55\,\frac{26.7}{0.036}$$

$$= \underline{\underline{12{,}275 \ \frac{W}{m^2\text{-}°C}}}$$

And:

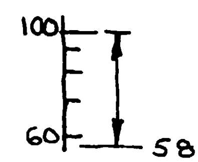
$$Q = \bar{h} A \Delta T = 12{,}275[1.82\pi(0.036)](404-394) = \underline{\underline{25{,}266W}}$$

7.23 Check the value of h given in Example 7.3 by using Reynold's analogy <u>directly</u> to calculate it. Which h do you deem to be in error, and by what percent.

Direct use of Reynold's analogy yields the Colburn equation. We have already made this comparison in Problem 7.5. The resulting deviation from the far more accurate Gnielinski equation was 26%.

7.26 Report the maximum percent scatter of data in Fig 7.14. What is happening in the fluid flow when the scatter is worst?

We identify the distance ↕ between the highest and lowest points at $Re_D \simeq 30{,}000$ and compare it with the log scale (as we see here:)



The error is such that $58(1+\text{scatter})(1+\text{scatter}) = 100$,

So: $\text{scatter} = \pm 0.31 = \underline{\underline{\pm 31\%}}$

The error, while not generally this bad, is still high in the range: $20{,}000 < Re_D < 300{,}000$. Figure 7.11 tells us that in this range, the conventional vortex street is gradually breaking down and becoming three-dimensional. When the b.l. on the cyl. finally becomes turbulent (and vortex shedding becomes unclear -- see Fig. 7.12), then the scatter reduces to about ±8%.

7.28 Freshly painted aluminum rods, 0.02 m in diameter are withdrawn from a drying oven at 150°C and cooled in a 3 m/s crossflow of air at 23°C. How long will it take to cool them to 40°C, so they can be handled?

We shall evaluate air properties at an average, average temperature of $\frac{1}{2}\left[\frac{150+23}{2} + \frac{40+23}{2}\right] = 57.25$ C = 330.4 K

$\nu = 1.982\,(10)^{-5}$, $k = 0.0282$, $Pr = 0.708$

for aluminum, $\rho c_p = 2707(905) = 2.45 \times 10^6\ \mathrm{J/m^2\text{-}°K}$, $k = 240\ \frac{\mathrm{W}}{\mathrm{m\text{-}°C}}$

Then: $Re_D = \dfrac{0.02\,(3)}{1.982\,(10)^{-5}} = 3027$ so we use eqn. (7.66)

$$\overline{Nu_D} = 0.3 + \frac{0.62\sqrt{3027}\ 0.708^{1/3}}{\left[1 + (0.4/0.708)^{2/3}\right]^{1/4}} = 27.6$$

and $\bar{h} = \overline{Nu_D}\,\dfrac{k}{D} = 27.6\,(0.0282)/0.02 = \underline{39.2\ \frac{\mathrm{W}}{\mathrm{m^2\text{-}°C}}}$

Next, we calculate $Bi = \dfrac{39.2\,(0.02)}{240} = 0.0033 \ll 1$, so we can assume lumped capacity.

$$T = \frac{\rho c V}{\bar{h} A} = \frac{\rho c D}{4\bar{h}} = \frac{2.45(10)^6\,0.02}{4\,(39.2\ )} = \underline{308\ \mathrm{sec}}$$

Then:

$$\left.\frac{T - T_\infty}{T_i - T_\infty}\right|_{T=50} = \frac{40-23}{150-23} = 0.134 = e^{-t/310}$$

Thus it will take $\underline{\underline{t = 623\mathrm{s} = 10\mathrm{min}, 23\mathrm{s}}}$ to cool the rods.

7.29 At what speed, u_∞, must 20°C air flow across an insulated tube before the insulation on it will do any good? The tube is at 60°C and 6 mm in diameter. The insulation is 12 mm in diameter with k = 0.08 W/m-°C. (Notice that we do not ask for the u_∞ for which the insulation will do the most harm.)

With reference to Fig. 2.14, we require that the sum of the thermal resistances of the insulated tube must exceed the thermal resistance around the uninsulated tube. So:

$$R_{t_{ins}} + R_{t_{\text{conv. for ins. tube}}} \geqslant R_{t_{\text{conv. for unins. tube}}} \quad ; \quad \frac{\ln r_o/r_i}{2\pi k_{ins}} + \frac{1}{2\pi r_o \bar{h}_{ins.}} \geqslant \frac{1}{2\pi r_i \bar{h}_{unins.}}$$

or:

$$\frac{\ln 2}{2\pi(0.08)} + \frac{1}{2\pi(0.006)\bar{h}_{ins}} \geqslant \frac{1}{2\pi(0.003)\bar{h}_{unins}} \quad ; \quad 1.379 + \frac{26.53}{\bar{h}_{ins.}} \geqslant \frac{53.05}{\bar{h}_{unins.}}$$

To calculate $\bar{h}_{ins}$ we shall evaluate properties at $T = 27°C$ ($T_{ins.} = 34°C$) and correct later if we must. ($\nu = 1.566 \cdot 10^{-5}$, $k = 0.02614$, $Pr = 0.711$)

To calculate $\bar{h}_{unins.}$ we evaluate at $(60+20)/2 = 40°C$ ($\nu = 1.69 \cdot 10^{-5}$, $k = 0.02707$, $Pr = 0.710$). Then using eqn. (7.68) we get:

$$\overline{Nu}_{D_{unins.}} = 0.3 + \frac{0.62\left(\frac{0.006}{1.69(10)^{-5}}\right)^{1/2} 0.710^{1/3}\sqrt{u_\infty}}{\left(1 + (0.4/0.710)^{0.667}\right)^{1/4}}\left[1 + \left(\frac{0.003}{1.69(0.282)}\right)^{1/2}\sqrt{u_\infty}\right]$$

$$= 0.3 + 9.15\sqrt{u_\infty}\left(1 + 0.0793\sqrt{u_\infty}\right)$$

&

$$\overline{Nu}_{D_{ins}} = 0.3 + \frac{0.62\left(\frac{0.012}{1.566 \cdot 10^{-5}}\right)^{1/2} 0.711^{1/3}\sqrt{u_\infty}}{\left(1 + (0.4/0.711)^{0.667}\right)^{1/4}}\left[1 + \left(\frac{0.012}{1.566(0.282)}\right)^{1/2}\sqrt{u_\infty}\right]$$

$$= 0.3 + 13.45\sqrt{u_\infty}\left(1 + 0.165\sqrt{u_\infty}\right)$$

Now solve by trial and error

u_∞ m/s	$\overline{Nu}_{D_{ins}}$	$\bar{h}_{ins} = \overline{Nu}_{D_i}\frac{0.02614}{0.012}$	$\overline{Nu}_{D_{unins}}$	$\bar{h}_{un} = \overline{Nu}_{D_u}\frac{0.02707}{0.006}$	$53.05/\bar{h}_{unins.}$	$1.379 + 26.53/\bar{h}_{ins.}$
1	15.97	34.79	10.18	45.9	1.156	2.142
0.5	10.92	23.79	7.133	32.18	1.648	2.494
0.25	7.58	16.51	5.056	22.81	2.32	2.986
0.2	6.76	14.72	4.537	20.47	2.59	3.181
0.1	4.78	10.40	3.266	14.74	3.60	3.929
0.05	3.42	7.45	2.382	10.75	4.94 =	4.94

Therefore, <u>the velocity must be at least 5.0 cm/s</u> if the insulation is to serve its function. (This gives $T_{outside}$ = 48.8°C so properties would better have been evaluated at (48.8 + 20)/2 = 34.4°C for $h_{insulation}$.

7.32 Evaluate $\overline{Nu}_D$ using Giedt's data for air flowing over a cylinder at $Re_D = 140{,}000$. Compare your result with the appropriate correlation, and with Figure 7.13

$$\overline{Nu}_D = \frac{1}{180}\int_0^{180} \frac{D}{k} h(\theta)\, d\theta = \frac{1}{180}\sum_i \left(\frac{Dh(\theta)}{k}\right)_i \delta\theta_i \quad \text{. Obtain data from Fig. 7.13}$$

$$= \frac{1}{180}\left[400(40) + 360(20) + 280(20) + 250(10) + 360(10) + 410(20) + 345(20) + 345(40)\right]$$

$\overline{Nu}_D = 354$ from Giedt's data

The appropriate correlation is eqn. (7.68). We don't know the temp. of the air, but the only property we need to evaluate is Pr & it is very insensitive to temp. Use $Pr = 0.711$ (for $T = 27°C$). Then:

$$\overline{Nu}_D = 0.3 + \frac{0.62(140{,}000)^{1/2}(0.711)^{1/3}}{\left[1+(0.4/0.711)^{2/3}\right]^{1/4}}\left[1+\left(\frac{140000}{282000}\right)^{1/2}\right] = \underline{310}$$

(If one erroneously used eqn. (7.65), he'd get $\overline{Nu}_D = 271$ which is low.)

The correlation underpredicts the data by 12.4%.

From Fig. 7.13 we read: at $Re_D = 140{,}000$, Giedt's data point gives $\{--\} = 451$ which is high but in the range of the other data. The correlation passes through $\{\cdots\} = 395$.

$$330 \leq \left\{(\overline{Nu}_D - 0.3)\left[1+\left(\frac{0.4}{0.711}\right)^{2/3}\right]^{1/4}\frac{1}{.711^{1/3}}\right\} \leq 460$$

7.33 A 25 mph wind blows across a 0.25in. telephone line. What is the pitch of the hum that it emits?

we don't know T_{air}, but between 0 and 100°F, $1.6(10)^{-5} < \nu\ \frac{m^2}{s} < 2.9(10)^{-5}$

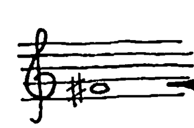
And 25 mph = 36.67 ft/s = 11.18 m/s , 0.25 in. = 0.0208 ft = 0.00635 m

Then $Re_D = \frac{u_\infty D}{\nu}$ so $2958 < Re_D < 4437$.

In this range (see Fig. 7.12) Str is virtually constant at 0.21.

Therefore: $0.21 = \frac{f_v D}{u_\infty}$; $f_v = 0.21(36.67)/0.0208 = \underline{370 \text{ cycle/sec}}$

[One half step in a tempered scale is a factor of $(2)^{1/12}$ or 1.0595 in frequency. We note that $440/(1.0595)^3 = 370$. Therefore we are 3 half tones below a concert A. The pitch is an F#.]



7.35 Consider the situation described in Problem 4.38 but suppose you do not know $\bar{h}$. Suppose instead that you know there is a 10 m/s crossflow of 27°C air over the rod. Then rework the problem.

With reference to the solution to Problem 4.38 we shall take the root temperature to be 122.4°C to evaluate properties. Then the average, average temp. on the rod is $\left(\frac{122.4+27}{2}+27\right)=50.8°C$. Let's evaluate properties at 325°C for simplicity's sake:

$\nu_{air}=1.814(10)^{-5}$, $k=0.02792$, $Pr=0.709$ and $Re_D=\frac{0.005(10)}{1.814(10)^{-5}}=2756$

Then, with the help of eqn. (7.66)

$$\overline{Nu}_D=0.3+\frac{0.62\sqrt{2756}\,(0.709)^{1/3}}{\left[1+(0.4/0.709)^{2/3}\right]^{1/4}}=25.8\quad,\quad \bar{h}=25.8\,\frac{0.02792}{0.005}=\underline{144\ \frac{W}{m^2\text{-}°C}}$$

This is within 4% of the original assumption of 150, so no reiteration is needed. Then in accordance with Problem 4.38, solution:

$$\Delta T=\frac{q_o}{k\,m}=\frac{q_o}{k\,m_{previous}}\sqrt{\frac{h_{previous}}{h_{new}}}=95.4\sqrt{\frac{150}{144}}=\underline{97.4°C}$$

Thus: $T_{base}=97.4+27=\underline{\underline{124.4°C}}$

7.36 A liquid whose properties are not known flows across a 40 cm diameter tube at 20 m/s. The measured heat transfer coefficient is 8000 W/m²°C. We can be fairly confident that Re_D is very large indeed. What would $\bar{h}$ be if D were 53 cm? What would $\bar{h}$ be if u_∞ were 28 m/s?

At large Re_D eqn.(7.68) reduces to: $\overline{Nu}_D=fn\left(\begin{smallmatrix}\text{physical}\\\text{properties}\end{smallmatrix}\right)Re_D$

or: $\bar{h}=fn(\text{physical properties})\,u_\infty$

Therefore: A change of diameter will not influence $\bar{h}$

And since $\bar{h}\sim u_\infty$, the new $\bar{h}$ will be:

$$\underline{\underline{\bar{h}=8000\,\frac{28}{20}=11{,}200\ \frac{W}{m^2\text{-}°C}}}$$

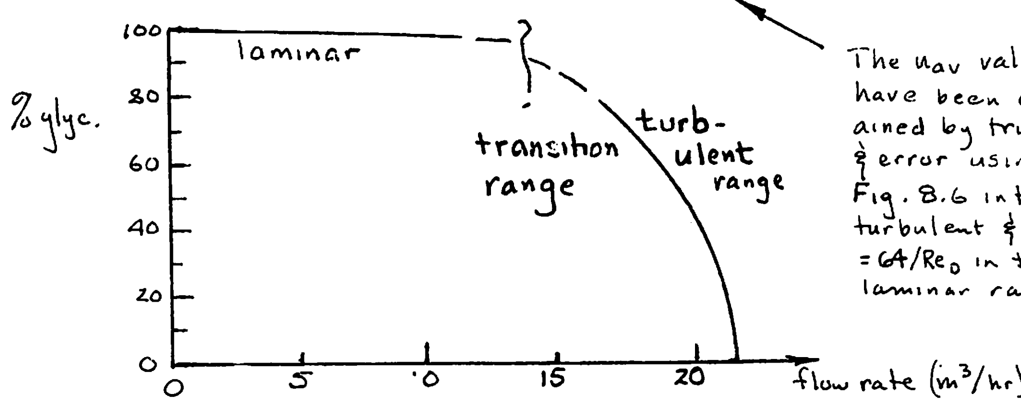
7.38 **Glycerin is added to water in a mixing tank at 20°C. The mixture discharges through a 4 m length of 0.04 m ID tubing under a constant 3 m head. Plot the discharge rate in m^3/hr as a function of composition.**

Using eqn. (8.20),

$$f = \frac{3\,m - \dfrac{u_{av}^2}{2g}}{\dfrac{4}{0.04}\,\dfrac{u_{av}^2}{2g}} = \frac{3 - 0.051\,u_{av}^2}{5.10\,u_{av}^2} = \begin{cases} f(Re_D) \text{ from Fig. 7.6} \\[4pt] \dfrac{64}{Re_D} = \dfrac{1600\,\nu}{u_{av}} \end{cases}$$

(Note to instructors: How many students will forget to add the velocity head to 3m? Perhaps you should remind them.)

% glyc.	ν m^2/s	$\frac{u_{av}D}{\nu} = Re_D$	The u_{av} consist with eqn. above, & Fig 7.6		flow rate $\frac{\pi}{4}(.04)^2 u_{av}\,3600$
0	$1.035(10)^{-6}$	38647 u_{av}	4.71	turbulent	21.31 m^3/hr
20	1.681 "	23795 u_{av}	4.62	"	20.90
40	3.467 "	11537 u_{av}	4.39	"	19.86
60	9.36 "	4274 u_{av}	3.93	"	17.78
80	$4.97(10)^{-5}$	805 u_{av}	(3.42 12.6	if turb.) if lam.)	(15.47 57.00)
100	0.00112	35.7 u_{av}	0.0329	laminar	0.15



The u_{av} values have been obtained by trial & error using Fig. 8.6 in the turbulent & $f = 64/Re_D$ in the laminar range.

7.40 Rework Problem 5.40 without assuming the Bi to be very large.

We need $\bar{h}$. Since Bi should not be small we shall evaluate properties close to the gas temp. -- at 277°C or 550°K -- and use eqn. (7.68). Then: $\nu = 4.456(10)^{-5}$, $k = 0.0426$, $Pr = 0.698$

$$Re_D = \frac{0.26(1)}{4.456(10)^{-5}} = 5835 \quad \& \quad \overline{Nu}_D = 0.3 + \frac{0.62\sqrt{5835}(0.698)^{1/3}}{1.14}\left[1+\left(\frac{5835}{282000}\right)^{5/8}\right]^{4/5} = \underline{39.74}$$

$$\bar{h} = 39.74\,\frac{0.0426}{0.26} = 6.51, \quad Bi^{-1} = \frac{0.68}{6.51(0.13)} = 0.803 \text{ (where we use } k = k_{H_2O})$$

This gives $Fo = 0.3$ so $t = 0.3\,(0.13)^2/1.35(10)^{-7} = \underline{37{,}555 \text{ sec}}$

Therefore the cooking time is considerably extended to $\underline{10.43 \text{ hrs}}$. The cooking should actually commence at about $\underline{5{:}30\text{AM}}$

(When this is done in Utah, the pig is started around 7:00 or 8:00. It cooks more quickly than we predict because the flame also heats a bed of coals which radiate additional heat to the pig.)

7.41 Water enters a 0.5 cm ID pipe at 24°C. The pipe walls are held at 30°C. Plot T_b against distance from entry, if u_{av} is 0.27 m/s, neglecting entry behavior in your calculation. (Indicate the entry region on your graph, however).

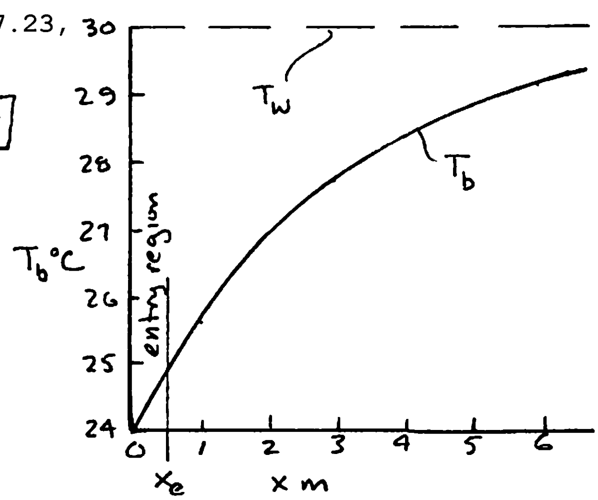
At 27°C: $\nu = 0.826(10)^{-6}$, $Pr = 5.65$, $Re_D = \dfrac{0.005(0.27)}{0.826(10)^{-6}} = 1634$ (laminar)

Then from eqns. 7.57, 7.58, and 7.23,

$$\frac{T_b - T_{b_{in}}}{T_w - T_{b_{in}}} = 1 - \exp\left[-\frac{\overset{=3.658}{Nu_D}}{Pr\,Re_D}\,\frac{4}{D}\,x\right]$$

$$\underline{T_b = 24 + 6\left[1 - \exp(-0.317x)\right]}$$

$$x_e = 0.05\,D\,Re_D = \underline{0.409\text{m}}$$



7.42 Devise a numerical scheme that will allow you to be able to find the velocity distribution and friction factor in a square duct of side length a. Set up a square grid of size N by N and solve the difference equations by hand for N = 2,3 and 4. Hint: First show that the velocity distribution is given by the solution to the equation

$$\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} = 1$$

where $\bar{u} = 0$ on the sides of the square and $\bar{u} = u\Big/\frac{a^2}{\mu}\frac{dp}{dz}$, $\bar{x} = \frac{x}{a}$ and $\bar{y} = \frac{y}{a}$. Then show that the friction factor, f, [equation (8.21)], is given by

$$f = \frac{-2}{\frac{\rho u_{av} a}{\mu} \iint \bar{u}\, d\bar{x}\, d\bar{y}}$$

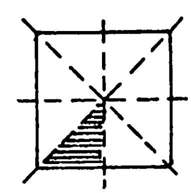
Note that the area integral can be evaluated as $\sum \bar{u}/N^2$.

<u>Solution:</u> By following the discussion preceding & following eqn. (8.6) we write the mom. eqn. for a square duct as:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{dp}{dx} \quad \text{or, using } \bar{x} = \frac{x}{a},\ \bar{y} = \frac{y}{a},\ \&\ \bar{u} = \frac{u}{\frac{a^2}{\mu}\frac{dp}{dx}}$$

we get: $\underline{\dfrac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \dfrac{\partial^2 \bar{u}}{\partial \bar{y}^2} = 1}$ with b.c.s $\bar{u} = 0$ on all sides.

Now, since the flow must be symmetrical about the diagonals & about the bisectors of the sides, we need only solve for flow in $(1/8)^{th}$ of the duct to know the entire flow field. However:



Laying out a square grid of size $\Delta\bar{x}$ & using central differences about a general point i, j, we get

$$\frac{\bar{u}_{i-1,j} + \bar{u}_{i+1,j} - 2\bar{u}_{i,j}}{\Delta\bar{x}^2} + \frac{\bar{u}_{i,j-1} + \bar{u}_{i,j+1} - 2u_{i,j}}{\Delta\bar{x}^2} = 1$$

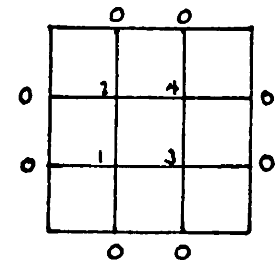
7.42 (Continued)

Now for good accuracy $\Delta\bar{x}$ must be $<< 1$. However, we only want to show the method here so we use a very coarse grid: $\Delta\bar{x} = 1/3$

$$\bar{u}_1 = \frac{\bar{u}_2 + \bar{u}_3}{4} - \frac{1}{4(9)}$$

$$\bar{u}_2 = \frac{\bar{u}_1 + \bar{u}_4}{4} - \frac{1}{4(9)}$$

etc.



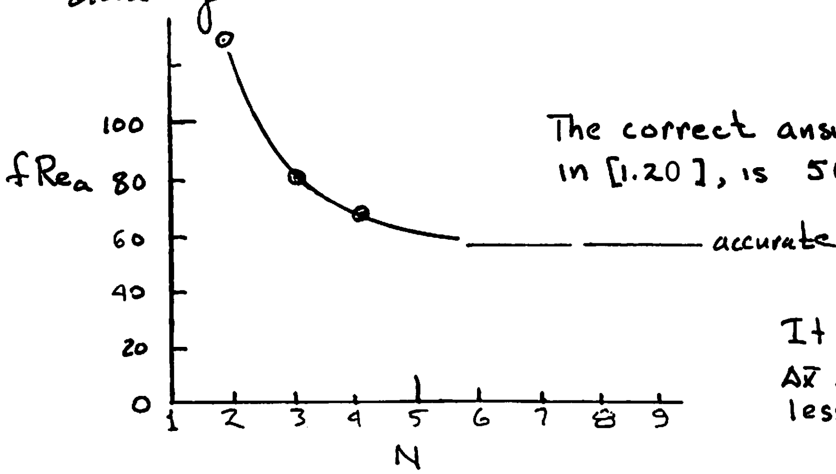
By inspection we find that $\bar{u}_1 = \bar{u}_2 = \bar{u}_3 = \bar{u}_4 = -\frac{1}{18}$ in this case.

The friction factor (eqn. (7.34)) is $f = \frac{2a}{\rho u_{av}^2}\frac{dp}{dz}$

but $u_{av} = \frac{1}{A}\sum u \Delta x^2$ so: $f\,\frac{\rho u_{av} a}{\mu} = \frac{2N^2}{-\sum \bar{u}}$

$$\underline{f\,Re_a = \frac{2(3)(3)}{4\left(\frac{1}{18}\right)} = 81}$$

This computation can be redone for larger N's (or smaller $\Delta\bar{x}$'s.) $\Delta\bar{x} = \frac{1}{4}$ can in fact still be done by hand with the result that $f\,Re = 69.424$.



The correct answer, given in [1.20], is 56.8.

It looks like $\Delta\bar{x}$ should be less than 1/5.

PROBLEM 7.51 Consider the water-cooled annular resistor of Problem 2.49 (Fig. 2.24). The resistor is 1 m long and dissipates 9.4 kW. Water enters the inner pipe at 47 °C with a mass flow rate of 0.39 kg/s. The water passes through the inner pipe, then reverses direction and flows through the outer annular passage, counter to the inside stream.

a) Determine the bulk temperature of water leaving the outer passage.
b) Solve Problem 2.49 if you have not already done so. Compare the thermal resistances between the resistor and each water stream, R_i and R_o.
c) Use the thermal resistances to form differential equations for the streamwise (x-direction) variation of the inside and outside bulk temperatures ($T_{b,o}$ and $T_{b,i}$) and an equation the local resistor temperature. Use your equations to obtain an equation for $T_{b,o} - T_{b,i}$ as a function of x.
d) Sketch qualitatively the distributions of bulk temperature for both passages and for the resistor. Discuss the size of: the difference between the resistor and the bulk temperatures; and overall temperature rise of each stream. Does the resistor temperature change much from one end to the other?
e) Your boss suggests roughening the inside surface of the pipe to an equivalent sand-grain roughness of 500 μm. Would this change lower the resistor temperature significantly?
f) If the outlet water pressure is 1 bar, will the water boil? *Hint:* See Problem 2.48.
g) Solve your equations from part (c) to find $T_{b,i}(x)$ and $T_r(x)$. Arrange your results in terms of $\text{NTU}_o \equiv 1/(\dot{m}c_p R_o)$ and $\text{NTU}_i \equiv 1/(\dot{m}c_p R_i)$. Considering the size of these parameters, assess the approximation that T_r is constant in x.

SOLUTION

a) The answer follows directly from the 1st Law, $Q = \dot{m}c_p\left(T_{b,\text{out}} - T_{b,\text{in}}\right)$:

$$\Delta T_b = Q/(\dot{m}c_p) = 9400/(0.39 \cdot 4180) = 5.77\ °\text{C}$$

so $T_{b,\text{out}} = 47 + 5.77 = \underline{52.8\ °\text{C}}$.

b) The inside thermal resistance, $R_i = 3.69 \times 10^{-2}$ K/W, is <u>23% greater</u> than the outside resistance, $R_o = 3.00 \times 10^{-2}$ K/W.

c) With eqn. (7.10), putting $(q_w P)_{\text{inside}} = (T_r - T_{b,i})/R_i L$ and $(q_w P)_{\text{outside}} = (T_r - T_{b,o})/R_o L$ where the tube length is $L = 1$ m:

$$\dot{m}c_p \frac{dT_{b,i}}{dx} = \frac{T_r - T_{b,i}}{R_i L} \tag{1}$$

$$-\dot{m}c_p \frac{dT_{b,o}}{dx} = \frac{T_r - T_{b,o}}{R_o L} \tag{2}$$

Recalling the solution of Problem 4.29, we can divide the resistance equation by L to obtain a local result (assuming that h is equal to $\overline{h}$ along the entire passage):

$$\frac{T_r - T_{b,i}}{R_i L} + \frac{T_r - T_{b,o}}{R_o L} = \frac{Q}{L} = \text{constant} \tag{3}$$

Each of $T_{b,i}$, $T_{b,o}$, and T_r are functions of x.

By adding eqn. (1) to eqn. (2), and then using eqn. (3),

$$-\dot{m}c_p \frac{d(T_{b,o} - T_{b,i})}{dx} = \frac{Q}{L}$$

and integrating (with $T_{b,o} = T_{b,i}$ at $x = L$), we find

$$\boxed{T_{b,o} - T_{b,i} = \frac{Q}{\dot{m}c_p}(1 - x/L)} \tag{4}$$

d) From working part (a) and Problem 2.49, we already know that the resistor will be much hotter than the water on either side (194 °C at the end where the water enters and exits). At any point, $T_r - T_b \gg T_{b,o} - T_{b,i}$, so that $T_r - T_{b,i} \simeq T_r - T_{b,o} \simeq$ constant, along the entire passage. From eqns. (1) and (2), then, the bulk temperature of each stream has a nearly straight line variation in x, but the outer passage temperature rises a bit faster because the thermal resistance on that side is lower. Similarly, eqn. (3) shows that the resistor temperature varies by no more than do the bulk temperatures.

e) Your solution to Problem 2.49 shows that the epoxy layers provide the dominant thermal resistance on each side. Roughness will make the convection resistance smaller, but convection resistance is only about 10% of the overall resistance. Your boss's idea will add cost and pressure drop, but it won't lower the resistor temperature much. (*Suggestion:* Find a diplomatic way to tell him that.)

f) The water will not boil if the highest temperature of the epoxy is below T_{sat}. The hottest point for the epoxy is in the outlet stream at the exit (where the bulk temperature is greatest). From the solution to Problem 2.49, using the voltage divider relation from Problem 2.48,

$$T_{\text{epoxy}} - T_{b,\text{outlet}} = (T_r - T_{b,\text{outlet}})\frac{R_{\text{conv}}}{R_{\text{outside}}} = (194 - 52.8)\frac{0.00307}{0.0300} = 14.4 \text{ K}$$

The water will not boil.

g) Rearranging eqn. (3) with eqn. (4):

$$T_r - T_{b,i} + (T_r - T_{b,i})\frac{R_i}{R_o} = QR_i - (T_{b,o} - T_{b,i})\frac{R_i}{R_o}$$

$$(T_r - T_{b,i})\left(1 + \frac{R_i}{R_o}\right) = QR_i - \frac{QR_i}{\dot{m}c_p R_o}(1 - x/L)$$

$$T_r - T_{b,i} = (QR_i)\left(\frac{R_o}{R_o + R_i}\right)\left[1 - \frac{1}{\dot{m}c_p R_o}(1 - x/L)\right] \tag{5}$$

From eqn. (3), we may estimate that $QR_i \approx (T_r - T_{b,i})/2$; thus, we can see that the second term on the right is very small and could be neglected entirely.

Upon substituting eqn. (5) into eqn. (1) we have:

$$\dot{m}c_p\frac{dT_{b,i}}{dx} = \frac{Q}{L}\left(\frac{R_o}{R_o + R_i}\right)\left[1 - \frac{1}{\dot{m}c_p R_o}(1 - x/L)\right]$$

Integration gives:

$$T_{b,i}(x) - T_{b,\text{in}} = \frac{Q}{\dot{m}c_p}\left(\frac{R_o}{R_o + R_i}\right)\left[\frac{x}{L} - \frac{1}{\dot{m}c_p R_o}\left(\frac{x}{L} - \frac{x^2}{2L^2}\right)\right]$$

Because the second term in the square brackets is small, we see that the bulk temperature has an essentially straight line variation.

More precisely, we may think of this arrangement as a heat exchanger, where $UA = 1/R_o$ so that

$$\mathrm{NTU}_o = \frac{UA}{\dot{m}c_p} = \frac{1}{\dot{m}c_p R_o} = \frac{1}{(3.00\times 10^{-2})(0.39)(4180)} = 0.020 \ll 1$$

From Chapter 3, we recall that a heat exchanger with very low NTU causes very little change in the temperature of the streams, as is the case here. Putting our result in terms of the outside and inside NTUs:

$$\boxed{T_{b,i}(x) - T_{b,\mathrm{in}} = (QR_i)\mathrm{NTU}_i\left(\frac{R_o}{R_o+R_i}\right)\left[\frac{x}{L} - \mathrm{NTU}_o\left(\frac{x}{L} - \frac{x^2}{2L^2}\right)\right]} \tag{6}$$

Substituting eqn. (6) into eqn. (5):

$$\boxed{T_r - T_{b,\mathrm{in}} = (QR_i)\left(\frac{R_o}{R_o+R_i}\right)\left\{1 - \mathrm{NTU}_o\left(1-\frac{x}{L}\right) - \mathrm{NTU}_i\left[\frac{x}{L} - \mathrm{NTU}_o\left(\frac{x}{L} - \frac{x^2}{2L^2}\right)\right]\right\}}$$

Since NTU_i has a similar value to NTU_o, the resistor temperature is indeed nearly constant, with variations on the order of $\mathrm{NTU}_0 = 0.02$.

8.1 Show that Π_4 is equivalent to $PrRe^2/Ja$.

The velocity that the condensing film would reach in free fall through a characteristic length is $\sqrt{g(\rho_f-\rho_g)L/\rho_f}$. Let's call this u_{char}. Then:

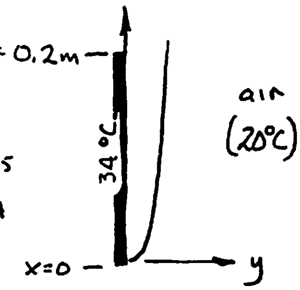
$$\Pi_4 = \frac{\rho_f(\rho_f-\rho_g)g\,h_{fg}L^3}{\mu k(T_{sat}-T_w)} = \frac{\rho_f u_c^2 h_{fg} L^2}{\nu k(T_{sat}-T_w)}$$

$$= \frac{u_c^2L^2}{\nu^2}\,\frac{\nu}{\alpha}\,\frac{h_{fg}}{c_p(T_{sat}-T_w)} = \frac{Re^2 Pr}{Ja}$$

8.2 For the figure shown, Plot

δ and h vs. x

T and u vs. y

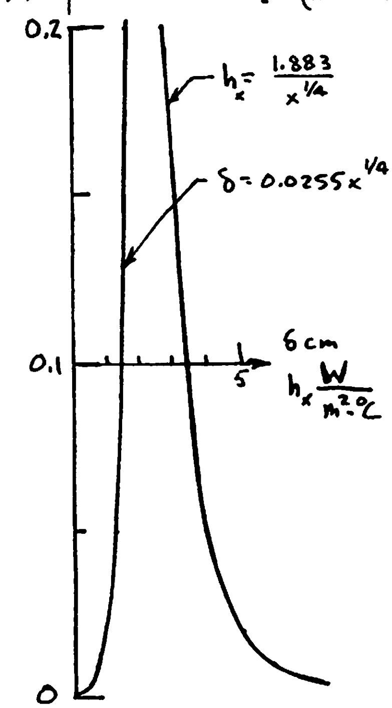


$T_{avg} = 27°C$
$Pr = 0.711$
$\nu = 1.566\times10^{-5}$
$k = 0.02614$
$\beta = \frac{1}{300} = 0.00333$

from eqns. (8.24) & (8.27)

$$\delta = 4.87\,x/Ra_x^{1/4}\,,\quad Nu_x = 0.3773\,Ra_x^{1/4}$$
$$h_x = 0.00986\,Ra_x^{1/4}/x$$

and $Ra_x^{1/4} = \left[\frac{9.8(0.00333)(14)}{(1.566(10)^{-5})^2}.711\right]^{1/4} x^{3/4} = 191\,x^{3/4}$



$$\frac{u}{u_c} = \frac{y}{\delta}\left(1-\frac{y}{\delta}\right)^2$$

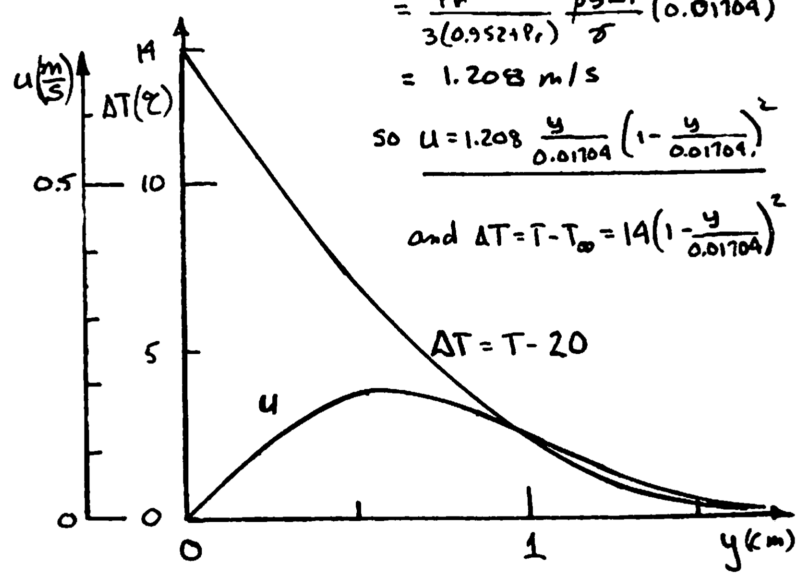
where $u_c = C_1\frac{\beta g\Delta T}{\nu}\delta^2$

$$= \frac{Pr}{3(0.952+Pr)}\frac{\beta g\Delta T}{\nu}(0.01704)^2$$

$$= 1.208 \text{ m/s}$$

so $u = 1.208\frac{y}{0.01704}\left(1-\frac{y}{0.01704}\right)^2$

and $\Delta T = T - T_\infty = 14\left(1-\frac{y}{0.01704}\right)^2$



8.3 Re-do the Squire-Eckert analysis neglecting inertia.

Omitting the inertial terms form the momentum equation, we reduce the equation after equation (8.20) to:

$$0 = \frac{1}{3} - C_1 \qquad \text{or} \qquad \underline{C_1 = 1/3}$$

eqn. (8.22) is unchanged, so we put this C_1 in it & get:

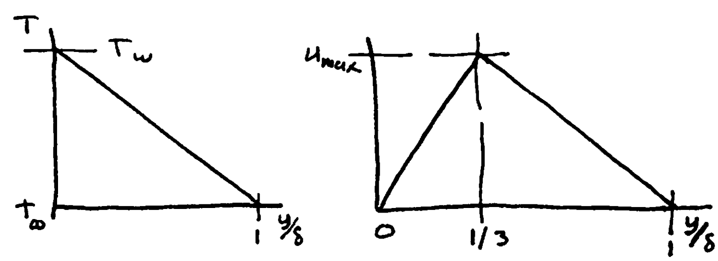
$$\delta^4 = \frac{240\,\nu^2}{\beta g\,\Delta T\,Pr}\,x \quad ; \quad \frac{\delta}{x} = 3.936\ Ra^{-1/4}$$

Then:

$$\underline{\underline{Nu_x = 2\frac{x}{\delta} = 0.508\ Ra_x^{1/4}}}$$

This is exactly the Squire-Eckert result for $Pr \gg 1$.

8.4 Predict Nu_x, using an integral method and the assumed profiles shown:



$$\frac{T - T_\infty}{T_w - T_\infty} = 1 - \frac{y}{\delta} \qquad \frac{u}{u_{max}} = 3\cdot\frac{y}{\delta}\ ,\ \frac{y}{\delta} \le \frac{1}{3}$$
$$= 3\left(1 - \frac{y}{\delta}\right)\ ;\ \frac{y}{\delta} \ge \frac{1}{3}$$

Thus:
$$\frac{d}{dx}\,\delta u_{max}^2 \underbrace{\left[\int_0^{1/3} 9\left(\frac{y}{\delta}\right)^2 d\left(\frac{y}{\delta}\right) + \int_{1/3}^{1} \frac{9}{4}\left(1 - \frac{y}{\delta}\right)^2 d\,\frac{y}{\delta}\right]}_{=1/3} = g\beta\Delta T\,\delta \int_0^1 \left(1 - \frac{y}{\delta}\right) d\,\frac{y}{\delta} - \frac{3\nu u_{max}}{\delta}$$

so:
$$\underline{\frac{1}{3}\frac{d}{dx}(\delta u_{max}^2) = \frac{g\beta\Delta T\,\delta}{2} - \frac{3\nu u_{max}}{\delta}} \qquad \text{mom. eqn.}$$

and the energy equation gives:

$$\frac{d}{dx}\,\delta u_{max}\Delta T \underbrace{\left[\int_0^{1/3} 3\frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) d\left(\frac{y}{\delta}\right) + \int_{1/3}^{1} \frac{3}{2}\left(1 - \frac{y}{\delta}\right)^2 d\,\frac{y}{\delta}\right]}_{=15/54} = + \frac{\alpha\Delta T}{\delta}$$

8.4 (continued)

or: $$\underline{\frac{5}{18}\frac{d(\delta u_{max})}{dx} = \frac{\alpha}{\delta}}$$ energy eqn.

But $u_{max} = C_1\delta^2$ so the mom. & en. eqns. become:

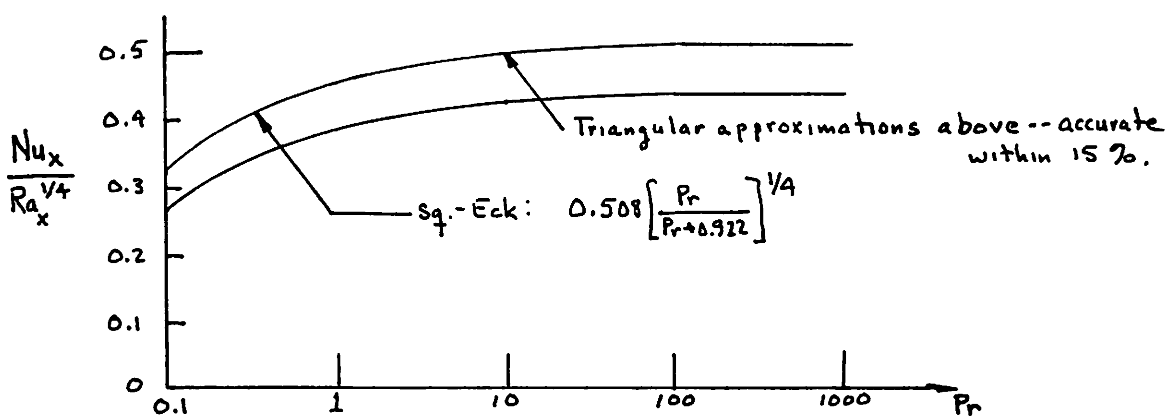
momentum: $$C_1^2\,\frac{1}{3}\frac{5}{4}\frac{d\delta^4}{dx} = \frac{g\beta\Delta T}{2} - 3\nu C_1 \qquad \text{or} \qquad \underline{\delta^4 = \left(\frac{6g\beta\Delta T}{5} - \frac{36\nu C_1}{5}\right)\frac{x}{C_1^2}}$$

energy: $$C_1\,\frac{5}{8}\frac{3}{4}\frac{d\delta^4}{dx} = \alpha \qquad \text{or} \qquad \underline{\delta^4 = \frac{24}{5}\frac{\alpha x}{C_1}}$$

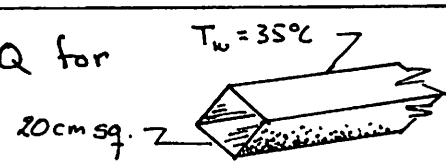
Equating these eqns. for δ^4, we get: $C_1 = \dfrac{g\beta\Delta T}{6\alpha(Pr + \frac{2}{3})}$, so:

$$\frac{\delta}{x} = \sqrt[4]{\frac{24\times 6}{5}\,\frac{\alpha\nu}{g\beta\Delta T x^3}\left(Pr + \frac{2}{3}\right)} = 2.317\,Ra_x^{-1/4}\left(\frac{Pr + 2/3}{Pr}\right)^{1/4}$$

and $$\underline{\underline{Nu_x = -\frac{\frac{dT}{dx}}{\Delta T}x = \frac{x}{\delta} = 0.432\left(\frac{Pr}{Pr + \frac{2}{3}}\right)^{1/4} Ra_x^{1/4}}}$$



8.5 Find $\bar{h}$ & Q for



$(T_{air} = 25°C)$

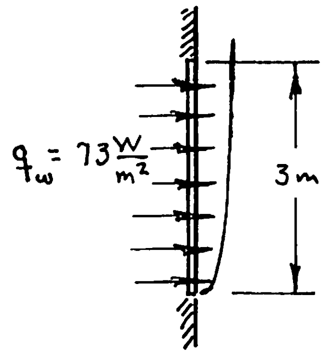
We know that $Nu_\tau = 0.52\,Ra_\tau^{1/4}$ where $\tau = 2(20)\,cm = 0.4\,m$. Thus (at $\bar{T} = 303°C$)

$$\bar{h} = 0.52\,\frac{k_{air}}{0.4m}\left(\frac{g\beta\Delta T(0.4)^3}{\nu\alpha}\right)^{1/4} = 0.52\,\frac{0.02635}{0.4}\left[\frac{9.8\,\frac{1}{25+273}\,(10)(0.4)^3}{(1.596)(2.246)}\right]^{1/4}$$

or $$\underline{\underline{\bar{h} = 3.00\,\frac{W}{m^2\text{-}°C}}}$$

and $$\underline{\underline{Q = \bar{h}A\Delta T = 3.00\,\frac{W}{m^2\,°C}\,\frac{4(0.2)\,m^2}{m}\,10°C = 24.0\,\frac{W}{m}}}$$

8.6 Heat heat flux from a 3 m high electrically heated panel in a wall is 75 W/m^2 in an 18^{o}C room. What is the average temperature of the panel? What is the temperature at the top? -- at the bottom?



$$Ra^*_L = \frac{g\beta q_w L^4}{k\nu\alpha} = \frac{(9.8/(273+18))(75)(3^4)}{0.02614(15.66)(.2203)} 10^{10}$$

where we assume $(\overline{T_w}+T_\infty)/2 = 300$. Then $Ra^*_L = 2.2 \times 10^{13}$

$$\frac{0.67\, Ra^{*\,1/4}_L}{\left[1+\left(\frac{0.492}{Pr}\right)^{9/16}\right]^{4/9}} = \frac{0.67\,(2182)}{\left(1+\left(\frac{0.492}{.708}\right)^{9/16}\right)^{4/9}} = 1122 = Nu_L^{5/4} - 0.68\, Nu_L^{1/4}$$

$\underline{Nu_L = 276}$

Then $\overline{\Delta T} = \frac{q_w}{Nu_L}\frac{L}{k} = \frac{75(3)}{276(.02614)} = \underline{31.2}$

This gives $\overline{T_w} = 319.5$°K and we assumed 309°C If we accept this as close enough

$$\overline{T_w} = 18 + 31.2 = \underline{\underline{49.2\,°C}}$$

and since this "average" is a mid-point value we go to Fig. 8.9 and write:

$$\frac{49.2}{C} = 0.8706 \quad , \quad C = 56.5$$

so $\qquad \Delta T = 56.5(x/L)^{1/5}$

at the leading edge $\Delta T = 56.5 \times 0 = 0$, $\underline{\underline{T_{x=0} = 18\,°C}}$

at the top, $\underline{\Delta T = 56.5\,°C}$ $\qquad \underline{\underline{T_{x=3} = 56.5\,°C}}$

8.7 Find pipe diameters and wall temperatures for which the film condensation heat transfer coefficients given in Table 1.1 are valid.

We must make some assumptions here since there are many circumstances under which these values would be obtainable. Let us take p = 1 atm. Then:

for water:

$h_{fg} = 2{,}257{,}000$ J/kg $\qquad c_p = 4.219 \times 10 \ \frac{kJ}{kg}$

$\rho_g = 0.577$ kg/m³

$\rho_f = 957.2$ "

$k_f = 0.6811$ W/m·°C

$\mu_f = 0.000278$ kg/m-s

for benzene:

$T_{sat} = 68°C$

$h_{fg} = 407000$

$\rho_g = 1.92$

$\rho_f = 827$

$k_f = 0.164$

$\mu_f = 0.000365$

$c_p = 1.74 \times 10$

Note to instructor: These properties are not available in the text. The student must go to conventional data sources to obtain them.

$$\bar{h} = 0.729\, k_f \left[\frac{\rho_f(\rho_f - \rho_g) g\, h_{fg}}{\mu_f k_f}\right]^{1/4} \frac{1}{(D\Delta T)^{1/4}} \left(1 + \frac{c_p}{h_{fg}} \Delta T\right)^{1/4}$$

$$15{,}000 = 8979 \left(\frac{1 + 0.00187\Delta T}{D\Delta T}\right)^{1/4} \quad \text{for water}$$

$$1700 = 1746 \left(\frac{1 + 0.0043\Delta T}{D\Delta T}\right)^{1/4} \quad \text{for benzene}$$

Some possible solutions:

water: $\Delta T = 3°C$, $D = 0.043$ m
$\Delta T = 6°C$, $D = 0.0216$ m
$\Delta T = 10°C$, $D = 0.013$ m

benzene: $\Delta T = 10°C$, $D = 0.116$ m
$\Delta T = 15°C$, $D = 0.079$ m
$\Delta T = 25°C$, $D = 0.049$ m

8.8 A 0.3 m high plate at 90°C condenses steam at 1 atm. Change the height or the temperature to values that will cause the laminar to turbulent transition to occur at the bottom.

From eqn. (8.72), turbulence occurs when:

$$\delta_{tu} = \sqrt[3]{[3\mu_f^2/\rho_f(\rho_f-\rho_g)g]450} \simeq \sqrt[3]{(3\nu_f^2/g)450}$$

$$= \sqrt[3]{[3(0.294)^2 10^{-12}/9.8]450} = \underline{0.0000228\ \text{m}}$$

Then, using eqn. (8.56) we get,

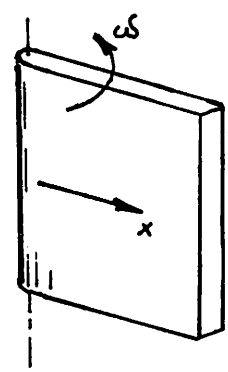
$$\frac{\delta_{tu}}{\delta_{30}} = \left(\frac{x_{tu}}{0.3\text{m}}\right)^{1/4} \; ; \; x_{turb} = 0.3\left(\frac{0.000228}{0.000103}\right)^4 = \underline{\underline{7.28\ \text{m}}}$$

or

$$\frac{\delta_{tu}}{\delta_{30_{cm}}} = \left(\frac{\Delta T_{turb}}{10°\text{C}}\right)^{1/4} \; ; \; \Delta T_{turb} = 10\left(\frac{0.000228}{0.000103}\right)^4 = \underline{\underline{240°\text{C}}}$$

We can't reach turbulence in a 0.3 cm length by cooling. The flow would freeze up first.

8.9 A cool plate spins in a synchronously rotating vapor, so $g(x) = \omega^2 x$. Find: Nu_L



$$g_{eff} = \frac{x\omega^{8/3}x^{4/3}}{x^{2/3}\int_0^x x^{1/3}dx} = \frac{4\omega^2}{3}x$$

so:

$$Nu_x = \left(\frac{\rho_f(\rho_f-\rho_g)4\omega^2 x^4 h'_{fs}}{4\mu k\Delta T\ 3}\right)^{1/4}$$

and

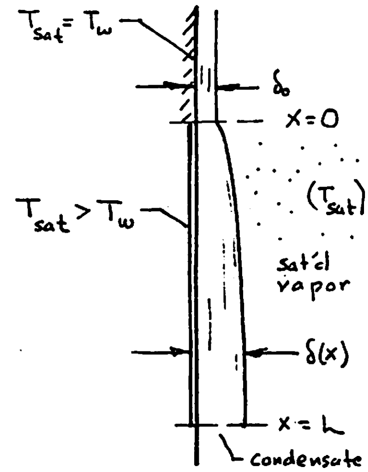
$$h = \left(\frac{\rho_f-\rho_g)\omega^2 k^3 h'_{fs}}{3\mu\Delta T}\right)^{1/4} = \text{constant} = \bar{h}$$

Thus

$$\underline{\underline{Nu_L = 0.760\left(\frac{\rho_f(\rho_f-\rho_g)\omega^2 h'_{fg}L^3}{\mu k\Delta T}\right)^{1/4}}}$$

8.10 For the flow shown, calculate

$\delta(x)$, Nu_x, and Nu_L



Eqn. (8.55) applies in this case.
We rewrite it as follows:

$$\underbrace{\delta^3 \frac{d\delta}{dx}}_{\frac{1}{4}\frac{d\delta^4}{dx}} = \frac{k\mu\Delta T}{\rho_f(\rho_f-\rho_g)g\,h_{fg}'}$$

And we integrate this using the b.c. $\delta(x=0) = \delta_o$:

$$\delta^4 - \delta_o^4 = \frac{k\mu\Delta T\,x}{\rho_f(\rho_f-\rho_g)g\,h_{fg}'} \quad \text{or} \quad \underline{\underline{\delta(x) = \left[\frac{4k\mu\Delta T\,x}{\rho_f(\rho_f-\rho_g)g\,h_{fg}'} + \delta_o^4\right]^{1/4}}}$$

so :

$$\underline{\underline{Nu_x = \frac{x}{\delta} = \left[\frac{4k\mu\Delta T}{\rho_f(\rho_f-\rho_g)g\,h_{fg}'\,x^3} + \left(\frac{\delta_o}{x}\right)^4\right]^{-1/4}}}$$

and

$$Nu_L = \frac{L}{k}\,\frac{1}{L}\int_0^L h(x)\,dx = \int_0^L \frac{dx}{\left[\frac{4k\mu\Delta T\,x}{\rho_f(\rho_f-\rho_g)g\,h_{fg}'} + \delta_o^4\right]^{1/4}}$$

$$= \frac{4}{3}\,\frac{\rho_f(\rho_f-\rho_g)g\,h_{fg}'}{4k\mu\Delta T}\left[\frac{4k\mu\Delta T\,x}{\rho_f(\rho_f-\rho_g)g\,h_{fg}'} + \delta_o^4\right]^{3/4}\Bigg]_{x=0}^{x=L}$$

$$\underline{\underline{Nu_L = \frac{4}{3}\left[\left(\frac{\rho_f(\rho_f-\rho_g)g\,h_{fg}'L^3}{4k\mu\Delta T}\right)^{1/3} + \left(\frac{\rho_f(\rho_f-\rho_g)g\,h_{fg}'\delta_o^3}{4k\mu\Delta T}\right)^{4/3}\right]^{3/4} - \frac{4\rho_f(\rho_f-\rho_g)g\,h_{fg}\,\delta_o^3}{3k\mu\Delta T}}}$$

8.11 Prepare a table of formulas of the form:

$$\bar{h}\ \mathrm{W/m^2{-}^oC} = C[\Delta T\ ^oC/L\ m]^{1/4}$$

for natural convection at normal gravity in air and in water at $T_\infty = 27^oC$. Assume that T_w is close to 27^oC. Your table should include results for vertical plates, horizontal cylinders, spheres, and possibly additional geometries. Do not include your calculations.

$$Ra_{H_2O} = \left(\frac{g\beta}{\alpha\nu}\right)\Delta T L^3 = \left(\frac{9.8(0.000275)}{1.462(0.826)\times 10^{-13}}\right)\Delta T L^3 = 2.232\times 10^{10}\ \Delta T L^3$$

$$Ra_{air} = \left(\frac{9.8\ \frac{1}{300}}{1.566(2.203)\times 10^{-10}}\right)\Delta T L^3 = 9.47\times 10^{7}\ \Delta T L^3$$

configuration	reference equation	simplified formula for $\bar{h}$: water	simplified formula for $\bar{h}$: air
vertical plate	eqn. (8.27)	$153\,(\Delta T/L)^{1/4}$	$1.414\,(\Delta T/L)^{1/4}$
horizontal cyl.	eqn. (8.28) (neglect lead const. restrict to larger values of Ra_D)	$109\,(\Delta T/D)^{1/4}$	$1.01\,(\Delta T/D)^{1/4}$
sphere	eqn. (8.32) (neglect lead const. restrict to larger Ra_D's.)	$101\,(\Delta T/D)^{1/4}$	$1.11\,(\Delta T/D)^{1/4}$
other situation where $\overline{Nu_D} = C\,Ra_D^{1/4}$		$C(235)\left(\frac{\Delta T}{D}\right)^{1/4}$	$C(2.58)\left(\frac{\Delta T}{D}\right)^{1/4}$

8.12 For what value of the Prandtl number is the condition:

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} = \frac{\beta g \Delta T}{\nu}$$

satisfied exactly in the Squire-Eckert b.l. solution?

In the context of eqn. (8.19) we saw that C_1 must be 1/4; but eqn. (8.23) tells us that:

$$C_1 = \frac{Pr}{3(\frac{20}{21} + Pr)} = \frac{1}{4}$$

so:

$$\frac{20}{21} + Pr = \frac{4}{3}\ Pr$$

Solving this, we obtain: $\underline{\underline{Pr = 2.86}}$

Problem 8.13 The side wall of a house is 10 m in height. The overall heat transfer coefficient between the interior air and the exterior surface is 2.5 W/m^{2}K. On a cold, still winter night $T_{\text{outside}} = -30\,°\text{C}$ and $T_{\text{inside air}} = 25\,°\text{C}$. What is $\overline{h}_{\text{conv}}$ on the exterior wall of the house if $\varepsilon = 0.9$? Is external convection laminar or turbulent?

Solution The exterior wall is cooled by both natural convection and thermal radiation. Both heat transfer coefficients depend on the wall temperature, which is unknown. We may solve iteratively, starting with a guess for T_w. We might assume (arbitrarily) that ⅔ of the temperature difference occurs across the wall and interior, with ⅓ outside, so that $T_w \approx (25+30)/3 - 30 = -11.7\,°\text{C} = 261.45$ K. We may take properties of air at $T_f \approx 250$ K, to avoid interpolating Table A.6:

Properties of air at 250 K

thermal conductivity	k	0.0226	W/m·K
thermal diffusivity	α	1.59×10^{-5}	m^2/s
kinematic viscosity	ν	1.135×10^{-5}	m^2/s
Prandtl number	Pr	0.715	

The next step is to find the Rayleigh number so that we may determine whether to use a correlation for laminar or turbulent flow. With $\beta = 1/T_f = 1/(250)\ \text{K}^{-1}$:

$$\text{Ra}_L = \frac{g\beta(T_w - T_{\text{outside}})L^3}{\nu\alpha} = \frac{(9.806)(-11.7+30)(10^3)}{(250)(1.59)(1.135)(10^{-10})} = 3.98 \times 10^{12}$$

Since, $\text{Ra}_L > 10^9$, we use eqn. (8.13b) to find $\overline{\text{Nu}}_L$:

$$\overline{\text{Nu}}_L = \left\{0.825 + \frac{0.387\ \text{Ra}_L^{1/6}}{\left[1+(0.492/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2$$

$$= \left\{0.825 + \frac{0.387(3.98\times10^{12})^{1/6}}{\left[1+(0.492/0.715)^{9/16}\right]^{8/27}}\right\}^2 = 1738$$

Hence

$$\overline{h}_{\text{conv}} = (1738)\frac{0.0226}{10} = 3.927\ \text{W/m}^2\text{K}$$

The radiation heat transfer coefficient, for $T_m = (261.45 + 243.15)/2 = 252.30$ K, is

$$h_{\text{rad}} = 4\varepsilon\sigma T_m^3 = 4(0.9)(5.6704\times10^{-8})(252.30)^3 = 3.278\ \text{W/m}^2\text{K}$$

The revised estimate of the wall temperature is found by equating the heat loss through the wall to the heat loss by convection and radiation outside:

$$(2.5)(25 - T_w) = (3.927 + 3.278)(T_w + 30)$$

so that $T_w = -15.8\,°\text{C}$, which is somewhat lower than our estimate. We may repeat the calculations with this new value (without changing the property data) finding $\text{Ra}_L = 3.09\times10^{12}$, $\overline{\text{Nu}}_L = 1799$, $\overline{h}_{\text{conv}} = 4.065$ W/m^{2}K, $T_m = 250.3$ K, and $h_{\text{rad}} = 3.201$ W/m^{2}K. Then

$$(2.5)(25 - T_w) = (4.065 + 3.201)(T_w + 30)$$

so that $T_w = -15.9\,°\text{C}$. Further iteration is not needed. Since the film temperature is very close to 250 K, we do not need to update the property data.

To summarize the final answer, $\underline{\overline{h}_{\text{conv}} = 4.07\ \text{W/m}^2\text{K}}$ and <u>most of the boundary layer is turbulent</u>.

8.14 Plot T_{sheet} vs. time for Ex. 8.2, if the sheets are 1 % carbon, 2 m long and 6mm thick ($w = 0.003$ m). The bath is water at 60°C and the sheets are introduced at 18°C. Compare the result with exponential response.

With reference to Example 8.2, with properties evaluated at (60 + 80)/2, or 39°C:

$$\text{define } B \equiv 0.678\,\frac{0.6253}{2}\left[\frac{4.96}{0.952+4.96}\right]^{1/4}\left[\frac{9.8(0.000371)2^3}{1.509(0.67)10^{-13}}\right]^{1/4} = \underline{148\,\frac{W}{m^2\text{-}°C^{5/4}}}$$

Then:

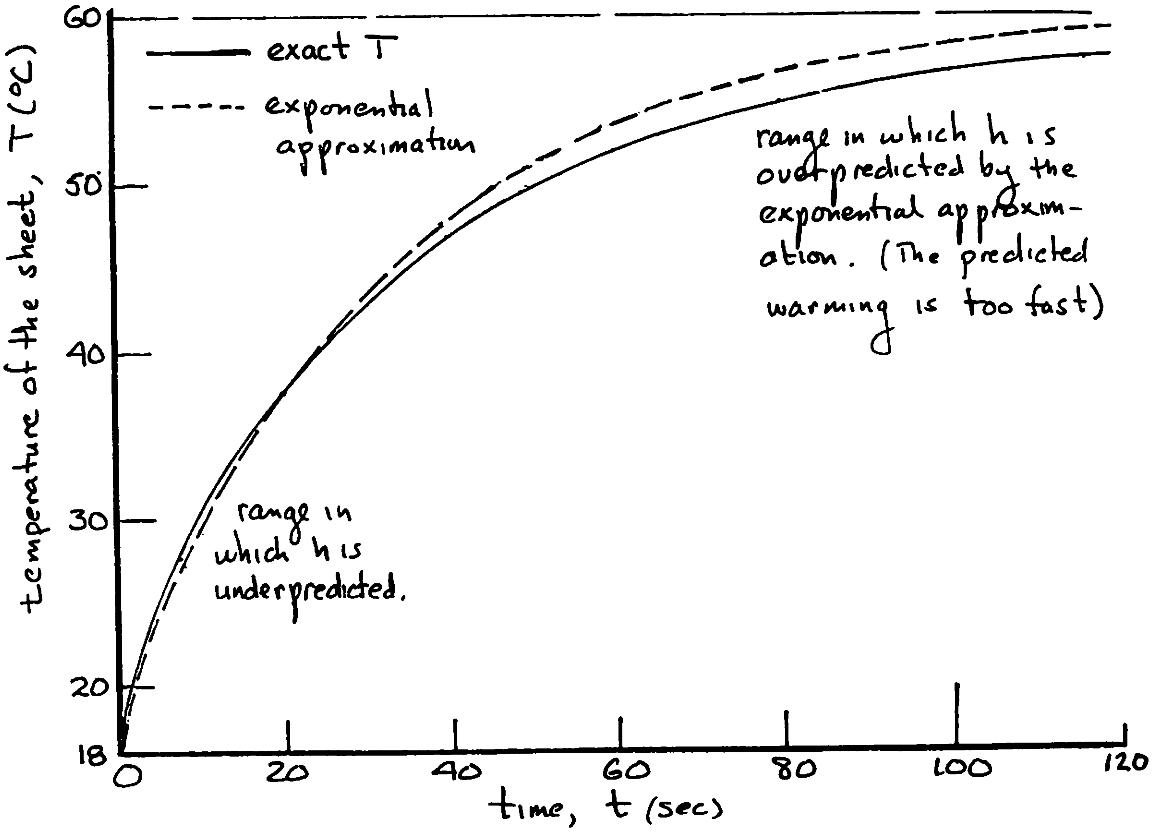
$$\underline{T = 60 - \left[\frac{1}{42^{1/4}} + \frac{B\ t}{4(7801)(473)(0.003)}\right]^{-4} = 60 - \frac{1}{[0.393+0.00334t]^4}}$$

The exponential response is: $\dfrac{T - T_\infty}{T_i - T_\infty} = e^{-t/\mathbf{T}}$

where $\mathbf{T} = \dfrac{\rho c V}{\bar{h} A} = \dfrac{7801(473)(w)}{B(39-18)^{1/4}} = 31.84\text{ sec}$; $\underline{T = 60 - 42\,\exp(-0.0314\,t)}$

(Continued, next page)

8.14 (continued)

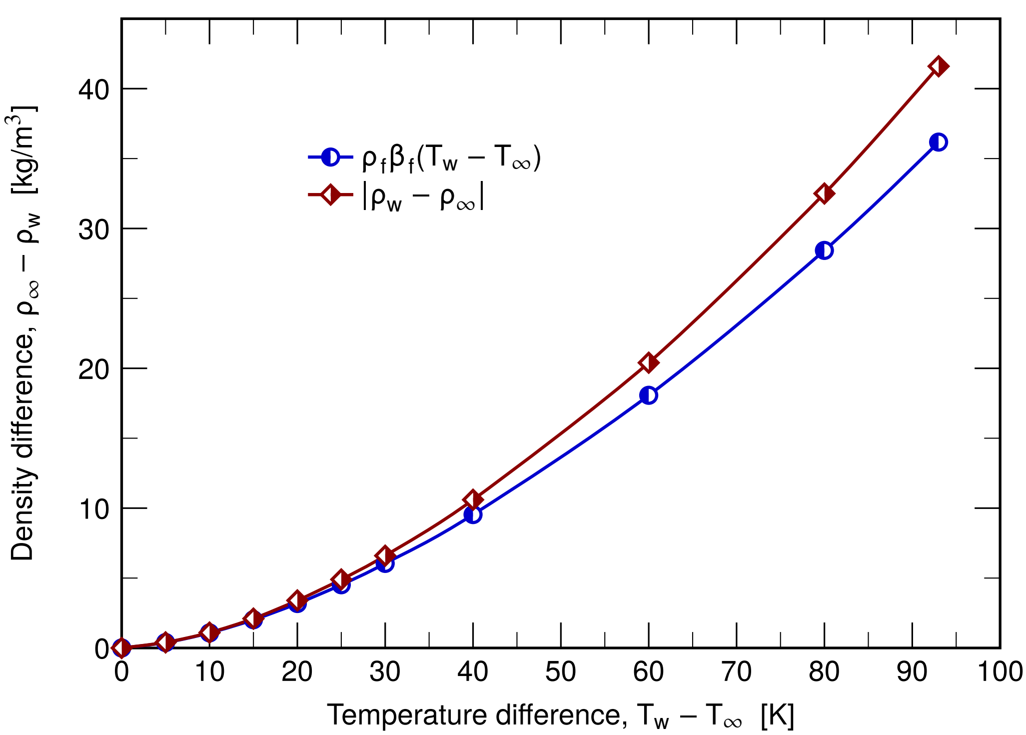


Problem 8.15 In eqn. (8.7), we linearized the temperature dependence of the density difference. Suppose that a wall at temperature T_w sits in water at $T_\infty = 7$ °C. Use the data in Table A.3 to plot $|\rho_w - \rho_\infty|$ and $|-\rho_f \beta_f (T_w - T_\infty)|$ for 7 °C $\leqslant T_w \leqslant$ 100 °C, where $(..)_f$ is a value at the film temperature. How well does the linearization work?

Solution With values from Table A.3, we may perform the indicated calculations and make the plot. The linearization is accurate to within 10% for temperature differences up to 40 °C, and within 13% over the entire range.

Properties of water from Table A.3

T [°C]	ρ [kg/m^3]	β [K^{-1}]	$(\rho_w - \rho_\infty)$	$-\rho_f \beta_f (T_w - T_\infty)$
7	999.9	0.0000436	0.0	0.000
12	999.5	0.000112	−0.4	−0.389
17	998.8	0.000172	−1.1	−1.08
22	997.8	0.000226	−2.1	−2.02
27	996.5	0.000275	−3.4	−3.18
32	995.0	0.000319	−4.9	−4.52
37	993.3	0.000361	−6.6	−6.05
47	989.3	0.000436	−10.6	−9.54
67	979.5	0.000565	−20.4	−18.1
87	967.4	0.000679	−32.5	−28.4
100	958.3	0.000751	−41.6	−36.2



8.16 A 77°C vertical wall heats air at 27°C. Find Ra_L, δ_{top}/L, and L, where the line in Fig. 8.3 ceases to be straight. Comment on the implications of your results.

The line in Fig. 8.3 begins to deviate from straightness, and flatten out, when:

$$Ra_L\left[1+\left(\frac{0.492}{Pr}\right)^{9/16}\right]^{-1.7778} \simeq 10^3$$

But for $\overline{T} = 325°K$, $Pr = 0.7085$, so $\underline{\underline{Ra_L \simeq 2884}}$

(This result could reasonably range from 10^3 to 10^4.)

but $$Ra_L = 2884 = \frac{g\beta\Delta T L^3}{\nu\alpha} = \frac{9.8\left(\frac{1}{300}\right)50}{1.814(2.561)10^{-10}} L^3$$

$$\underline{\underline{L = 0.00936\text{ m} = 0.936\text{ cm}}}$$

Find $\frac{\delta_{top}}{L}$:

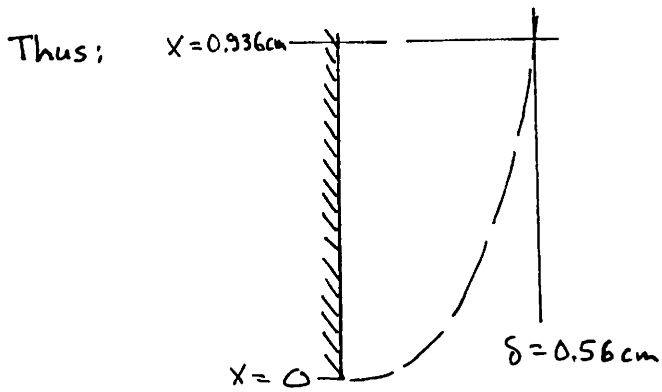
$$Nu_x\Big|_{x=L} = 2\frac{L}{\delta} = \frac{3}{4}\overline{Nu}_L$$

$$\frac{\delta_{top}}{L} = \frac{8}{3\overline{Nu}_L}$$

but $$\overline{Nu}_L = 0.68 + 0.67\frac{Ra_L^{1/4}}{\left[1+\left(\frac{0.492}{Pr}\right)^{9/16}\right]^{4/9}} = 0.68 + 0.67\,(10^3)^{1/4} = \underline{4.45}$$

8.16 (continued)

Thus;



$$\frac{\delta_{top}}{L} = \frac{8}{3(4.95)} = \underline{\underline{0.60}}$$

The b.l. looks something like this -- quite thick. Thus the deviation from the linear relationship reflects the breakdown of the b.l. assumptions.

8.17 A horizontal 0.08 m diameter pipe, at 150°C on the inside, has 85 % magnesia insulation with a 0.11 O.D. What is the heat loss if T_∞ = 17°C?

First we have to guess the outside temperature to evaluate properties. h on the outside should be low -- around 6 W/m²-°C so we go to equation (2.25) and calculate

$$Q = \frac{\Delta T}{\frac{1}{2\pi h r_o} + \frac{\ln r_o/r_i}{2\pi \underbrace{k}_{\text{for } 80\% \text{ mag.}}}} = \frac{150-17}{\frac{1}{2\pi(6)(0.055)} + \frac{\ln(0.055/0.04)}{2\pi(0.071)}} = 111\ \frac{W}{m}$$

Therefore ΔT across $\bar{h} = \frac{Q}{\bar{h}(\pi D)} = \frac{111}{6(\pi)(0.11)} = 53.5$, $T_{\text{outside mag.}} = 70.5\,°C$

we then evaluate properties at $\frac{70.5+17}{2} = 44\,°C$ or $317\,°K$

Then :

$$\frac{Ra_L^{1/4}}{\left[1+\left(\frac{0.559}{Pr}\right)^{9/16}\right]^{4/9}} = \frac{\left[\frac{9.8\frac{1}{273+17}(70.5-17)(0.11)^3}{1.735(2.447)\,10^{-10}}\right]^{1/4}}{\left[1+\left(\frac{0.559}{0.710}\right)^{9/16}\right]^{4/9}} = 36.9$$

$$\overline{Nu}_D = 0.36 + 0.518(36.9) = 19.5\ , \quad \bar{h} = 19.5\frac{0.02614}{0.11} = 4.63\ \frac{W}{m^2\text{-}°C}$$

This gives an outside temp. of $\frac{99.3}{4.63\,\pi(0.11)} = 62°C$. It should suffice to correct the Rayleigh no. by a factor of $\left(\frac{62.0-17}{70.5-17}\right)^{1/4}$ or 0.958. Then $\bar{h}$ will become $\underline{4.44\ W/m^2\text{-}°C}$ which we shall use:

$$Q = \frac{150-17}{\frac{1}{4.44(2\pi)(0.055)} + \frac{\ln(0.055)/(0.04)}{2\pi(0.071)}} = \frac{133}{0.652+0.714} = \underline{\underline{97.3\frac{W}{m}}}$$

8.18 How much heat is needed to keep a horizontal wire, 10^{-5} m in diam., at 40°C in 10°C water.

eqn. (8.28): $$\overline{Nu}_D = 0.36 + \frac{0.518}{\left[1+\left(\frac{0.559}{Pr}\right)^{9/16}\right]^{4/9}}\left[\underbrace{\frac{g\beta\Delta T D^3}{\nu\alpha}}_{\frac{9.8(0.00021)(30)10^{-15}}{1.454(0.885)\times10^{-13}}}\right]^{1/4}$$

Pr: 6.11

where we evaluate properties at 298°K

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.6057}{0.00001}\,0.4337 = 26{,}269\ \frac{W}{m^2\,°C}$$

So $$Q = \bar{h}\pi D\Delta T = 26{,}269(\pi)10^{-5}(30) = \underline{\underline{24.8\ \frac{W}{m}}}$$

8.19 A 20 cm vertical run of 0.5 cm diam. tubing carries condensing vapor at 60°C. The air outside is at 27°C. What is the heat loss?

Neglect the resistance of the tubing and of $h_{cond.}$. They aren't specified, and they would have to be small.

First treat the tube like a vertical wall, π(0.005 m) wide and 0.2 m high. From equation (8.13a), using $\bar{T}$ = 316.5°K:

$$\overline{Nu}_L = 0.68 + 0.67\left[\frac{g\beta\Delta T L^3}{\nu\alpha}\right]^{1/4}\left[1+\left(\frac{0.492}{Pr}\right)^{9/16}\right]^{-4/9}$$

$$= 0.68 + 0.67\left[\frac{9.8\left(\frac{1}{300}\right)33(0.2)^3}{1.402(2.44)\times10^{-10}}\right]^{1/4}\left[1+\left(\frac{0.492}{0.709}\right)^{0.5625}\right]^{-.444} = 37.11$$

$$\bar{h} = \frac{k}{L}\overline{Nu}_L = \frac{0.02731}{0.2}\,37.11 = \underline{5.06\ \frac{W}{m^2\text{-}°C}}$$

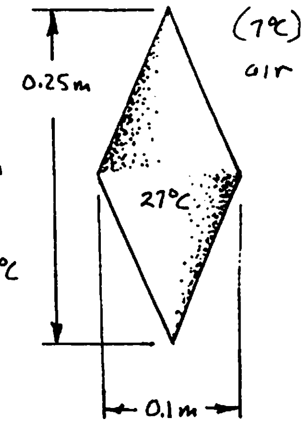
Now we must correct $\bar{h}$ for curvature using Fig. (8.7)

$$\frac{2\sqrt{2}}{(Ra_L/Pr)^{1/4}}\frac{L}{R} = \frac{2\sqrt{2}(0.2)}{77.22(0.0025)} = 2.93,\ \text{so}\ \frac{\bar{h}_{actual}}{\bar{h}_{cyl.}} = 1.8$$

The actual value of $\bar{h}$ is $1.8(5.06) = \underline{9.11\ \frac{W}{m^2\text{-}°C}}$ and

the heat loss is: $\bar{h}A\Delta T = 9.11\left[\pi(0.005)(0.2)\right](33) = \underline{\underline{0.944\,W}}$

8.20 How much heat is removed from the body shown.



$$\tau = \frac{0.25\,m}{\cos(\tan^{-1}\frac{10}{25})}$$

$$= 0.2693\ m$$

evaluate properties at 17°C

$$\overline{Nu}_\tau = 0.52\, Ra_\tau^{1/4}$$

$$\simeq 0.52 \left[\frac{9.8 \frac{1}{280}\, 20\, (0.2693)^3}{1.477(2.072)\cdot 10^{-10}}\right]^{1/4}$$

$$= 42.5$$

so: $\bar{h} = 42.5 \dfrac{0.02536}{0.2693} = \underline{4.00 \dfrac{W}{m^2\text{-}°C}}$

The area of the cone is $2 \times (\frac{1}{2}$ perimeter of base)(lateral edge)

$$= \frac{0.2693}{2}\, \pi (0.1) = 0.0423\ m^2$$

so $Q = \bar{h} A \Delta T = 4(0.0423)(20) = \underline{\underline{3.38\ W}}$

PROBLEM 8.22 You are asked to design a vertical wall panel heater, 1.5 m high, for a dwelling. What should the heat flux be if no part of the wall should exceed 33 °C? How much heat goes to the room if the panel is 7 m wide with $\varepsilon = 0.7$? *Hint*: Natural convection removes only about 200 W depending on what room temperature you assume.

Assume $T_\infty = 23°C$ (73.4°F) as a maximum value. Then $\Delta T_{max} = 10°C$.
From Fig. 8.9, $\Delta T_{max} = 10°C$ gives $\overline{\Delta T} = 0.833 \times 10 = \underline{8.33°C}$.

To get q_w we must now solve eqn. (8.43a) by trial and error:

$$\left(\frac{q_w L}{k\overline{\Delta T}}\right)^{5/4} - 0.68\left(\frac{q_w L}{k\overline{\Delta T}}\right)^{1/4} = \frac{0.67\left[\frac{g\beta q_w L^4}{k\nu\alpha}\right]^{1/4}}{\left[1+\left(\frac{0.492}{Pr}\right)^{9/16}\right]^{4/9}}$$

Evaluate properties at 28°C ≃ 300°K so:

$$\left(\frac{q_w\, 1.5}{0.02614(8.33)}\right)^{1.25} - 0.68\left(\frac{q_w\, 1.5}{0.02614(8.33)}\right)^{0.25} = \frac{0.67\left[\frac{9.8\frac{1}{296}1.5^4 q_w}{0.02614(1.566)(2.203)}10^{10}\right]^{1/4}}{\left\{1+\left(\frac{0.492}{0.711}\right)^{0.5625}\right\}^{0.444\ldots}}$$

or

$$11.16\, q_w^{1.25} - 1.102\, q_w^{0.25} = 190\, q_w^{0.25}$$

or

$$11.16\, q_w = 191.1 \qquad \text{so} \qquad \underline{q_w = 17.12\ \frac{W}{m^2}}$$

(This corresponds with $\overline{h} = \dfrac{17.12}{8.33} = 2.05\ \dfrac{W}{m^2\text{-}°C}$.)

so

$$Q = q_w A = 17.12(1.5)(7) = \underline{\underline{180\ W}}$$

Since the wall temperature is known, the radiation loss can be computed separately because T_m does not change much along the length of the wall:

$$T_m = (T_\infty + \overline{T}_{\text{wall}})/2 = (23 + 23 + 8.33)/2 + 273.15 = 300.3\ \text{K}$$

and then

$$q_{\text{rad}} = h_{\text{rad}} T_m^3 (\overline{T}_{\text{wall}} - T_\infty) = 4(0.7)(5.670 \times 10^{-8})(300.3)^3(8.33) = 35.8\ \text{W/m}^2$$

Thus, radiation carries an additional $(35.8)(1.5)(7) = 376$ W, for a total panel heating power of $376 + 180 = 556$ W.

8.23 A 0.14 cm high wall is heated by condensation of steam at one atm. What will happen to $\overline{h}$ and Q if the steam is replaced with an organic vapor?

$$\frac{\overline{h}_{organic}}{\overline{h}_{steam}} = \frac{k_o}{k_s}\left[\frac{\frac{\rho_o}{\rho_s}\left(\frac{\rho_o-\rho_{g_o}}{\rho_s-\rho_{g_s}}\right)\frac{h_{fg_o}}{h_{fg_s}}\frac{\left[1+\frac{c_p(T_s-T_w)}{h_{fg}}\right]_o}{\left[1+\frac{c_p(T_s-T_w)}{h_{fg}}\right]_s}}{\frac{\mu_o}{\mu_s}\frac{k_o}{k_s}\frac{T_{s_o}-T_w}{T_{s_s}-T_w}}\right]^{1/4}$$

We can probably neglect ρ_g at 1 atm and the $c_p(T_s-T_w)/h_{fg}$ terms will contribute little. Thus:

$$\underline{\underline{\frac{\overline{h}_o}{\overline{h}_s} \simeq \left(\frac{k_o}{k_s}\right)^{3/4}\left[\left(\frac{T_{s_s}-T_w}{T_{s_o}-T_w}\right)\frac{\nu_s}{\nu_o}\frac{\rho_o}{\rho_s}\frac{h_{fg_o}}{h_{fg_s}}\right]^{1/4}}}$$

8.23 (continued) Finally, since $Q = hA(T_s - T_w)$:

$$\frac{Q_o}{Q_s} \simeq \left(\frac{k_o}{k_s}\,\frac{T_{s_o}-T_w}{T_{s_s}-T_w}\right)^{3/4}\left(\frac{\nu_s}{\nu_o}\,\frac{\rho_o}{\rho_s}\,\frac{h_{fg\,o}}{h_{fg\,s}}\right)^{1/4}$$

These expressions give the factors by which $\bar{h}$ and Q will change, once the instructor specifies the particular fluid. The student should remember that T_{sat}, as well as the other thermal properties, will change when the fluid is changed.

8.24 A 0.01 m diam. tube, 0.27 m long, runs horizontally through saturated steam. Plot Q vs. T_{tube} for $50 < T_{tube} < 150^\circ C$.

For natural convection, evaluating properties at $\frac{125+100}{2} = 113^\circ C = 386$ K

$$Ra_D = \frac{9.8(0.0029)\,0.01^3}{(2.295)(2.201)/10^{10}}\,\Delta T = 56.3\,\Delta T$$

so: $$\overline{Nu}_D = 0.36 + \frac{0.518\,Ra_D^{1/4}}{\left[1+\left(\frac{0.559}{Pr}\right)^{9/16}\right]^{4/9}} = \underline{0.36 + 1.12\,\Delta T^{1/4}}$$

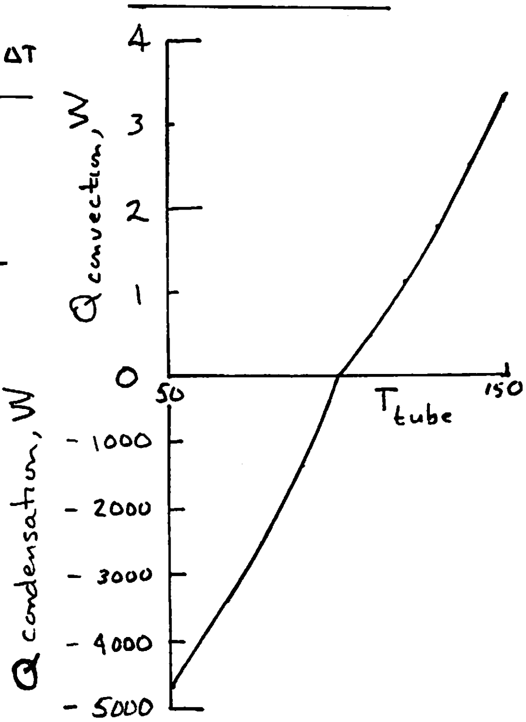
$\quad$ ↳ 1.02

And for film condensation, $h'_{fg} = 2.257\times10^6\left[1+\left(0.683-\frac{0.228}{1.75}\right)Ja\right] = 2.257\times10^6(1-0.001\,\Delta T)$, so we can use an average value (for $\Delta T = 25^\circ C$) of 2,313,000

$$\overline{Nu}_D = 0.728\left[\frac{(\rho_f)\Delta\rho\, g\, h'_{fg}\, D^3}{\mu\, k\, (T_{sat}-T_w)}\right]^{1/4} = 0.728\left[\frac{958(9.8)(2.313)}{2.77\,(0.684)10^{-7}}\right]^{1/4}\frac{1}{\Delta T^{1/4}}$$

$$\underline{\overline{Nu}_D = \frac{426}{\Delta T^{1/4}}}$$

ΔT °C	$\overline{Nu}_D$: $0.36+1.12\Delta T^{1/4}$	$\overline{Nu}_D$: $\frac{426}{\Delta T^{1/4}}$		$Q = \frac{k}{D}\overline{Nu}_D A\Delta T$ $= k_{\text{water or steam}}\,\pi(0.27)\,Nu_D\,\Delta T$
50	3.34	—	$Q = 0.02\,\overline{Nu}_D\Delta T$	3.28 W
40	3.19	—		2.50
30	2.99	—		1.76
20	2.72	—		1.075
10	2.37	—		0.463
-10	—	240	$Q = -0.58\,\overline{Nu}_D\Delta T$	-1389 W
-20	—	201		-2337
-30	—	182		-3167
-40	—	169		-3930
-50	—	160		-4646



Notice that:

$Q_{cond.} \gg Q_{conv.}$

$Q_{cond.} \sim \Delta T^{3/4}$

$Q_{conv.} \sim \Delta T^{5/4}$

8.25 A plate, 2m high, condenses steam at 1 atm. Calculate ΔT at which: a) Nusselt's solution loses accuracy; b) The film becomes turbulent.

$$\Gamma_c = \frac{\rho_f(\rho_f-\rho_g)g\delta^3}{3\mu^2} \simeq \frac{g\delta^3}{3\nu^2} = 6 \text{ for (a) and } 450 \text{ for (b)}$$

but $\nu = 0.29 \times 10^{-6}$ so $\delta = \left(\frac{3\Gamma_c\nu^2}{g}\right)^{1/3} = 0.00008106 \text{ m (a)}$

$= 0.000342 \text{ m (b)}$

Now:

$$\delta = \left[\frac{4k\Delta T\mu L}{\rho_f(\rho_f-\rho_g)g\, h'_{fg}}\right]^{1/4} \simeq \left[\frac{4k\Delta T\nu L}{\rho_f g\, h_{fg}}\right]^{1/4}$$

We'll go back & use h'_{fg} if ΔT is large.

$$= \left[\frac{4(0.6811)(0.29)10^{-6}}{958(9.8)2257(10)^3}\right]^{1/4} \Delta T^{1/4} = 0.0000782\Delta T^{1/4}$$

(a) $0.00008106 = 0.0000782\Delta T^{1/4}$; $\underline{\underline{\Delta T = 1.15°C}}$

when Nusselt's solution losses accuracy.

(b) $0.000342\text{m} = 0.0000782\Delta T^{1/4}$; $\Delta T = 366°C$

The flow can never become turbulent. It will freeze first.

8.26 A reflux condenser has $\alpha = 18°$, $d = 0.8$, $D = 6$ cm. At 30°C it condenses steam at 1 atm. What is $\bar{h}$? (Evaluate properties at 65°C)

To use Fig. 8.14, compute: $B = \frac{\rho_f-\rho_g}{\rho_f}\frac{c_p\Delta T}{h_f}\frac{\tan^2\alpha}{\Pr} = \frac{979.4-0.6}{979.4}$

$$= \frac{4186(70)\tan^2 18}{2{,}257{,}000(1.72)} = 0.008$$

$$d/D = 0.8/6 = 0.1333$$

So: $\mathrm{Nu}_L = \left[\frac{(\rho_f-\rho_g)g\, h'_{fg}(d\cos\alpha)^3}{\nu k\Delta T}\right]^{1/4} \times 0.727$;

but $h'_{fg} = h_{fg}\left(1+\left[0.683+\frac{0.228}{Pr}\right]Ja\right) = h_{fg}\left(1+\left[0.683+\frac{0.228}{2.65}\right]Ja\right) = (1+0.77\,Ja)h_{fg}$

$$\mathrm{Nu}_L = 0.727\left[\frac{978.8(9.8)2.257(10)^6\left(1+0.77\frac{4186(100-30)}{2{,}257{,}000}\right)}{0.435(10)^{-6}(0.6585)(100-30)}(0.008\cos 18°)^3\right]^{1/4}$$

$= 110$

Then: $\bar{h} = \frac{k}{d\cos\alpha}\overline{\mathrm{Nu}_L} = \frac{0.6585(110)}{0.008\cos 18°} = \underline{\underline{9512\ \frac{W}{m^2\text{-}°C}}}$

8.27 A 0.05m helix of 0.005 m diam. tubing carries 15°C water through saturated steam at 1 atm. Specify α and the number of coils if 6 kg/hr of steam are to be condensed. $\bar{h}_{inside} = 600 W/m^2\text{-}°C$.

First establish an approximate T_{wall} assuming an ordinate from Fig. 8.14.

$$h_{cond}(T_{sat}-T_w) = \frac{k}{d\cos\alpha}\underbrace{0.729}_{\text{assumed}}\left[\frac{(\rho_f-\rho_g)h'_{fg}\,g(d\cos\alpha)^3}{\nu k(T_{sat}-T_w)}\right]^{1/4}(T_{sat}-T_w) = \bar{h}_i(T_w-T_i)$$

For openers, call $h'_{fg} \simeq h_{fg}$ and $\cos\alpha \simeq 1$. Then:

$$\frac{0.6811}{0.005}(\cos\alpha)^{-1/4}\left[\frac{958(2{,}257{,}000)9.8(0.005)^3}{0.29\times10^{-6}(0.6811)}\right]^{1/4}(100-T_w)^{3/4} = 600(T_w-15)$$

$$77.23(100-T_w)^{3/4} = T_w-15 \;;\; \underline{T_w \simeq 98.88°C}$$

Now do an accurate computation based on this estimate. Base properties on $T_w \simeq 100°C$:

$$B = \frac{958.3-0.6}{958}\;\frac{4219(18)}{2{,}257{,}000}\;\frac{\tan^2\alpha}{1.72} = 0.002\tan^2\alpha \simeq 0,\text{ and } \frac{d}{D} = 0.1$$

Then from Fig. 9.14 the lead const. = 0.727

So: $$\frac{0.6811}{0.005}\left[\frac{958(2{,}257{,}000)9.8(0.005)^3}{0.29(10)^{-6}(0.6811)}1.16^3\right]^{1/4}\cos^{-1/4}\alpha = 600(83.88) \quad \text{(A)}$$

$$\cos^{-1/4}\alpha = 0.9997, \text{ not possible}$$

Pick $\underline{T_w = 1.07°C}$. Then:

$$\cos^{-1/4}\alpha = 0.9997\,\frac{83.93}{83.88}\left(\frac{1.116}{1.07}\right)^{3/4},\; \underline{\underline{\alpha = 28°}}$$

So: $$\bar{h} = [\text{LHS of (A)}]\left(\frac{1.07}{1.116}\right)^{3/4} = 47{,}625\;\frac{W}{m^2\text{-}°C}$$

Then
$$\dot{m} = \frac{Q}{h_{fg}} = \bar{h}\,\frac{\text{length}(\pi d)\Delta T}{h_{fg}} = \frac{6}{3600}\,\frac{kg}{s}, \text{so } \underline{\text{length} = 0.783 \text{ m}}$$

Finally:
$$\text{no. of coils} = \frac{\text{length}}{\pi D/\cos\alpha} = \frac{0.783\cos 28°}{\pi(0.05)} = \underline{\underline{4.4}}$$

8.29 What is the maximum speed of air in the natural convection b.l., in Example 8.3?

first find where u maximizes in y/δ using eqn. (8.18)

$$\frac{d(u/u_c)}{d(y/\delta)} = 0 = 1 - 4\frac{y}{\delta} + 3\left(\frac{y}{\delta}\right)^2 \quad \text{or} \quad \left(\frac{y}{\delta}\right)^2 - \frac{4}{3}\left(\frac{y}{\delta}\right) + \frac{1}{3} = 0$$

Thus:

$$\frac{y}{\delta} = \frac{2}{3} \pm \sqrt{\frac{4}{9} - \frac{3}{9}} = 1 \text{ or } \frac{1}{3}; \quad \frac{y}{\delta} = \frac{1}{3} \text{ gives } u_{max}.$$

Now using: $u_c(x) = C_1 \dfrac{\beta g \Delta T}{2\nu}\delta^2$ and $C_1 = Pr/3\left(\frac{20}{21} + Pr\right)$

we get:

$$u_{max} = \frac{Pr}{3\left(\frac{20}{21} + Pr\right)} \frac{\beta g \Delta T}{2\nu} \delta^2 \left[\frac{y}{\delta}\left(1 - \frac{y}{\delta}\right)^2\right]_{\frac{y}{\delta} = \frac{1}{3}}$$

Using numbers from Example 8.3 we obtain:

$$u_{max} = \frac{0.711}{3(0.952 + 0.711)} \frac{0.00348(9.8)(40-14)}{1.566 \times 10^{-5}} (0.0172)^2 \left[\frac{1}{3}\left(\frac{2}{3}\right)^2\right]$$

$$\underline{\underline{= 0.354 \text{ m/s}}} \longleftarrow$$

8.31 A large industrial process requires that water be heated by a large cylindrical heater using natural convection. The water is at 27°C. The cylinder is 5 m in diameter and it is kept at 67°C. First find $\bar{h}$. Then suppose D is doubled (D=10 m). What is the new $\bar{h}$? Explain the similarity of these answers in the turbulent natural convection regime.

at 47°C:

$$Pr = 3.67 \quad \text{and} \quad Ra_L = \frac{9.8\,(0.000435)(67-27)5^3}{0.566\,(1.541)\times 10^{-6-7}} = 2.44\times 10^{14}$$

So we use equation (8.29) and obtain:

$$\overline{Nu_L} = \left[0.60 + 0.387\Bigg[\underbrace{\frac{2.44\,(10)^{14}}{\left[1+\left(\frac{0.559}{3.67}\right)^{9/16}\right]^{16/9}}}_{1.437\times 10^{14}}\Bigg]^{1/6}\right]^2 = 7951$$

$$\bar{h} = 7951\,\frac{k}{L} = 7951\,\frac{0.6367}{5} = \underline{\underline{1012\ W/m^2\text{-}°C}} \leftarrow L=5\,m$$

If L is doubled we have

$$\overline{Nu_L} = \left[0.60 + 0.387\left(1.437\times 10^{14}\times 2^3\right)^{1/6}\right]^2 = 15{,}840$$

$$h = 15{,}840\,\frac{0.6367}{10} = \underline{\underline{1009\ W/m^2\text{-}°C}} \leftarrow L=10\,m$$

almost no change

We note that at high Ra, eqn. (8.29) reduces to:

$$\overline{Nu_L} = \frac{\bar{h}L}{k} = \frac{0.387^2}{\left[1+\left(\frac{0.559}{Pr}\right)^{9/16}\right]^{16/9}}\,Ra_L^{1/3}$$

so

$$\bar{h} = fn(Pr)\times\left(\frac{g\beta\Delta T\,k}{\nu/\rho c_p}\right)^{1/3} \neq fn(L)$$

The 1/3 power dependence of $\overline{Nu_L}$ on Ra_L that occurs in <u>turbulent</u> natural convection causes $\bar{h}$ to be <u>independent</u> of length in this regime!

8.32 A vertical jet of liquid, of diameter, d, and moving at velocity, u_∞, impinges on a horizontal disc rotating ω rad/s. There is no heat transfer in the system. Develop an expression for $\delta(r)$, where r is the radial coordinate on the disc. Contrast the r dependence of δ with that of a condensing film on a rotating disc and explain the difference qualitatively.

Nusselt's expression for the mass flow rate in the film is valid:

$$\dot{m}\ \frac{\text{kg}}{\text{m}} = \frac{\rho_f(\rho_f-\rho_g)}{3\mu}\, g\, \delta^3(r)$$

However, in this case, ρ_f is the liquid density and ρ_g is the air density, ρ_{air}. The "gravity" is now $\omega^2 r$. Then the total mass flow is:

$$\dot{M} = \rho_f u_\infty \frac{\pi}{4} d^2 = \dot{m}(2\pi r) = \frac{2\pi}{3}\frac{\rho_f(\rho_f-\rho_{air})}{\mu}\omega^2 r^2 \delta^3(r)$$

we solve this for $\delta(r)$:

$$\boxed{\delta(r) = \sqrt[3]{\frac{3}{8}\frac{\mu u_\infty d^2}{(\rho_f-\rho_{air})\omega^2}}\ \frac{1}{r^{2/3}}}$$

The film thickness is uniform during condensation on a rotating disc (see discussion following eqn. (8.70).) because condensation causes the film to accrue liquid. Thus $\dot{m}$ increases as r^2, and this accretion just compensates the natural thinning that must occur as the sheet spreads.

But in this case, $\delta \sim r^{-2/3}$ because no fluid is added as the film spreads out.

8.33 We have seen that, if properties are constant, $h \sim \Delta T^{1/4}$ in natural convection. If we consider the variation of properties as T_w is increased over T_∞, will h depend more or less strongly on ΔT in air? -- in water?

We see that h in natural convection varies as $k/(\nu\alpha)^{1/4}$. We then find that this quantity increases strongly in water -- especially at lower values of T_∞ -- so h depends more strongly than as $\Delta T^{1/4}$ on ΔT. In the case of air $k/(\nu\alpha)^{1/4}$ is a constant within ± 13 % over the entire range of properties given in the book. It drops off only slightly with increasing temperature so the dependency of h on ΔT is only a little less strong than $\Delta T^{1/4}$. If T_w were less than T_∞, these trends would be reversed.

8.34 A film of liquid falls along a vertical plate. It is initially saturated and it is surrounded by saturated vapor. The film thickness at the top is δ_o. If wall temperature, T_w, is slightly above T_{sat}, derive expressions for $\delta(x)$, Nu_x, and x_f--the distance at which the plate becomes dry. Calculate x_f if the fluid is water at 1 atm., if T_w = 105°C, and δ_o = 0.1 mm.

Equation (8.54) still applies, but the sign is reversed, thus:

$$k\frac{T_w - T_{sat}}{\delta} = -h_{fg}\frac{d\dot{m}}{dx} = -\frac{h_{fg}(\rho_f - \rho_g)}{\nu}g\,\delta^2\frac{d\delta}{dx}$$

So:

$$\frac{4k\Delta T\nu}{h_{fg}'(\rho_f-\rho_g)g} = -\frac{d\delta^4}{dx}$$

Integrating from $\delta(x=0)=\delta_o$ to $\delta(x)$ we get:

$$\frac{4k\Delta T\nu x}{h_{fg}'(\rho_f-\rho_g)g} = \delta_o^4 - \delta^4(x) \quad \text{or} \quad \underline{\underline{\delta(x) = \left[\delta_o^4 - \frac{4k\Delta T\nu x}{h_{fg}'(\rho_f-\rho_g)g}\right]^{1/4}}}$$

Then:

$$\underline{\underline{Nu_x = \frac{x}{\delta(x)} = \left[\left(\frac{\delta_o}{x}\right)^4 - \frac{4k\Delta T\nu}{h_{fg}'(\rho_f-\rho_g)gx^3}\right]^{-1/4}}}$$

and x_f is the value of x at which $\delta(x) = 0$

$$\underline{\underline{x_f = \frac{g(\rho_f-\rho_g)h_{fg}'\delta_o^4}{4k\nu\Delta T}}}$$

For the specified case we set $h_{fg} \simeq h_{fg}'$ and get

$$x_f = \frac{9.8(957.2-0.6)(2257000)(0.0001)^4}{4(0.6811)(5)(0.290)10^{-6}}$$

the plate will dry out when $\underline{\underline{x_f = 0.5356\text{ m}}}$

8.35 In a particular solar collector, dyed water runs down a vertical plate in a laminar film, with thickness, δ_o, at the top. The sun's rays pass through parallel glass plates (see Section 11.6) and deposit q_s W/m^2 in the flowing water film. Assume the water to be saturated at the inlet and the plate behind it to be insulated. Develop an expression for $\delta(x)$ as the water evaporates. Develop an expression for the maximum length of wetted plate, and provide a criterion for the laminar solution to be valid.

Equation (8.54) applies to this problem, but we must replace $k(T_w - T_{sat})/\delta$ with $-q_w$. Thus:

$$-q_w = h_{fg}\frac{d\dot{m}}{dx} = \frac{h_{fg}(\rho_f - \rho_g)}{\nu}\, g\,\delta^2 \frac{d\delta}{dx}$$

So:

$$\frac{3 q_w \nu}{g\, h_{fg}(\rho_f - \rho_g)} = -\frac{d\delta^3}{dx}$$

Integrating from $\delta(x=0) = \delta_o$ to $\delta(x)$ we get

$$\frac{3 q_w \nu x}{g\, h_{fg}(\rho_f - \rho_g)} = \delta_o^3 - \delta^3_{(x)} \quad \text{or} \quad \underline{\underline{\delta(x) = \left[\delta_o^3 - \frac{3 q_w \nu x}{g\, h_{fg}(\rho_f - \rho_g)}\right]^{1/3}}}$$

The film will dry out at $x = x_f$ when $\delta(x_f) = 0$.

$$\underline{\underline{x_f = \frac{g\, h_{fg}(\rho_f - \rho_g)}{3 q_w \nu}\,\delta_o^3}}$$

and this solution will only be valid when $\Gamma_c \leq 450$. Thus we write at the top of the plate:

$$\Gamma_c = \frac{(\rho_f - \rho_g) g\, \delta_o^3}{3\nu} \leq 450\mu \quad \text{or} \quad \underline{\underline{\delta_o \leq \sqrt[3]{\frac{1350\nu\mu}{g(\rho_f - \rho_g)}}}}$$

8.36 What heat removal flux can be achieved at the surface of a horizontal 0.01 mm diameter electrical resistance wire in still 27°C air if its melting point is 927°C?

Evaluate β at 27°C and the other properties at $(927+27)/2$, or 477°C = 750°K: $\nu = 7.43(10)^{-5}$, $\alpha = 10.57(10)^{-5}$, $k = 0.054$, $Pr = 0.703$.

$$Ra_D = \frac{9.8(1/300)900(0.00001)^5}{7.43(10.57)10^{-5-5}} = 3.744(10)^{-6}$$

Using equation (8.29) -- applicable for $Ra_D \geqslant 10^{-6}$ -- we get:

$$\overline{Nu_D} = \left\{0.6 + 0.387\left[\frac{3.744(10)^{-6}}{(1+(0.559/0.703)^{0.5625})^{1.778}}\right]^{1/6}\right\}^2 = 0.4096$$

Then: $\bar{h} = \overline{Nu_D}\frac{k}{D} = 0.4096\frac{0.054}{10^{-5}} = \underline{2212 \frac{W}{m^2\text{-}°C}}$

Thus: $q = 2212(927-27) = \underline{1,990,700\ W/m^2}$

This is an incredibly high heat flux. Natural convection, which normally inefficient, becomes remarkably effective when the diameter is very small.

PROBLEM 8.37 A 0.03 m O.D. vertical pipe, 3 m in length with $\varepsilon = 0.7$, carries refrigerant through a 24°C room at low humidity. How much heat does it absorb from the room if the pipe wall is at 10°C?

Evaluate properties at $(10°C+24°C)\frac{1}{2} = 17°C$. $\nu = 1.477(10)^{-5}$, $\alpha = 2.207(10)^{-5}$, $Pr = 0.713$, $k = 0.0254$

Then $\overline{Ra_L} = \frac{9.8(1/297)(24-10)3^3}{2.207(1.477)10^{-5-5}} = 3.826(10)^{10}$

eqn. (8.27)

$$\overline{Nu_L} = 0.68 + 0.67(3.826\times10^{10})^{1/4}\left[1+\left(\frac{0.492}{0.713}\right)^{0.5625}\right]^{-0.4444} = \underline{228}$$

$$\bar{h}_{flat\,plate} = 228\,\frac{0.0254}{3} = \underline{1.933\ W/m^2\text{-}°C}$$

Correct for curvature using Fig. 8.7: $\frac{2\sqrt{2}}{(Ra_L/Pr)^{1/4}}\frac{L}{R} = \frac{2\sqrt{2}}{\left(\frac{3.826(10)^{10}}{0.713}\right)^{1/4}}\frac{3}{0.015} = 1.18$

so $\bar{h}_{cyl}/\bar{h}_{plate} = 1.37$ and

$$\bar{h}_{cyl} = 1.37(1.933) = \underline{2.65 W/m^2\text{-}°C}$$

$$Q = \bar{h}_{cyl} A \Delta T = 2.65(3)(\pi)(0.03)(24-10) = \underline{\underline{10.5 W}}$$

But $h_{rad} = 4\varepsilon\sigma T_m^3 = 4(0.7)(5.670\times10^{-8})(17+273)^3 = 3.88\ W/m^2K$

Adding the natural convection and thermal radiation heat transfer coefficients, we can compute Q:

$$Q = (\bar{h}_{cyl} + h_{rad})A\,\Delta T = (2.65+3.88)(3\pi)(0.03)(24-10) = \underline{25.8\ W}$$

8.38 A 1 cm OD tube at 50°C runs horizontally in 20°C air. What is the critical radius of 85% magnesium insulation on the tube?

From eqn. (2.27) we have: $r_{crit} = \dfrac{k_{mag}}{\bar{h}} = \dfrac{k_{mag}}{k_{air}} \dfrac{2r_c}{\overline{Nu_D}}$

& using eqn. (8.28):

$$\overline{Nu_D} = 0.36 + \frac{0.518\, Ra_D^{1/4}}{\left[1+\left(\frac{0.559}{Pr}\right)^{0.5625}\right]^{4/9}} = 2\sqrt{\frac{k_{mag}}{k_{air}}}$$

We'll evaluate properties at 27°C ($T_{wall} = 34$°C) & hope that we won't have to re-iterate.

$$0.36 + \frac{0.518}{\left[1+\left(\frac{0.559}{0.711}\right)^{0.5625}\right]^{4/9}}\left[\frac{9.8\,\frac{1}{293}}{1.566(2.203)}10^{10}\right]^{1/4}\left[\Delta T(2r_c)^3\right]^{1/4} = 2\sqrt{\frac{0.067}{0.0264}}$$

$$0.36 + 38.89\left([2r_c]^3\Delta T\right)^{1/4} = 3.202$$

So $\underline{2r_c = 0.03055/\Delta T^{1/3}}$.

Furthermore: $q = \dfrac{k_{mag}(30-\Delta T)}{\ln(r_c/0.005)} = \bar{h}\,\Delta T$

but: $\bar{h} = \dfrac{k_{air}}{2r_c}\left(\overline{Nu_D}\right) = \dfrac{k_{air}}{2r_c}\left[2\sqrt{\dfrac{k_{mag}}{k_{air}}}\right] = \dfrac{1}{r_c}\sqrt{k_{mag}k_{air}}$

so: $\dfrac{30}{\Delta T} - 1 = \dfrac{\ln(r_c/0.005)}{r_c}\sqrt{\dfrac{k_{air}}{k_{mag}}} = \underline{\dfrac{30}{(0.03055)^3}(2r_c)^3 = \dfrac{\ln(r_c/0.005)}{r_c}0.625}$

so we solve for r_c by trial & error: $\underline{\underline{r_c = 0.01745\,m}}$ ←

This gives: $\Delta T = (0.03055/2[0.01745])^3 = \underline{0.67\,°C}$

Thus, we evaluated properties at a temperature 6-1/2°C above the right value. Further calculation would not be worth the trouble.

8.40 A horizontal electrical resistance heater, 1 mm in diameter, releases 100 W/m in water at 17°C. What is the wire temperature?

We modify eqn. (**8.28**), using $Ra_L = Ra^*_D / \overline{Nu}_D$, and get:

$$\overline{Nu}_D^{\,5/4} - 0.36\,\overline{Nu}_D^{\,1/4} = \frac{0.518}{\left[1+\left(\frac{0.559}{Pr}\right)^{0.5625}\right]^{4/9}} \left[\frac{g\beta q_w D^4}{k\nu\alpha}\right]^{1/4}$$

Guess $T_w = 37°C$. Then, at 27°C, $\nu = 0.826\times10^{-5}$, $\alpha = 1.462\times10^{-7}$, $Pr = 5.65$ & $k = 0.6084$, And $q_w = 100/\pi(.001) = 3183\ W/m^2$

Then: $$\overline{Nu}_D^{\,5/4} - 0.36\,\overline{Nu}_D^{\,1/4} = 0.4654\left[\frac{9.8(0.000275)(31830)10^{-12}}{0.826(1.462)10^{-12}\,0.6084}\right]^{1/4} = 1.53$$

By trial & error we get $\overline{Nu}_D = 1.70$, so

$$\bar{h} = 1.70\frac{k}{D} = 1.70\frac{0.6084}{0.001} = \underline{1034\ W/m^2\text{-}°C}$$

Then:

$$Q = \bar{h}A\Delta T, \quad 100 = 1034(\pi[0.001])\Delta T$$

$$\Delta T = 30.78\ °C$$

so $T_w = 17 + 30.78 = \underline{\underline{47.78\ °C}}$

The properties should have been evaluated at 32.4°C instead of at 27°C. This is not enough difference to warrant a recalculation. However, if we did the recalculation we'd get:

$$T_w = \underline{\underline{46.64°C}}$$

Which is less than 1°C improvement.

8.41 Solve Problem 5.39 using the correct formula for the heat transfer coefficient.

We shall evaluate the properties of water at $(47+27)/2 = 37°C$:

$\nu = 0.696(10)^{-6}$, $\alpha = 1.502(10)^{-7}$, $Pr = 4.66$, $k = 0.6226$, $\beta = 0.000355$

Then: $Ra_D = \dfrac{9.76(0.000355)(0.03)^3 \Delta T}{0.698(1.502)10^{-13}} = 69{,}400\ \Delta T$ so eqn. (8.29)

yields: $\underline{\underline{\bar{h} = \frac{k}{D}\overline{Nu}_D = \left[2.733 + 10.45\,\Delta T^{1/6}\right]^2}}$

This is exactly the value given in Problem 5.39. Therefore its solution applies here.

8.43 A 0.25 mm diameter platinum wire, 0.2 m long, is to be held horizontally at 1035°C. It is black. How much electric power is needed? Is it legitimate to treat it as a constant wall temperature heater, in calculating the convective part of the heat transfer? The surroundings are at 20°C and the surrounding room is virtually black.

$$Q_{rad} + Q_{conv} = \pi(0.00025)(0.2)\left[\sigma(1308^4 - 293^4) + \frac{k_{air}}{D}\overline{Nu}_D \Delta T\right]$$

evaluate properties at $(1035+20)/2 = 527.5°C \simeq 800°K$.

$\nu = 8.26(10)^{-5}$, $\alpha = 11.73(10)^{-5}$, $k = 0.0569$, $Pr = 0.704$

$$Ra_D = \frac{9.8\left(\frac{1}{800}\right)0.00025^3(1035-20)}{8.26(11.73)10^{-10}} = 0.0200$$

$$\overline{Nu}_D = 0.36 + 0.518(0.02)^{1/4}\Big/\left[1+\left(\frac{0.559}{.704}\right)^{0.5625}\right]^{0.4444} = \underline{0.507}$$

so:

$$Q = \left[165,546 + \underbrace{\frac{0.0569}{0.00025}0.507(1015)}_{117,124}\right]\pi(0.00025)(0.2) = \underline{\underline{44.4W}}$$

$$Bi_{conv} = \frac{\frac{117124}{1035-20}0.00025}{84} = 0.000343$$

$$Bi_{total} = \frac{117124+165546}{1035-20}0.00025/84 = 0.000829$$

In either case $Bi \ll 1$ so $T_w = const.$ is valid.

8.44 A vertical plate, 11.6 m long, condenses saturated steam at one atmosphere. We want to be sure that the film stays laminar. What is the lowest allowable plate temperature and what is $\overline{q}$ at this temperature?

Let us save work by adapting a result from Example 8.6:

$$\delta_{bottom} = 0.000138\,L^{1/4}\left(\frac{\Delta T}{10}\right)^{1/4} = 0.0001432\,\Delta T^{1/4} = \left(\underbrace{\frac{3\nu\mu}{(\rho_f-\rho_g)g}}_{\text{from eqn. (9.68)}} Re_c\right)^{1/3}$$

Then using $Re_c = 450$, we get:

$$\Delta T = \left[\frac{1}{14.32(10)^{-5}}\left(\frac{3[(0.290)10^{-6}]^2}{9.8}450\right)^{1/3}\right]^4 = \underline{6.23°C} \qquad T_{w_{lowest}} = \underline{\underline{93.77°C}}$$

$$\overline{q} = \frac{4}{3}\frac{k\Delta T}{\delta} = \frac{4}{3}0.681\frac{6.23^{3/4}}{0.0001432} = \underline{\underline{25,004\ W/m^2}}$$

8.45 a) Show that $\Theta_{\xi\xi} = m^2L^2\Theta^{5/4}$ for a straight fin in natural convection.
b) develop an iterative method to solve the equation assuming an insulated tip.
c) Solve the resulting difference equations.

a) With $Nu \sim Gr^{1/4}$, $\bar{h} = \bar{h}_0[(T-T_\infty)/(T_0-T_\infty)]^{5/4}$ where $\bar{h}_0$ is $\bar{h}$ at $T=T_0$. The given equation follows immediately from eqn. (4.30) and the subsequent non-dimensionalisation.

b) Using central differences for the second derivative, we have

$$\Theta_i^{k-1} - [2 + m^2L^2\,\delta\xi^2(\Theta_i^{k-1})^{1/4}]\Theta_i^k + \Theta_i^{k-1} = 0$$

$$\Theta_0^k \equiv 1.0$$

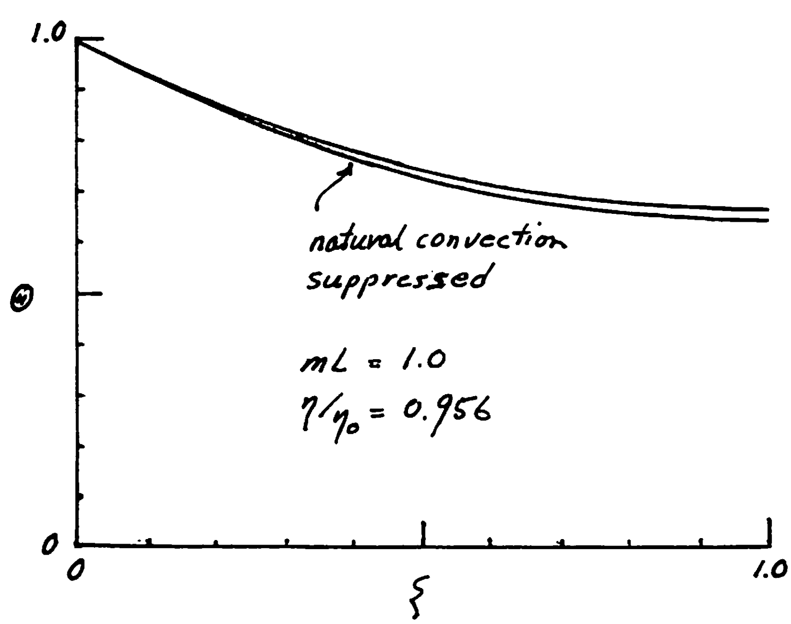
$$2\Theta_{n-1}^k - [2 + m^2L^2\,\delta\xi^2(\Theta_n^{k-1})^{1/4}]\Theta_n^k = 0$$

(To suppress natural convection, we change the exponent $\frac{1}{4}$, to zero)

c) Sample output from a BASIC program that solves these equations is given below. The program itself is on the next page.

```
Fin with natural convection heat exchange
Natural convection suppressed

mL= 1
R= 0
Number of iterations is 2

Step size is X1/L= .02
Efficiency= .761625234908
```

```
Fin with natural convection heat exchange

mL= 1
R= .000218334308
Number of iterations is 5

Step size is X1/L= .02
Efficiency= .728340253762
```



8.45 (continued)

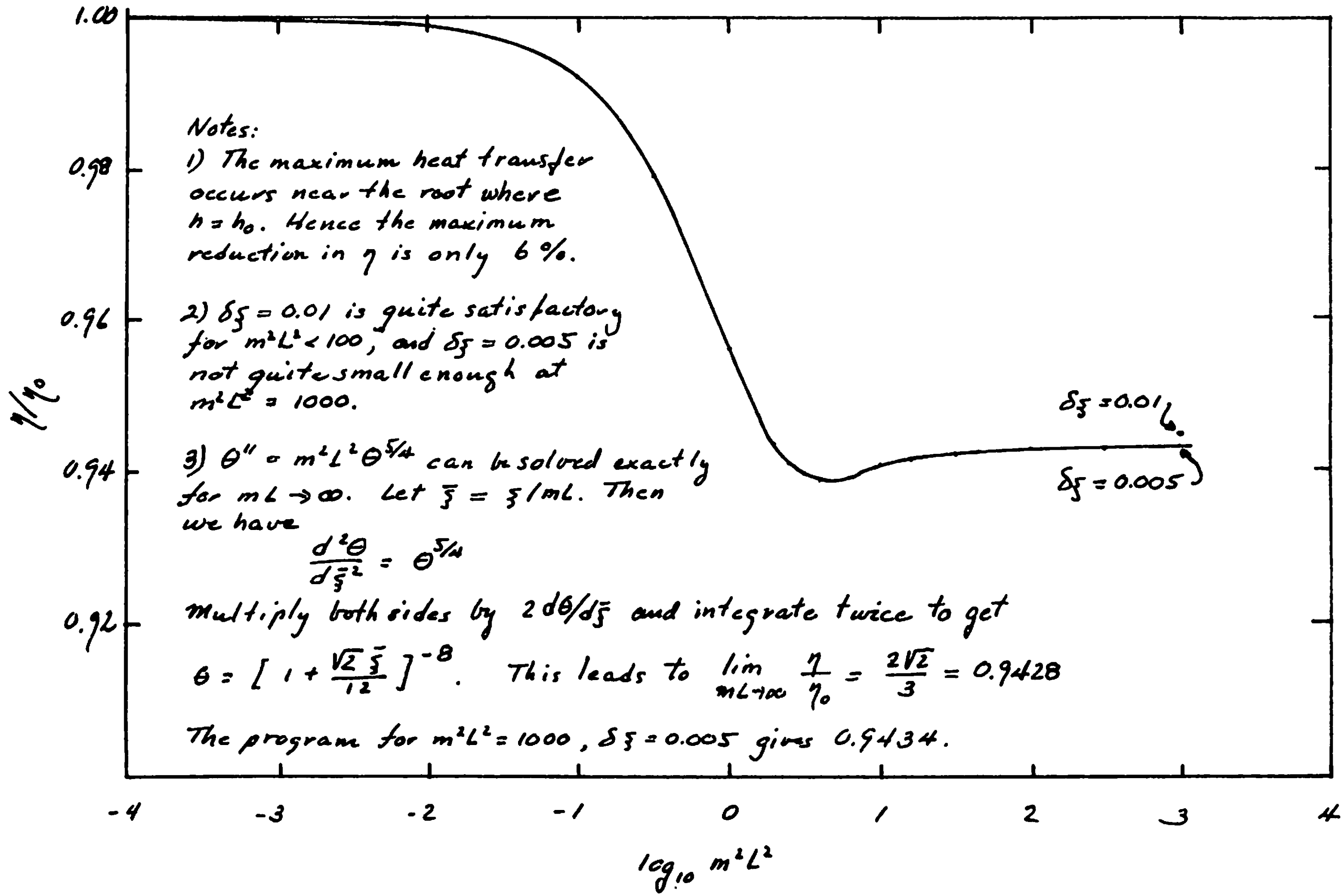
This old solution was carried out on an HP-85 calulator. Today we would certainly use more a more modern means of calculation.

```
10 ! "FIN WITH NATURAL CONVECTION HEAT EXCHANGE"
20 ! T0=temp from previous iteration;T1=temp from current iteration;
30 ! A,B,C,R are coefficients in the difference equations
40 ! D is an intermediate value in the tridiagonal algorithm.
50 DIM T0(200),T1(200),B(200)
60 DIM A(200),C(200),D(200),R(200)
70 ! N is the number of spatial units
80 DISP "ENTER (ml),N"
90 INPUT A1,N
100 ! E1 is the min. sum of ABS(T1(I)-T0(I))
110 E1=.0001*N/10
120 X1=1/N
130 D=A1*X1^2
140 Z=0
150 FOR I=0 TO N
160 A(I)=1 @ B(I)=-(2+D) @ C(I)=1 @ R(I)=0
170 T1(I)=1 @ T0(I)=1
180 NEXT I
190 N1=0
200 A(1)=0 @ C(N)=0 @ A(N)=2
210 R(1)=-1
220 N1=N1+1
230 R(1)=-1
240 FOR I=2 TO N
250 R(I)=0
260 NEXT I
270 GOSUB 400
280 R=0
290 FOR I=1 TO N
300 R=R+ABS(T1(I)-T0(I))
310 ! Z=0 suppresses natural convection
320 T0(I)=T1(I)
330 IF Z=0 THEN GOTO 350
340 B(I)=-(2+A1*X1^2*SQR(SQR(T1(I))))
350 !
360 NEXT I
370 DISP "R=";R
380 IF R<E1 THEN DISP "R=";R @ GOSUB 580
390 GOTO 200
400 ! Tridiagonal algorithm
410 FOR I=0 TO N
420 T1(I)=T0(I)
430 NEXT I
440 N2=N-1
450 D(1)=C(1)/B(1)
460 R(1)=R(1)/B(1)
470 FOR I=2 TO N
480 D(I)=B(I)/A(I)-D(I-1)
490 R(I)=(R(I)/A(I)-R(I-1))/D(I)
500 D(I)=C(I)/A(I)/D(I)
510 NEXT I
520 T1(N)=R(N)
530 FOR I=2 TO N
540 I1=N+1-I
550 T1(I1)=R(I1)-D(I1)*T1(I1+1)
560 NEXT I
570 RETURN
580 PRINT
590 PRINT "Fin with natural convection heat exchange"
600 IF Z=0 THEN PRINT "Natural convection suppressed"
610 PRINT @ PRINT "mL=";A1
620 PRINT "R=";R
630 PRINT "Number of iterations is";N1
640 PRINT
650 PRINT "Step size is X1/L=";X1
660 F=0
670 FOR I=0 TO N
680 IF Z=1 THEN F=F+T1(I)^1.25
690 IF Z=0 THEN F=F+T1(I)
700 NEXT I
710 IF Z=1 THEN F=F-(T1(0)^1.25+T1(N)^1.25)/2
720 IF Z=0 THEN F=F-(T1(0)+T1(N))/2
730 F=F*X1
740 PRINT "Efficiency=";F
750 SCALE 0,1,0,1
760 XAXIS 0,.1
770 YAXIS 0,.1
780 MOVE 0,1
790 FOR I=1 TO N
800 IDRAW X1,T1(I)-T1(I-1)
810 NEXT I
820 GRAPH
830 IF Z=0 THEN Z=1 @ GOTO 150
835 COPY
840 END
```

Note:
The difference equations are written as

$$A_i\Theta_{i-1}+B_i\Theta_i+C_i\Theta_{i+1}=R_i$$

9.45 (continued)



Notes:

1) The maximum heat transfer occurs near the root where $h = h_0$. Hence the maximum reduction in η is only 6%.

2) $\delta\xi = 0.01$ is quite satisfactory for $m^2L^2 < 100$, and $\delta\xi = 0.005$ is not quite small enough at $m^2L^2 = 1000$.

3) $\Theta'' = m^2L^2\Theta^{5/4}$ can be solved exactly for $mL \to \infty$. Let $\bar{\xi} = \xi/mL$. Then we have

$$\frac{d^2\Theta}{d\bar{\xi}^2} = \Theta^{5/4}$$

Multiply both sides by $2\,d\Theta/d\bar{\xi}$ and integrate twice to get

$\Theta = \left[1 + \frac{\sqrt{2}\,\bar{\xi}}{12}\right]^{-8}$. This leads to $\lim_{mL\to\infty} \frac{\eta}{\eta_0} = \frac{2\sqrt{2}}{3} = 0.9428$

The program for $m^2L^2 = 1000$, $\delta\xi = 0.005$ gives 0.9434.

8.46 Find the temperature of a black sphere in equilibrium with air at 20°C and surroundings at 1000 °K.

Equation 8.32 gives

$$\bar{h} = \frac{k}{D}\left[2 + 0.43\ Ra^{1/4}\right]$$

The equation we need to solve is

$$T = T_\infty + \frac{\mathcal{F}\sigma}{\bar{h}}\left(T_s^4 - T^4\right)$$

We first guess that the properties in the expression for $\bar{h}$ can be evaluated at 500°K. This gives

$$\frac{g\beta D^3}{\nu\alpha} = \frac{9.8(293)^{-1}(2\times10^{-2})^3}{3.798\times10^{-5}(5.438\times10^{-5})} = 129.6(°C)^{-1}$$

Ra will be $< 10^5$ if $\Delta T < 772$ °C, or $T < 1065$°K.

This will always be the case, so

$$\bar{h} = 3.95 + 2.87\left(T - T_\infty\right)^{1/4}$$

or
$$T = 293 + \frac{0.56697\times10^{-8}\left(10^{12} - T^4\right)}{3.95 + 2.87\left(T - 293\right)^{1/4}}$$

The solution to this equation is 601.43 °K, so $T_{film} = 447$°K

If the properties are evaluated at 450 °K, we have

$$\bar{h} = 3.63 + 3.14\left(T - T_\infty\right)^{1/4}$$

and $T = 591.35$°K. An additional iteration with $T_{film} = 440$°K gives $T = \underline{\underline{584.54\ °K.}}$

Note: The iteration process described in footnote 2 of chapter 6 diverges if the initial guess for T is too close to either 293 or 1000.

PROBLEM 8.53 An inclined plate in a piece of process equipment is tilted 30° above horizontal. The plate is 20 cm long in the inclined plane and 25 cm wide. The plate is held at 280 K by a liquid flowing past its underside. The liquid is cooled by a refrigeration system capable of removing 12 W, but if the heat load exceeds 12 W, the temperature of both the liquid and the plate will begin to rise. The upper surface of the plate is in contact with ammonia vapor at 300 K and a varying pressure. An engineer suggests that an increase of the bulk temperature of the liquid will signal that the pressure has exceeded a level of about $p_{\text{crit}} = 551$ kPa.

a) Explain why the gas's pressure will affect the heat transfer to the coolant. What is the significance $p_{\text{crit}} = 551$ kPa?
b) Suppose that the pressure is 255.3 kPa. What is the heat transfer rate (W) from the gas to the plate, if the plate temperature is $T_w = 280$ K? Will the coolant temperature rise?
c) Suppose that the pressure rises to 1062 kPa. What is the heat transfer rate if the plate is still at $T_w = 280$ K? Will the coolant temperature rise?

For gaseous ammonia at 255.3 kPa and 290 K: $\beta = 0.0040\ \text{K}^{-1}$, $\rho = 1.86\ \text{kg/m}^3$, $c_p = 2314$ J/kgK, $\mu = 9.75 \times 10^{-6}\ \text{kg/m}^3$, and $k = 0.0247$ W/m·K. Take other data from Appendix A.

SOLUTION

a) If the vapor's pressure were to exceed $p_{\text{sat}}(280\ \text{K}) = 551$ kPa, the vapor would condense on the plate. The vapor cannot condense at lower pressures, and heat transfer would be by natural convection only. The heat transfer coefficient in condensation is more than 100 times greater than for natural convection, so the heat load would be dramatically higher for pressures of 551 kPa or more, causing the refrigeration loop to overheat.

b) At 255.3 kPa, the saturation temperature is $T_{\text{sat}} = 260\ \text{K} < 280$ K; condensation will not occur. The film temperature in the vapor is 290 K (which corresponds to the data given). From the given data, $\nu = 5.24 \times 10^{-6}\ \text{m}^2/\text{s}$ and $\alpha = 5.74 \times 10^{-6}\ \text{m}^2/\text{s}$

Replacing g with an effective gravity $g \cos 60°$, the Rayleigh number is

$$\text{Ra}_L = \frac{g \cos 60° \beta \Delta T L^3}{\nu \alpha} = \frac{(9.81)(1/2)(0.0040)(20)(0.2)^3}{(5.24 \times 10^{-6})(5.74 \times 10^{-6})} \simeq 1.04 \times 10^8$$

The Nusselt number, from eqn. (8.13a) and using effective gravity, is

$$\begin{aligned}\overline{\text{Nu}}_L &= 0.68 + 0.67\,\text{Ra}_L^{1/4} \left[1 + \left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{-4/9} \\ &= 0.68 + 0.67(1.04 \times 10^8)^{1/4} \left[1 + \left(\frac{0.492}{0.912}\right)^{9/16}\right]^{-4/9} \\ &\simeq 54.0\end{aligned}$$

Then,

$$h = \overline{\text{Nu}}_L \frac{k}{L} = 54.0\left(\frac{0.0247}{0.2}\right) = 6.67\ \text{W/m}^2\text{K}$$

and the heat transfer is

$$Q = hA(T_\infty - T_w) = (6.67)(0.2)(0.25)(300 - 280) \simeq 6.67\ \text{W} < 12\ \text{W}$$

and the plate and liquid temperatures will not rise.

c) At a pressure of 1062 kPa, the saturation temperature is $T_{\text{sat}} = 300$ K > 280 K; condensation occurs. The Nusselt number, from eqn. (8.62b) and using effective gravity, is

$$\overline{\text{Nu}}_L = 0.9428 \left[\frac{\rho_f(\rho_f - \rho_g)\text{g}\cos 60° h'_{fg} L^3}{\mu k (T_{\text{sat}} - T_w)} \right]^{1/4} = 1814$$

Here μ, k, and ρ_f are properties of the liquid at a film temperature of 290 K, and ρ_g is for saturated ammonia vapor at 300 K. A simple calculation shows that Ja = 0.078 and so with eqn. (8.61), $h'_{fg} \cong h_{fg}(1.040) = 1204$ kJ/kg. Then

$$\overline{\text{Nu}}_L = 0.9428 \left[\frac{(614.7)(614.7 - 8.244)(9.81)(0.5)(1204 \times 10^3)(0.2)^3}{(1.39 \times 10^{-4})(0.488)(20)} \right]^{1/4} = 1898$$

The heat transfer coefficient is

$$h = \overline{\text{Nu}}_L \frac{k}{L} = 4632 \text{ W/m}^2\text{K}$$

The heat transfer rate is

$$Q = hA(T_{\text{sat}} - T_w) = (4632)(0.2)(0.25)(20) = 4632 \text{ W} \gg 12 \text{ W}$$

and the plate and liquid temperatures will rise.

***Comment*:** The ammonia vapor is part (b) is superheated, but still not far from saturation conditions. The vapor does not behave like an ideal gas in this range. The property data given are from the NIST Webbook, https://webbook.nist.gov/chemistry/fluid/.

Problem 8.54 The film Reynolds number Re_c in eqn. (8.72) was based on the thickness, δ. Show that the Reynolds number would be four times larger if it were based on the hydraulic diameter of the film.

Solution The hydraulic diameter is defined in eqn. (7.60) as

$$D_h \equiv \frac{4A_c}{P}$$

where A_c is the cross-sectional area and P is the passage's *wetted* perimeter. For a unit width of falling film having a local thickness δ

$$D_h = \frac{4(1)(\delta)}{(1)} = 4\delta$$

because the *wetted* perimeter is the part of the film in contact with the wall, *excluding* the free surface: $P = 1$.

The film Reynolds number from eqn. (8.72) is

$$\mathrm{Re}_c = \frac{\rho u_{\mathrm{av}} \delta}{\mu} = \frac{\Gamma_c}{\mu}$$

which takes the film thickness as the length scale for the Reynolds number. If instead we define the Reynolds number using the hydraulic diameter, we have

$$\mathrm{Re}_{\mathrm{wp}} = \frac{\rho u_{\mathrm{av}} D_h}{\mu} = \frac{4\rho u_{\mathrm{av}} \delta}{\mu} = \frac{4\Gamma_c}{\mu}$$

Thus

$$\mathrm{Re}_{\mathrm{wp}} = 4\,\mathrm{Re}_c \quad \xleftarrow{\text{Answer}}$$

Problem 8.55 A characteristic length scale for a falling liquid film is $\ell = (\nu^2/g)^{1/3}$. If the Nusselt number for a laminar film condensing on plane wall is written as $\mathrm{Nu}_\ell \equiv h\ell/k$, derive an expression for Nu_ℓ in terms of Re_c. Show that, when $\rho_f \gg \rho_g$, $\mathrm{Nu}_\ell = \left(3\mathrm{Re}_c\right)^{-1/3}$.

Solution Starting with eqns. (8.58) and (8.72), we have

$$\mathrm{Nu}_x = \frac{hx}{k} = \frac{x}{\delta} \tag{8.58}$$

and

$$\mathrm{Re}_c = \frac{\rho_f\left(\rho_f - \rho_g\right) g\delta^3}{3\mu^2} = \frac{\rho_f \Delta\rho\, g\delta^3}{3\mu^2} \tag{8.72}$$

Then, by replacing x by ℓ

$$\mathrm{Nu}_\ell = \frac{h\ell}{k} = \frac{\ell}{\delta}$$

and, by rearranging Re_c,

$$\delta = \left(\frac{3\mu\nu}{g\Delta\rho}\,\mathrm{Re}_c\right)^{1/3}$$

So

$$\boxed{\mathrm{Nu}_\ell = \left(\frac{\nu^2}{g}\right)^{1/3}\left(\frac{g\Delta\rho}{3\mu\nu}\right)^{1/3}\mathrm{Re}_c^{-1/3} = \left(\frac{\Delta\rho}{3\rho_f}\right)^{1/3}\mathrm{Re}_c^{-1/3}}$$

and when $\rho_f \gg \rho_g$, $\Delta\rho \simeq \rho_f$ so

$$\boxed{\mathrm{Nu}_\ell \simeq (3\mathrm{Re}_c)^{-1/3}} \quad \text{for } \rho_f \gg \rho_g$$

Problem 8.57 Perform the integration for $\overline{h}$ in Example 8.8 and obtain eqn. (8.67). *Hint*: Recall that the gamma function, $\Gamma(z)$, is a tabulated special function. It may be shown that [8.42,§9.51]:

$$\int_0^{\pi/2} \cos^{2m-1}\theta \sin^{2n-1}\theta \, d\theta = \frac{\Gamma(m)\Gamma(n)}{2\,\Gamma(m+n)} \quad \text{for } m, n > 0$$

Solution The integral in question is

$$\overline{h} = \frac{2}{\pi D}\int_0^{\pi D/2} \frac{1}{\sqrt{2}}\frac{k}{x}\left[\frac{(\rho_f-\rho_g)h'_{fg}x^3}{\nu k(T_{\text{sat}}-T_w)}\,\frac{x g_e(\sin 2x/D)^{4/3}}{\int_0^x (\sin 2x/D)^{1/3}\,dx}\right]^{1/4} dx$$

This integral may look formidable, but it is in fact merely messy. Let us start by lumping all the constants

$$\mathbb{C} \equiv \frac{2}{\pi D}\frac{k}{\sqrt{2}}\left[\frac{(\rho_f-\rho_g)h'_{fg}g_e}{\nu k(T_{\text{sat}}-T_w)}\right]^{1/4}$$

so that

$$\begin{aligned}\overline{h} &= \mathbb{C}\int_0^{\pi D/2}\left[\frac{(\sin 2x/D)^{4/3}}{\int_0^x (\sin 2x/D)^{1/3}\,dx}\right]^{1/4} dx \\ &= \mathbb{C}\int_0^{\pi D/2}(\sin 2x/D)^{1/3}\left[\int_0^x (\sin 2x/D)^{1/3}\,dx\right]^{-1/4} dx\end{aligned}$$

where the factors in x canceled out. Now define

$$f(x) \equiv (\sin 2x/D)^{1/3}$$

and we have

$$\overline{h} = \mathbb{C}\int_0^{\pi D/2} f(x)\left[\int_0^x f(x)\,dx\right]^{-1/4} dx$$

Further, we can take advantage of the derivative of an integral. Let

$$F(x) \equiv \int_0^x f(x)\,dx$$

Then

$$\frac{dF}{dx} = f(x)$$

So,

$$\overline{h} = \mathbb{C}\int_0^{\pi D/2}\frac{dF}{dx}\left[F(x)\right]^{-1/4}dx = \frac{4}{3}\mathbb{C}\left[F(x)\right]^{3/4}\Big|_0^{\pi D/2} = \frac{4}{3}\mathbb{C}\left[F(\pi D/2)\right]^{3/4}$$

Now, with our previous definitions

$$F(\pi D/2) = \int_0^{\pi D/2} (\sin 2x/D)^{1/3}\,dx$$

which corresponds to the integral given in the problem statement for $m = 1/2$ and $n = 2/3$, where we put $\theta = 2x/D$:

$$F(D/2) = \int_0^{\pi D/2} (\sin 2x/D)^{1/3}\, dx = \frac{D}{2}\int_0^{\pi} (\sin\theta)^{1/3}\, d\theta = D\int_0^{\pi/2} (\sin\theta)^{1/3}\, d\theta = D\,\frac{\Gamma(1/2)\,\Gamma(2/3)}{2\,\Gamma(7/6)}$$

From tabulated results, we find that

$$\Gamma(1/2) = \sqrt{\pi}, \quad \Gamma(2/3) = 1.3541\cdots, \quad \Gamma(7/6) = \frac{1}{6}\Gamma(1/6) = \frac{1}{6}(5.5663\cdots)$$

Collecting all this:

$$\overline{h} = \frac{4}{3}\mathbb{C}\left[D\,\frac{\Gamma(1/2)\,\Gamma(2/3)}{2\,\Gamma(7/6)}\right]^{3/4} = \frac{4\sqrt{2}k}{3\pi D}\left[\frac{(\rho_f - \rho_g)h'_{fg}g_e D^{3/4}}{\nu k(T_{\text{sat}} - T_w)}\right]^{1/4}\left[\frac{6\sqrt{\pi}(1.3541)}{2(5.5663)}\right]^{3/4}$$

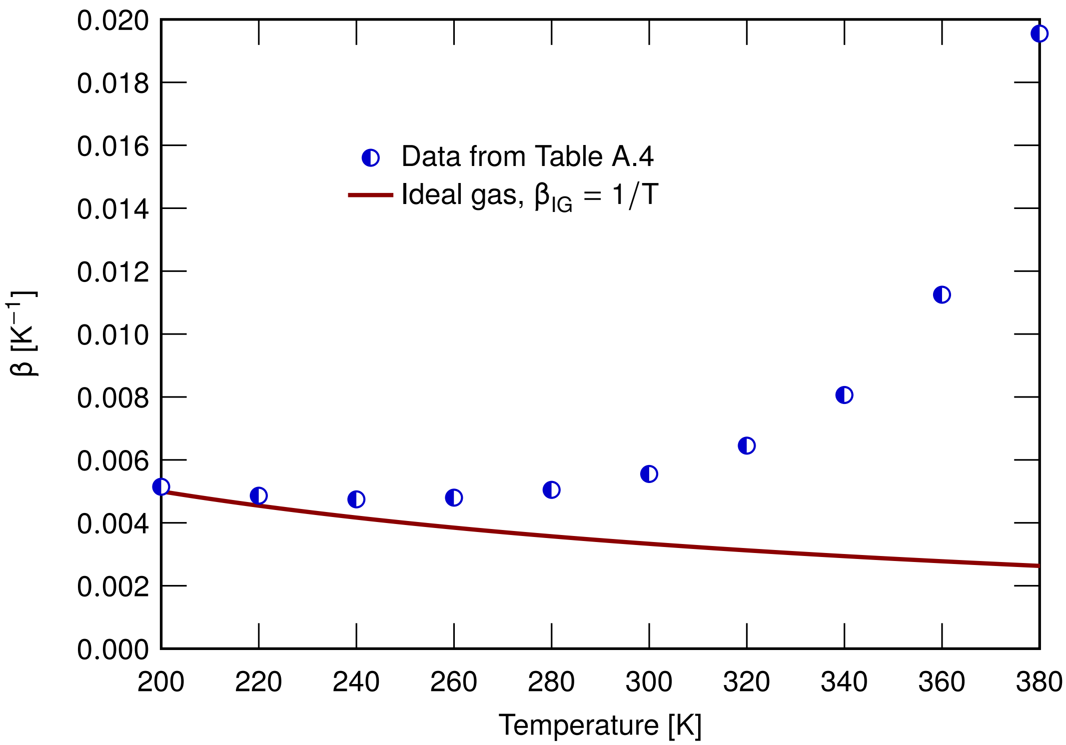
and finally

$$\overline{\text{Nu}}_D = \frac{\overline{h}D}{k} = \left\{\frac{4\sqrt{2}}{3\pi}\left[\frac{3\sqrt{\pi}(1.3541)}{(5.5663)}\right]^{3/4}\right\}\left[\frac{g_e(\rho_f - \rho_g)h'_{fg}D^3}{\nu k(T_{\text{sat}} - T_w)}\right]^{1/4}$$

$$= 0.7280\left[\frac{g_e(\rho_f - \rho_g)h'_{fg}D^3}{\nu k(T_{\text{sat}} - T_w)}\right]^{1/4} \quad \xleftarrow{\text{Answer}}$$

PROBLEM 8.59 Using data from Tables A.4 and A.5, plot β for saturated ammonia vapor for 200 K $\leqslant T \leqslant$ 380 K, together with the ideal gas expression $\beta_{IG} = 1/T$. Also calculate $Z = P/\rho RT$. Is ammonia vapor more like an ideal gas near the triple point or critical point temperature?

SOLUTION

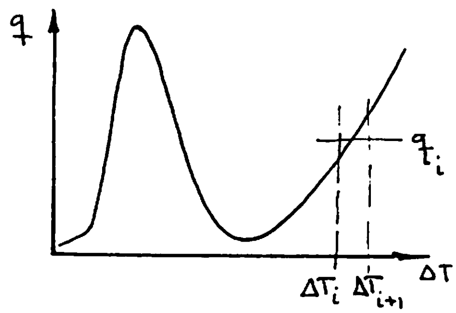


With p and ρ from Table A.5, and using $R = R^\circ/M_{NH_3} = 8314.5/17.031 = 488.2$ J/kg-K, we find Z as below. For an ideal gas, $Z = 1$.

T [°C]	Z	T [°C]	Z
200	0.9944	300	0.8788
220	0.9864	320	0.8263
240	0.9722	340	0.7606
260	0.9505	360	0.6784
280	0.9198	380	0.5716

Saturated ammonia vapor only behaves like an ideal gas for temperatures close the triple point temperature (195.5 K) and is highly non-ideal in the vicinity of the critical point temperature (405.4 K). This behavior underscores the importance of using data for β when dealing with vapors near saturation conditions.

9.1 Water boils, according to the graphical relation in Fig. 9.2, on a 1.27 cm thick copper slab which starts out at 650^{o}C. Plot T_{slab} vs. time, indicating the regime of boiling and noting the temperature at which the cooling is most rapid.



$$Bi = \frac{hL}{k} \simeq \frac{0.0127\,h}{376} < 1$$ as long as h is less than 29,600 $\frac{W}{m^2\text{-}°C}$

from eqn. (1.19)

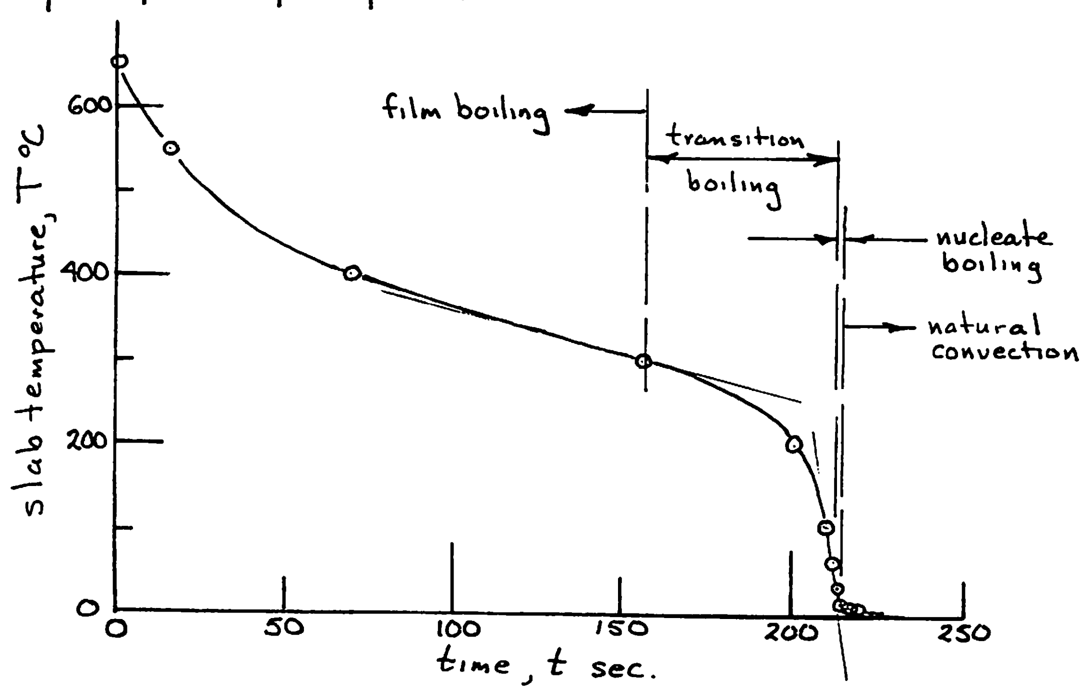
$$\frac{Q}{A} = q = \frac{\delta\left(\frac{\rho c V}{A}[T - T_{sat}]\right)}{\delta t}$$

or

$$\delta t = \rho c L \frac{\delta \Delta T}{q} = 43{,}667 \frac{\delta \Delta T}{q}$$

Now we use this eqn & Fig 9.2 to calculate:

ΔT_{i+1} °C	ΔT_i °C	q_i $\frac{W}{m^2\text{-}°C}$	δt sec	$t_{i+1} = t_i + \delta t$	
550	400	4.5×10^5	14.56	14.56	
400	300	0.8 "	54.59	69.14	
300	200	0.5 "	87.33	156.5	q_{min}
200	150	" "	43.67	200.2	
150	100	2.8 "	7.8	208.	
100	60	7.5 "	2.3	210.3	
60	30	11.4 "	1.15	211.45	q_{max}
30	10	7.8 "	1.12	212.57	
10	4	2.3 "	1.14	213.71	$q_{boiling\ inception}$
4	2	0.3 "	2.9	216.62	
2	1	0.15 "	2.9	219.53	



9.2 Predict q_{max} for horizontal cylinders for the cases in Fig. 10.3b and indicate the fraction of q_{max} in each case.

(a) 0.0322 cm diam. in methanol with g = 98 m/s^2
(b) 0.164 cm diam. in benzene with g = 9.8 m/s^2

first find R': $R' = R\sqrt{g(\rho_f-\rho_g)/\sigma}$

	methanol	benzene
T_{sat}	64°C	80°C
σ	18.7 dyne/cm	21.3 dyne/cm
ρ_f	.814 gm/cm³	.816 gm/cm³
ρ_g	.00117 "	.00249 "
h_{fg}	1114 kJ/kg	391 kJ/kg

$$R'_{meth.} = \frac{.0322}{2}\sqrt{\frac{9800(0.813)}{18.7}} = \underline{0.332}$$

$$R'_{Benz.} = \frac{0.164}{2}\sqrt{\frac{980(0.814)}{21.3}} = \underline{0.502}$$

$$q_{max_{Z_{meth}}} = \frac{\pi}{24}\left(1.17\frac{kg}{m^3}\right)^{1/2} 1{,}114{,}000\frac{J}{kg}\sqrt[4]{98\frac{m}{s^2}\, 0.0187\frac{kg}{s^2}\left(813\frac{kg}{m^3}\right)} = \underline{980{,}000\ \frac{W}{m^2}}$$

$$q_{max_{Z_b}} = \frac{\pi}{24}(2.44)^{1/2}\, 391000\sqrt[4]{9.8(.0213)(814)} = \underline{288{,}600\ \frac{W}{m^2}}$$

From Fig. 9.13, upper left-hand corner, we read:

$$\left(\frac{q_{max}}{q_{max_Z}}\right)_{\substack{Meth.\\ R'=.332}} = 1.2 \qquad \text{so} \qquad q_{max_{Meth}} = \underline{\underline{1{,}176{,}000\ W/m^2}}$$

$$\left(\frac{q_{max}}{q_{max_Z}}\right)_{\substack{Benz.\\ R'=0.502}} = 1.1 \qquad \text{so} \qquad q_{max_{Benz}} = \underline{\underline{318{,}000\ W/m^2}}$$

Fig. 9.3b shows methanol at 1,040,000 W/m^2 or 88.4 % of q_{max}.
Fig. 9.3b shows benzene at 350,000 W/m^2 or 90.7 % of q_{max}.

9.3 Water at 70°C is depressurized until it is subcooled 30°C. Find the pressure at this point and the diameter of the critical nucleus.

$p_{sat.}$ at 70°C = 31,170 N/m^2, $\underline{\underline{p_{sat}\ \text{at}\ 40^oC = 7375\ N/m^2}}$

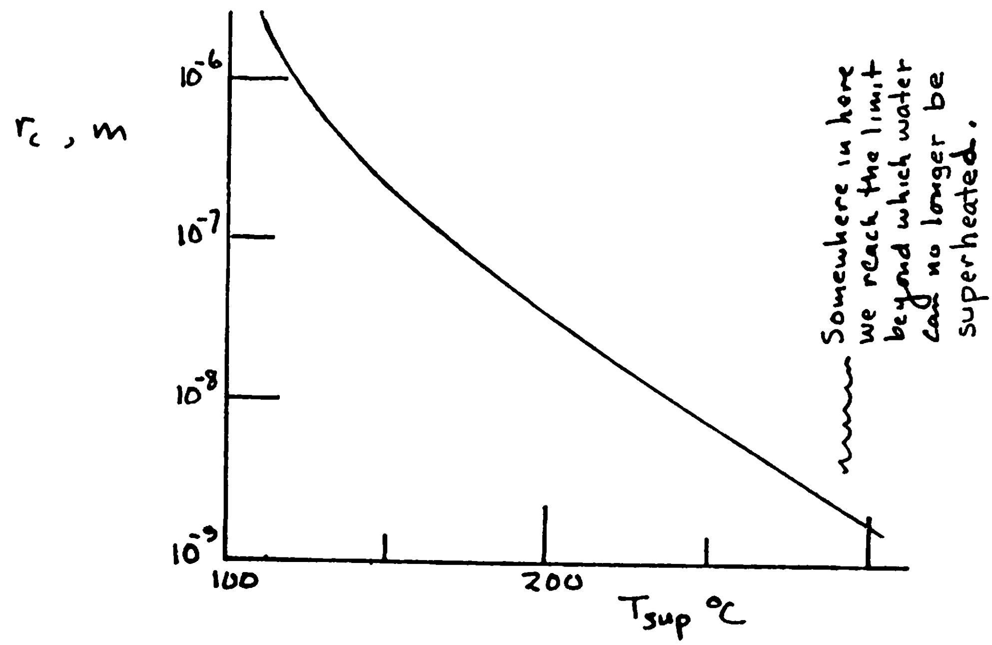
$$r_c = \frac{2\sigma_{at\ 70^oC}}{31{,}170 - 7375} = 2\,\frac{65.49\,\frac{dyne}{cm}\left(10^{-3}\frac{N/m}{dyne/cm}\right)}{23{,}795\ N/m^2}$$

diameter of nucleus = $2r_c$ = $\underline{\underline{1.1(10)^{-6}\ m}}$

9.4 Plot r_c vs. liquid superheat for water at 1 atm.

$$r_c = \frac{\sigma}{p_{sat\ at\ T_{sup}} - 10^5} \quad ; \quad \sigma = 0.2358\left(1 - \frac{T_{sup}}{647.2}\right)^{1.256}\left[1 - 0.625\left(\frac{T_{sup}}{647.2}\right)\right] \ \frac{N}{m}$$

T_{sup}, °C/°K	ΔT °C = $T_{sup} - 100$	σ $\frac{N}{m}$	$p_{sat\ at\ T_{sup}}$ (N/m²)	r_c m	r mm	r Å
110/383	10	0.0482	1.433×10^5	2.23×10^{-6}	0.00223	
130/403	30	0.0423	2.701 "	0.50 "	0.00050	
160/433	60	0.0342	6.18 "	0.132 "	0.000132	
200/473	100	0.0246	15.55 "	0.0338 "		338
250/523	150	0.0147	39.78 "	7.58×10^{-9}		76
300/573	200	0.00694	85.92 "	1.63×10^{-11}		16



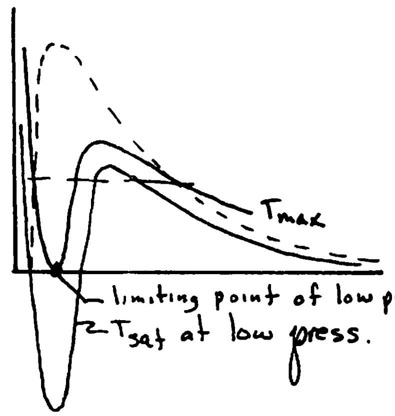
9.5 Why does bumping occur in a test tube, but not in a teakettle?

The test-tube is very smooth so $(r_c)_{test\text{-}tube} \ll (r_c)_{teakettle}$. It follows that, since $r_c = 2\sigma/(p_{sat.\,at\,T_{nucl.}} - p_{sat})$ is small, $p_{sat\ at\ T_{nucl.}}$ is large. Thus T_{nucl} is also much higher in the test-tube than in the tea-kettle.

It is beyond our scope here, but the thermodynamic availability is a measure of the damage that a superheated liquid can do when it nucleates. We can show [Jour. Ht. Transfer, Feb. 1981, Vol. 103, pp. 61-64] that the avalability rises as $(T_{sup} - T_{sat})^2$.

Thus Δa (and the possible damage) increase strongly with superheat.

9.6 Use van der Waals' equation to estimate how much superheat water can sustain at low pressure.



In reduced form the van der Waals equation is:

$$p_r = \frac{8T_r}{3(1-v_r)} - \frac{3}{v_r^2}$$

set $p_r \simeq 0$ and solve for $T_{r_{max}}$:

$$\underline{T_{r_{max}} = \frac{9}{8}\frac{1-v_r}{v_r^2}}$$

but at the limiting point: $\frac{\partial p_r}{\partial v_r} = 0 = -\frac{8T_r}{3(1-v_r)^2} + \frac{6}{v_r^3}$

substitute $T_{r_{max}}$: $\quad 0 = -\frac{8}{3}\frac{1}{(1-v_r)^2}\frac{9}{8}\frac{1-v_r}{v_r^2} + \frac{6}{v_r^3}$

or: $\quad 0 = -\frac{1}{1-v_r} + \frac{2}{v_r} \quad ; \quad v_r = \frac{2}{3}$

Thus: $\quad T_{r_{max}} = \frac{9}{8}\frac{1/3}{(2/3)^2} = \frac{27}{32}$

$$T_{max_{H_2O}} = \frac{27}{32}T_c = \frac{27}{32}647.2 = 546^\circ R$$

So, at 1 atm, the limiting superheat is $\Delta T = (546 - 373) = \underline{\underline{173^\circ C}}$

(The measured extremes are just a little greater than this.)

9.7 Find c in $n \sim \Delta T^c$ such that the result is consistent with Berenson's curves in Fig. 9.14 and Yamagata's equation:

$$q \sim n^{1/3}\Delta T^{1.2}$$

From the log-log plots in Fig. 9.14 we measure the slopes in the nucleate boiling range. Call this slope, d. The 5 values are 6, 5.7, 5.3, 4, 2.2. Then:

$$q \sim \Delta T^d \sim \Delta T^{c/3}\Delta T^{1.2} \quad \text{or} \quad c = 3d - 1.2$$

Thus the 5 values of c in $n = \Delta T^c$ are:

$\underline{\underline{c = 16.8}}$; $\underline{\underline{c = 15.9}}$; $\underline{\underline{c = 14.7}}$; $\underline{\underline{c = 10.8}}$; and $\underline{\underline{c = 5.4}}$

9.8 Suppose C_{sf} for a given surface is reported as being 50 % higher than is really is. How much error will this contribute to the calculated q?

from eqn. (9.4) we have: $q \sim (\Delta T / C_{sf})^3$

Then $q_{calculated} \sim \frac{1}{1.5^3}\left[\frac{\Delta T}{C_{sf_{correct}}}\right]^3$

or $q_{calculated} = 0.296\, q_{correct}$

Thus the calculation is $\underline{\underline{\text{low by } 70\%}}$

9.9 Water at 100 atm boils on a nickel heater. $\Delta T = 6^oC$. Find q and h.

Properties at $T_{sat} = 310°C$: $\rho_f = 690\ kg/m^3$ $\quad h_{fg} = 1325000\ J/kg$
$\rho_g = 54.7$ " $\quad c_p = 5600\ J/kg°C$
$\mu_f = 0.0000875\ kg/m\text{-}s$ $\quad P_r = 1.02$
$\sigma = 0.0117\ kg/s^2$ $\quad C_{sf} = 0.006$

Then, from eqn. (9.4)

$$(c_p \Delta T / h_{fg} P_r^{1})^3 = \frac{C_{sf}}{\mu h_{fg}} \sqrt{\frac{\sigma}{g(\rho_f - \rho_g)}}\ q$$

So:

$$q = \left(\frac{5600(6)}{1325000(1.02)}\right)^3 \frac{0.0000875(1.325)10^6}{(0.006)^3} \Big/ \sqrt{\frac{0.0117}{9.8(635.3)}}$$

$$q = \underline{\underline{6.017(10)^6\ \frac{W}{m^2}}}$$

and. $$h = \frac{q}{\Delta T} = \frac{6.017(10)^6}{6} = \underline{\underline{1,003,000\ \frac{W}{m^2\text{-}°C}}}$$

This is very high.

9.10 Compute q_{max} for saturated water at 1 atm on a flat plate -- very large in extent -- at $g/g_e = 1/6$ and 10^{-4}.

At earth-normal gravity: $q_{max} = 1,260,000\ W/m^2$ (Example 10.5)

Thus, at $\frac{g}{g_e} = \frac{1}{6}$

$$q_{max} = 1,260,000 \sqrt[4]{\frac{1}{6}} = \underline{\underline{805,000\ \frac{W}{m^2}}}$$

And at $\frac{g}{g_e} = 10^{-4}$

$$q_{max} = 1,260,000 \sqrt[4]{\frac{1}{10^{-4}}} = \underline{\underline{126\,000\ \frac{W}{m^2}}}$$

Since, in accordance with eqn. (9.11)

$$q_{max\ flat\ plate} \sim g^{1/4}$$

9.11 Water boils on a 0.001 m radius copper wire. Plot as much of the boiling curve as you can, for this case.

We go through the regimes of the boiling curve, one at a time, starting with natural convection.

$$\overline{Nu_D} = 0.36 + \frac{0.518 Ra_D^{1/4}}{\left[1+\left(\frac{0.559}{Pr}\right)^{9/16}\right]^{4/9}} = 0.36 + \underbrace{\frac{0.518\left[\frac{9.8(0.00072)(0.002)^3}{1.653(0.292)10^{-13}}\right]^{1/4}}{\left[1+\left(\frac{0.559}{1.74}\right)^{9/16}\right]^{4/9}}} \Delta T^{1/4}$$

$$q = \overline{Nu_D}\frac{k}{D}\Delta T = 122\Delta T + 844\,\Delta T^{5/4}$$

ΔT °C	q $\frac{W}{m^2}$
1	966
2	2251
3	3332
4	5262
5	6920

The nucleate boiling heat flux is given by eqn. (9.4)

$$q = \left[\frac{c_p \Delta T}{h_{fg} Pr\, C_{sf}}\right]^3 \mu h_{fg}\sqrt{\frac{g\Delta\rho}{\sigma}} = \left[\frac{4218}{2.286(10)^6 1.74(0.013)}\right]^3 0.265(10)^{-6}\, 2.286(10)^6 \sqrt{\frac{9.8(958.5)}{0.0589}}\Delta T^3$$

$$q = 124\,\Delta T^3$$

ΔT °C	q $\frac{W}{m^2}$
3	3358
4	7958
5	15544
10	124000
20	994,900

The peak heat flux at $R' = R\sqrt{\frac{9.8(958)}{0.0589}} = 0.399$ is given by eqn. (9.20)

$$q_{max} = \left[\frac{1}{1.14}\underbrace{q_{max_F}}_{\text{get from Example 10.5}}\right]\frac{0.94}{(R')^{1/4}} = \frac{1.26}{1.14}(10)^6\frac{0.94}{(0.399)^{1/4}} = 1,307,000\ \frac{W}{m^2}$$

The minimum heat flux for $R' = 0.399$ is

$$q_{min} = 0.515\left[\frac{18}{R'^2(2R'^2+1)}\right]^{1/4}\left[0.09\rho_g h_{fg}\sqrt[4]{\frac{\sigma g(\rho_f-\rho_g)}{(\rho_f+\rho_g)^2}}\right]$$

$$= 0.0464\left[\frac{18}{0.399^2(2[0.399]^2+1)}\right]^{1/4}(0.597)(2,257,000)\sqrt[4]{\frac{0.0589(9.8)(958)}{959^2}}$$

$$q_{min} = 29,800\ W/m^2$$

In the film boiling regime $h = h_{f.b.} + \frac{3}{4}h_{rad}$. Thus:

$$q = \frac{k\Delta T}{D}\left\{\left(0.661+\frac{0.243}{R'}\right)R'^{1/4}\, 0.62\left[\frac{(\rho_f-\rho_g)g h'_{fg}D^3}{\nu_g k_g \Delta T}\right]^{1/4}\right\} + \frac{3}{4}\sigma\left(T_w^4 - T_{sat}^4\right)$$

where we assume $\epsilon = 1$ and in which:

$$h'_{fg} = h_{fg}\left(1+\left[0.968-\frac{0.163}{Pr_g}\right]Ja\right) = 2,257,000\left(1+\left[0.968-\frac{0.163}{1.052}\right]\frac{2030\,\Delta T}{2,257,000}\right)$$

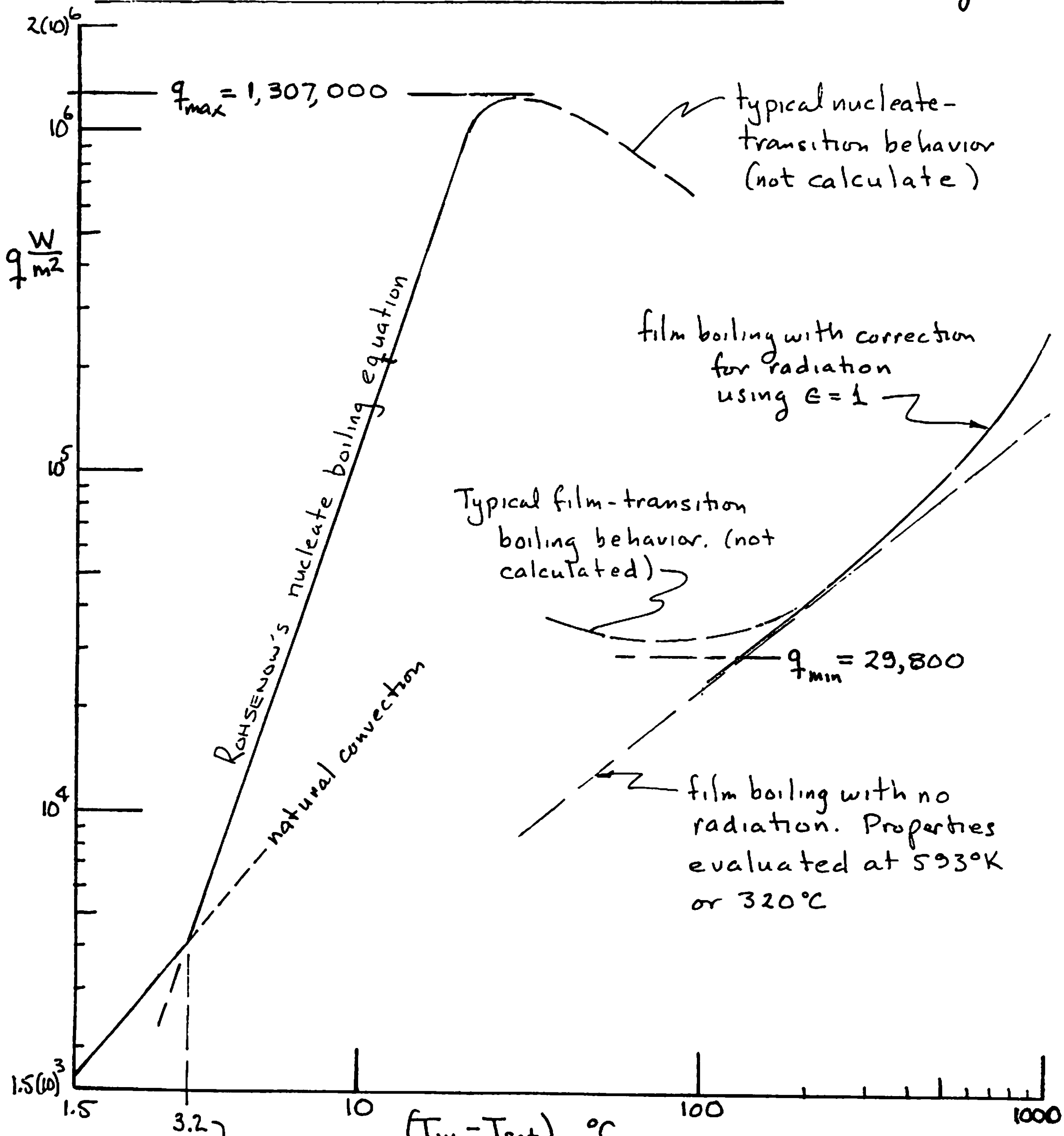
$$= 2,257,000\left(1+0.00075\,\Delta T\right)$$

9.11 (continued)

$$q = \frac{0.0237}{0.002}\left(0.661 + \frac{0.243}{0.399}\right)0.399^{1/4}(0.62)\left[\frac{958(9.8)2.257(10)^6(1+0.00075\Delta T)0.002^3}{5.884\times 10^{-5}(0.04)}\right]^{1/4}\Delta T^{3/4}$$

$$+0.75(5.67)10^{-8}\left(T_w^4 - 1.936(10)^{10}\right)$$

$$\underline{\underline{q = 683(1+0.00075\Delta T)^{1/4}\Delta T^{3/4} + 0.4253(10)^{-8}\left(T_w^4 - 1.936(10)^{10}\right)}} \quad \text{for film boiling}$$



$\Delta T_{inception}$ occurs where $(122\Delta T + 844\Delta T^{5/4}) = 124\,\Delta T^3$

Trial & error gives $\Delta T_{incept.} = 3.2°C$ or $5.8°F$.

9.12 This problem is the same as 9.11 with the following exceptions: Since the heater is a sphere, we use equation (8.32) instead of eqn. (8.28) for natural convection. The peak heat flux is given by eqn. (9.22) instead of (9.20). Film boiling is still given by $h = h_{f.b} + 0.75h_{rad.}$, and $h_{rad.}$ is still the same. But we no longer need include a curvature correction in calculating $h_{f.b.}$ because a 0.03 m sphere is sufficiently large not to need it.) There is no reliable eqn. for q_{min} in this case.

9.13 Predict q_{max} for a small flat plate with only one jet on it.

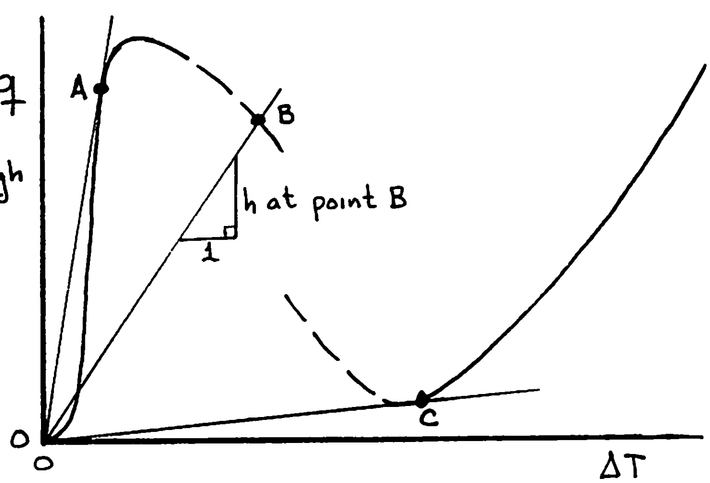
$$q_{max} = \underbrace{(1.14\, q_{max_Z})}_{q_{max} \text{ for a flat plate}} \frac{(A_{heater})_{actual}}{(A_{heater})_{ideal}}$$

but $(A_{heater})_{ideal} = \lambda_{d_1}^2$, therefore $\dfrac{q_{max}}{q_{max_Z}} = \dfrac{1.14}{\lambda_{d_1}^2} A_{heater}$

9.14 Show how to locate points of miximum and minumum h during pool boiling.

Use Fig. 9.2

$h = \dfrac{q}{\Delta T}$ = slope of lines which pass through the origin & touch the curve



(Notice that this can only be done on linear coordinates. Fig. 9.2 is semi-logarithmic. Therefor we must locate points A & C by trial & error in Fig. 9.2) we get:

$$h_{max} = \frac{960{,}000}{2} = 48{,}000 \frac{W}{m^2\text{-}^\circ C} \qquad h_{min} = \frac{170{,}000}{250} = 680 \frac{W}{m^2\text{-}^\circ C}$$

9.15 A 0.002 m diam. jet of saturated water flows normal to a 0.015 m diam. disc, at 1 m/s. How much energy can the disc dissipate?

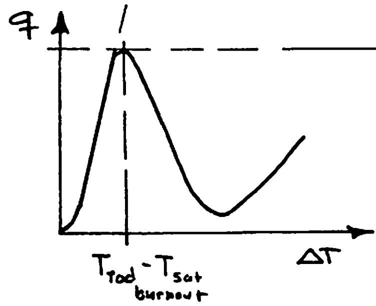
$\dfrac{\rho_f}{\rho_g} = \dfrac{958.3}{0.597} = 1605$ so eqn. (9.41) gives $A = 0.329$. Then eqn. (9.40) gives:

$$q_{max} = 2.939 \rho_g h_{fg} u_{jet} \left(\frac{0.002}{0.015}\right)^{1/3} \left(\frac{1000\,[1605]}{\rho_f u_j D/\sigma}\right)^A$$

$$= 2.939(0.597)(2.257\cdot 10^6)\,1\,(0.1333)^{1/3}\left(\frac{1{,}605{,}000}{958.3(1)(0.015)/0.0589}\right)^{0.329} = 3.65\times 10^7 \frac{W}{m^2}$$

So the maximum heat dissipation is $Q_{max} = q_{max} \frac{\pi}{4} D^2 = 6{,}448\ W$

9.16 Saturated water at 1 atm. boils on a 0.005 m diam. rod of platinum. What is T_{rod} at burnout?



$q_{max} = (0.94/R'^{1/4})\ q_{max_z}$ (eqn. 9.20))

$$R' = \sqrt{\frac{g(\rho_f - \rho_g)}{\sigma}}R = \sqrt{\frac{9.8(957.6)}{0.0589}}(0.0025)$$

$$= 0.9979$$

$$q_{max_z} = \frac{\pi}{24}(0.597)^{1/2}(2257000)\sqrt[4]{0.0589(9.8)(957.6)}$$

$$= 1,107,000\ \frac{W}{m^2}$$

so $q_{max} = (0.94/R'^{1/4})\ q_{max_z} = \underline{1,041,000\ \frac{W}{m^2}}$

from eqn. (9.4)

$$T_{rod_{burnout}} = T_{sat} + \frac{C_{sf}\,Pr\,h_{fg}^{2/3}}{c_p\,\mu^{1/3}}\left(\frac{\sigma}{g(\rho_f-\rho_g)}\right)^{1/6} q_{max}^{1/3}$$

$$= 100 + \frac{0.013(1.72)(2,257,000)^{2/3}}{4219(0.29\times10^{-6}\times957.2)^{1/3}}\left(\frac{0.0589}{9.8(957.6)}\right)^{1/6}(1.04\times10^{6})^{1/3} = \underline{\underline{119.2^\circ C}}$$

9.19 Verify the form of eqn.(9.8) using dimensional anlysis.

$u_g = fn(\delta, \rho_g, \lambda_H)$, 4 variables in m,s, and kg. Thus we look for 4-3, or 1 π-group. Let's write that like a Weber number:

$$\pi = \frac{\rho_g u_g^2 \lambda_H}{\sigma} = \text{const.} \quad \text{or} \quad \underline{\underline{u_g = C\sqrt{\frac{\sigma}{\rho_g \lambda_H}}}}$$

eqn.(9.8) is of this form with $C \equiv \sqrt{2\pi}$

9.20 Compare the value of q_{max} implied by data for pool boiling from a 1 in. diam. sphere in Problem 5.6, with the appropriate prediction.

The measured value of q_{max} can be obtained using the expression derived in the solution of Problem 5.6.

$$q_{max} = \bar{h}\Delta T_{sat} = 0.712 \frac{\text{Btu}}{\text{ft}^2\text{-°F}} \left(\frac{dT}{dt} \frac{\text{°F}}{\text{s}} \right)_{max}$$

From the figure associated with Problem 5.6 we read

$$\left.\frac{dT}{dt}\right|_{max} = 102 \frac{\text{°F}}{\text{s}} \quad \text{so} \quad q_{max} = 72.62 \frac{\text{Btu}}{\text{ft}^2\text{-s}} = \underline{\underline{261{,}446 \frac{\text{Btu}}{\text{ft}^2\text{-hr}}}}$$

9.20 (continued)

Now for this sphere, $R' = \dfrac{0.0254\text{ m}/2}{\sqrt{\dfrac{0.0589}{9.8(958.2-0.6)}}\text{ m}} = \underline{5.07}$

Therefore we use eqn. (9.21):

$$q_{max} = 0.84\, q_{max_Z} = 0.84\, \frac{q_{max_F}}{1.14} = 0.737\, q_{max_F}$$

where q_{max_F} is given in Example 9.5 as $1{,}260{,}000\ \dfrac{W}{m^2}$

So:

$$q_{max} = 0.737(1{,}260{,}000)/3.154 = \underline{\underline{294{,}426\ \frac{\text{Btu}}{\text{ft}^2\text{-hr}}}}$$

In this case the measurement is 11% below the prediction.

9.22 Verify equation (9.53) which gives δ for a condensing film subject to a shear stress, τ_δ.

We first integrate eqn. (8.50) twice and get:

$$\frac{du}{dy} = \frac{\rho_f-\rho_g}{\rho_f}\, gy + C_1 \quad \text{and} \quad u = -\frac{\rho_f-\rho_g}{\rho_f}\frac{y^2}{2} + C_1 y + C_2$$

The first b.c., $u(y=0) = 0$ gives C_2 & the second $\left.\frac{\partial u}{\partial y}\right|_{y=\delta} = \frac{\tau_\delta}{\mu}$ gives $C_1 = \frac{\tau_\delta}{\mu} + \frac{\rho_f-\rho_g}{\rho_f} g\delta$. Thus

$$u = \frac{(\rho_f-\rho_g)g\delta^2}{2\mu}\left[2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2\right] + \frac{\tau_\delta}{\mu} y$$

Then equation (8.53) gives

$$\dot{m} = \int_0^\delta \rho_f u\,dy = \frac{\rho_f-\rho_g}{3\nu}\, g\,\delta^3 + \frac{\tau_\delta}{2\nu}\,\delta$$

and equation (8.54) becomes:

$$\frac{k\Delta T}{h'_{fg}\delta} = \frac{d\dot{m}}{dx} = \left[\frac{\rho_f-\rho_g}{\nu} g\delta^2 + \frac{\tau_\delta}{\nu}\delta\right]\frac{d\delta}{dx}$$

which we integrate, subject to $\delta(x=0) = 0$:

$$\frac{2k\Delta T\nu}{h'_{fg}}\,dx = (\rho_f-\rho_g)g\,\delta^2 d\delta^2 + \tau_\delta\sqrt{\delta^2}\,d\delta^2$$

or

$$\underline{\underline{\frac{4k\Delta T\nu x}{g(\rho_f-\rho_g)h'_{fg}} = \delta^4 + \frac{4}{3}\frac{\tau_\delta}{g(\rho_f-\rho_g)}\delta^3}} \longleftarrow \text{eqn. (9.53)}$$

Now if $\tau_\delta = -\frac{3g(\rho_f-\rho_g)}{4}\delta$ equation (9.53) reduces to:

$$\frac{4k\Delta T\nu x}{g(\rho_f-\rho_g)h'_{fg}} = 0$$

Which means that only $\Delta T = 0$ will work (otherwise the film must grow to a larger value of δ so τ_δ no longer equals $-(3g(\rho_f-\rho_g)/4)\delta$.)

9.23 A 0.07m O.D. pipe is at 40 °C. Saturated steam at 80°C blows across it. Plot $\bar{h}_{cond.}$ for $0 \le Re_D \le 10^6$.

We are given the following expression for flow over a cylinder, (where we evaluate μ_f and k_f at 60°C, and the other properties at 80°C):

$$\bar{h} = 0.64\frac{k_f}{D}\sqrt{\frac{\rho_g u_\infty D}{\mu_f}\left[1+\left(1+1.69\,\frac{g\,h'_{fg}\,\mu_f D}{u_\infty^2 k_f (T_{sat}-T_w)}\right)^{1/2}\right]}\ , \qquad \text{note: } u_\infty \equiv Re_D\frac{\nu_f}{D}$$

where $h'_{fg} = 2,308,000\left(1+\left[0.683+\frac{0.228}{4.2}\right]\frac{2063(40)}{2,308,000}\right) = 2,369,000$

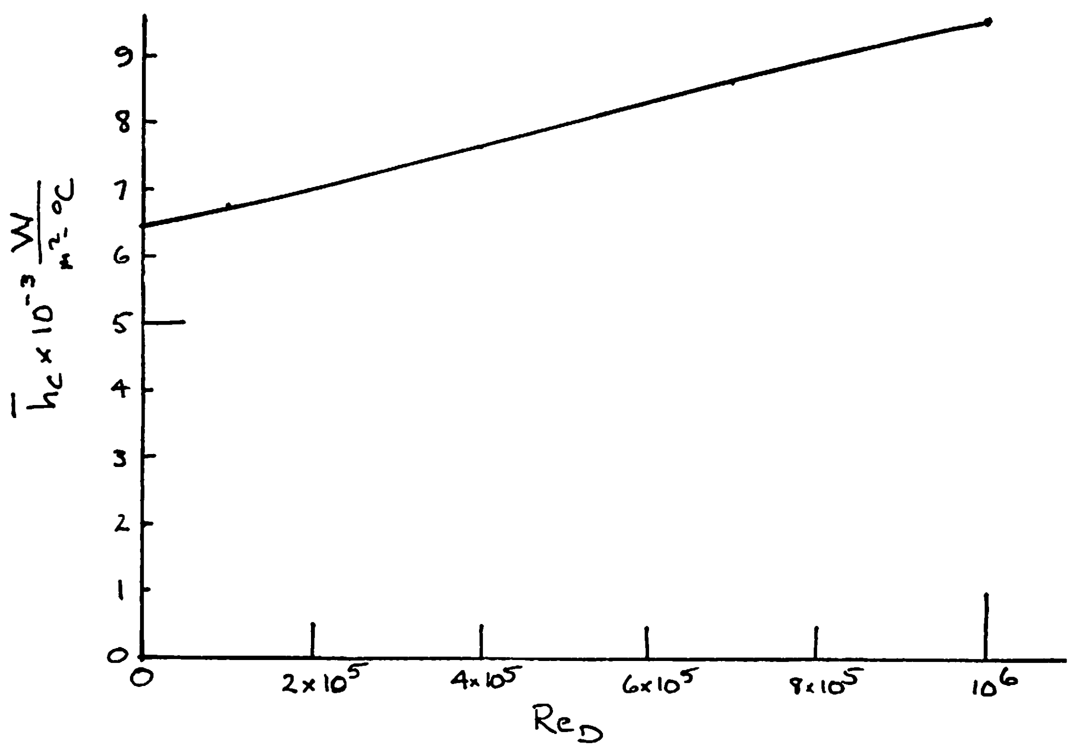
so:

$$\bar{h} = 0.64\frac{0.651}{0.07}\sqrt{1+\left(1+1.69\,\frac{9.8(2,369,000)(0.000355)}{Re_D^2\,\frac{(3.61\times10^{-7})^2}{0.07^3}\,0.651(40)}\right)}\ Re_D^2$$

$$= 5.952\sqrt{1+\left(1+\frac{1.408\times10^{12}}{Re_D^2}\right)^{1/2}}\ Re_D^{1/2}$$

When $Re_D \Rightarrow 0$, $\bar{h} \Rightarrow 6480\ W/m^2\text{-}^\circ C$, or eqn. (9.55) reduces to: $\bar{h} = 0.7297\frac{k_g}{D}\sqrt[4]{\frac{g\,h'_{fg}\,\rho_f D^3}{\nu_f\,k_f\,\Delta T}}$

which is virtually the same as equation (8.67) for static condensation.



9.24 a) Suppose you have pits of roughly 0.002mm diameter in a metallic heater surface. At about what temperature might you expect water to boil on that surface, if the pressure is 20 atm.

b) Measurements have shown that water at atmospheric pressure can be superheated about 200°C above its normal boiling point. Roughly how large an embryonic bubble would be needed to trigger nucleation in water in such a state.

a) $T_{sat} = 213°C = 486.2°K$; $T_{reduced} = \frac{486.2}{647.2} = 0.7512$

$$\sigma = 235.8(1-.7512)^{1.256}(1-.625(1-.7512)) = \underline{34.70 \text{ dyne/cm}} = \underline{34.7\frac{mN}{m}}$$

$$\text{then } \Delta p = \frac{2(\sigma)}{R} = \frac{2(34.7)\frac{mN}{m}}{.000001\,m} = 69{,}400\ \frac{N}{m^2} = \underline{10.064psi}$$

T_{sat} at $(20(14.7) + 10.064)$psia is $\underline{419.8°F}$

$\Delta T = \underline{\underline{4.60°F}}$ or $\underline{\underline{2.56°C}}$ ← much less ΔT is needed to drive boiling at elevated pressures

b) $p_{sat}(300°C) = p_{sat}(572°F) = \underline{1246.6psia}$

$R = \frac{2\sigma}{\Delta p}$ but what is σ? Probably it should be evaluated at 300°C or

$$T_r = \frac{300+273}{647.2} = 0.885$$

$$\sigma = 235.8(1-.885)^{1.256}(1-.625(.115)) = 15.57\frac{dyne}{cm}$$

$$= \underline{0.001067\frac{lbf}{ft}}$$

$$R = \frac{2(0.001067)}{(1246.6-14.7)(144)} = 1.203\text{x}10^{-8}ft$$
$$= 1.443\text{x}10^{-7}ft$$
$$= 3.666\text{x}10^{-6}mm$$
$$= \underline{\underline{36.6\text{Å}}}$$

And that is very small indeed.

9.25 Obtain the dimensionless functional form of the pool boiling q_{max} equation, and the q_{max} equation for flow boiing on external surfaces, using dimensional analysis.

The pool boiling result is worked out fully in the solution of Problem 4.28. It takes the form:

$$\underbrace{\frac{\pi}{24}\frac{q_{max}}{q_{max_Z}}}_{\text{this called the "Kutateladze" No.}} = f(L')$$

All solutions for q_{max} in Table 9.3 take this form.

For external flows we have:

$$\underset{J/m^2\cdot s}{q_{max}} = fn\Big(\underset{kg/m^3}{\rho_g},\ \underset{kg/m^3}{\rho_f},\ \underset{J/kg}{h_{fg}},\ \underset{kg/s^2}{\sigma},\ \underset{m}{L},\ \underset{m/s}{u_\infty}\Big)$$

There are 7 variables in J, m, kg, & s or 7-4 = 3 Π-groups:

$$\Pi_1 = \frac{q_{max}}{\rho_g h_{fg} u_\infty} \qquad \Pi_2 = \frac{\rho_f}{\rho_g} \qquad \Pi_3 \equiv We_L = \frac{\rho_g u_\infty^2 L}{\sigma}$$

Thus:

$$\frac{q_{max}}{\rho_g h_{fg} u_\infty} = fn\left(\frac{\rho_f}{\rho_g},\ We_L\right)$$

We see that the flow boiling burnout expressions in the text take this form unless there is an additional characteristic length in the problem. (See the expression for q_{max} when a jet of diameter, d, impinges on a disc of diameter, D. This introduces an additional group d/D.) (See also the Katto flow boiling burnout correlation form.)

9.26 A (magical?) additive to water increases σ tenfold at 1 atm. By what factor will it improve q_{max} during pool boiling on: (a) infinite flat plates and (b) small horizontal cylinders; and (c) when a jet impinges on a disc.

a) from eqn. (9.11) $\dfrac{q_{max}(\sigma_{high})}{q_{max}(\sigma_{low})} = \left(\dfrac{\sigma_{high}}{\sigma_{low}}\right)^{1/4} = (10)^{1/4} = \underline{\underline{1.78}}$

b) from eqn. (9.20) $\dfrac{q_{max}(\sigma_h)}{q_{max}(\sigma_l)} = \left(\dfrac{\sigma_h}{\sigma_l}\right)^{1/4}\left(\dfrac{R'(\sigma_l)}{R'(\sigma_h)}\right)^{1/4} = (10)^{1/4}\left(\dfrac{\sigma_h}{\sigma_l}\right)^{1/8} = 10^{3/8} = \underline{\underline{2.37}}$

c) from eqn. (9.40) $\dfrac{q_{max}(\sigma_h)}{q_{max}(\sigma_l)} = \left(\dfrac{We_D(\sigma_l)}{We_D(\sigma_h)}\right)^{A} = \left(\dfrac{\sigma_h}{\sigma_l}\right)^{A}$

and from eqn. (9.41) we get, for $\rho_f/\rho_g = 957.2/0.597 = 1603 \equiv r$,
$\underline{A = 0.329}$. Thus:

$$\frac{q_{max}(\sigma_h)}{q_{max}(\sigma_l)} = 10^{A} = 10^{0.329} = \underline{\underline{2.133}}$$

9.27 Steam a 1 atm. is blown at 26 m/s over a 1 cm OD cylinder at 90°C. What is $\bar{h}$? Suggest a physical process within the cylinder that could sustain this temperature in this flow.

$$h'_{fg} = 2{,}257{,}000\left(1+\left[0.683+\frac{0.228}{1.72}\right]\frac{4219(10)}{2{,}257{,}000}\right) = 2{,}291{,}408$$

$$\overline{Nu_D} = 0.64\left\{\frac{26(0.01)}{0.29(10)^{-6}}\left[1+\left(1+1.69\,\frac{9.8(2.291)10^6(0.0002776)(0.01)}{26^2(0.6811)(100-90)}\right)^{1/2}\right]\right\}^{1/2}$$

$= 858.4$, so $\bar{h} = \overline{Nu_D}\,k/D = 858.4(0.6811)/0.01$

$= \underline{\underline{58{,}466\ \mathrm{W/m^2\text{-}°C}}}$

This means that we need a powerfully effective heat removal process in the cylinder -- enough to carry q = 584,660 W/m^2 away from the surface. Nucleate boiling to water at less than 1 atm. could do it, especially at high velocity. The right liquid -- one that is very cold and moves at high velocity -- might be made to do it.

9.28 The water shown in Fig. 9.17 is at one atmosphere and the nichrome heater can be approximated as nickel. What is $T_w - T_{sat}$?

$q_{FC} = \dfrac{\Delta T k}{L}\overline{Nu_L}$ where we scale L = 18 cm from the photo, & use eqn. (6.68) for $\overline{Nu_L}$.

$$q_{FC} = \frac{\Delta T(0.6817)}{0.18}\,0.664\left[\frac{0.18(0.52)}{0.29(10)^{-6}}\right]^{1/2}1.72^{1/3} = \underline{1712\,\Delta T}$$

And from eqn. (9.4):

$$q_B = \frac{\Delta T^3}{C_{sf}^3}\frac{\mu c_p^3}{h_{fg}^2 Pr^3}\sqrt{\frac{g\Delta\rho}{\sigma}} = \frac{\Delta T^3}{0.006^3}\;\frac{957.2(0.29)10^{-6}\,4219^3}{2.257^2\,10^{12}\;1.72^3}\sqrt{\frac{9.8(957.2-0.6)}{0.0589}}$$

$q_B = \underline{1485\,\Delta T^3}$

Then, noting that q is high we use the limiting form of eqn. (9.37), namely $q = \sqrt{q_B q_{FC}}$:

$$480000 = \sqrt{1712(1485)}\,\Delta T^2\ ,\quad \underline{\underline{\Delta T = 17.55\,°C}}$$

This is quite low. It gives $\bar{h}$ = 27,700 W/m^2-°C. The process is very efficient.

9.29 For film boiling on horizontal cylinders, eqn. (9.6a) is modified with Fig. 9.3d to: $\lambda_d = 2\pi\sqrt{3}\left([g(\rho_f-\rho_g)/\sigma] + 2/(\mathrm{diam.})^2\right)^{-1/2}$. If ρ_f is 748 kg/m^3 for saturated acetone, compare this λ_d, and the flat plate value,

$$\left.\begin{aligned}&22\text{ gage} \Rightarrow 0.000643\text{ m}\\ &T_{sat_{acetone}} = 56\ °C\\ &\text{so}\quad \sigma = 0.020\ \mathrm{kg/s^2}\end{aligned}\right\}\ \text{so}\quad \lambda_d = \frac{2\pi\sqrt{3}}{\sqrt{\dfrac{9.8(748)}{0.020}+\dfrac{2}{0.000643^2}}} = \underline{\underline{0.00477\text{m}}} = \underline{\underline{0.477\text{ cm}}}$$

(over)

9.29 (continued)

In Fig. 9.3d we find 4 wavelengths in 2.92 cm so

$\lambda_{d_{expt'l.}} = \underline{\underline{0.73\ cm}}$

deviation from theory $= \dfrac{0.73-0.977}{0.73} = \underline{35\%}$

$\lambda_{d_{plane}} = \dfrac{2\pi\sqrt{3}}{\sqrt{\dfrac{9.8(793)}{0.020}}} = 0.018m = \underline{\underline{1.8cm}}$ which is 147% above the expt'l. value & 277% above the data.

9.30 Water at 47°C flows through a 13 cm diameter thin-walled tube at 8 m/s. Saturated water vapor, at one atmosphere, flows across the tube at 50 m/s. Evaluate T_{tube}, U, and q.

Guess $T_{tube} = 67°C$ for property evaluation. Then $\overline{T}_{H_2O} = 57°C$ & $\overline{T}_{film} = 83.5°C$

in the pipe $\nu = 0.493\times10^{-6}$, $k = 0.6477$, $Pr = 3.14$, $\mu_b = 0.00056$, $\mu_w = 0.000411$

in the film $\nu = 0.346\times10^{-6}$, $k = 0.6743$

(This guess is already a second iteration. we don't present the first one here.)

Now we use eqn. (7.41) $Re_D = \dfrac{8(0.13)}{0.493(10)^{-6}} = 2.11(10)^6$

$$f/8 = 1/8[1.82\log_{10}Re_D - 1.64]^{-2} = 0.001283$$

$$\overline{Nu}_D = \frac{0.001283(2.11\times10^6)3.14}{1.07 + 12.7(0.03582)(3.14^{2/3}-1)}\left(\frac{0.00056}{0.000411}\right)^{0.25} = 5774$$

$$\overline{h}_{H_2O\ in\ tube} = 5774(0.6477)/0.13 = \underline{28,768\ W/m^2\text{-}°C}$$

& for condensation outside, use eqn. (9.56) -- use of eqn. (9.55) would be more accurate; but only by about 1% at this very high Re_D (see below)

$$Re_D = \frac{0.13(50)}{0.346(10)^{-6}} = 1.879(10)^7$$

$$\overline{Nu}_{D_{film}} = 0.64\sqrt{2(1.879)10^7} = 3,923\quad,\quad \overline{h}_{film} = 3923\frac{0.6743}{0.13} = \underline{20,348\ \frac{W}{m^2\text{°C}}}$$

Then:

$$U = \frac{1}{\frac{1}{28,768} + \frac{1}{20,348}} = \underline{\underline{11,918\ W/m^2\text{-}°C}}$$

&

$$q = U\Delta T = 11,918(100-47) = \underline{\underline{631,663\ W/m^2}}$$

&

$$q = h_{film}(100 - T_{tube});\quad T_{tube} = 100 - \frac{631,663}{20,348} = \underline{\underline{69°C}}$$

This temperature is within 2°C. Further iteration is not needed.

9.31 A 1 cm diameter, thin-walled tube carries liquid metal through saturated water at one atmosphere. The throughflow of metal is increased until burnout occurs. At that point the metal temperature is 250°C and h inside the tube is 9600 W/m^2-°C. What is the wall temperature at burnout?

$R' = R/\sqrt{\sigma/g(\rho_f-\rho_g)} = 0.005/\sqrt{0.0589/9.8(958.2-0.6)} = 1.996$, so we use eqn. (9.19) for burnout.

$$q_{max} = 0.9\, q_{max_{\text{flat plate}}} = 0.9\underbrace{(1,260,000)}_{\text{See Example 9.5}} = 1,134,000 \text{ W/m}^2$$

Then: $1,134,000 = 9600(250 - T_{wall})$ so $\underline{\underline{T_w = 132°C}}$

9.32 At about what velocity of liquid metal flow does burnout occur in Problem 9.31 if the metal is mercury?

The Nusselt no. at q_{max} is $\dfrac{9600(0.01)}{6.12} = 15.69$ so reading from Fig. 7.9,

$Pe_D = 2200 = u_\infty D/\alpha = u_\infty(0.01)/5.49\times10^{-6}$ so $\underline{\underline{u_\infty = 1.2 \text{ m/s}}}$

9.33 Explain, in physical terms, why equations (9.23) and (9.25) instead of differing by a factor of two, are almost equal. How do these equations change when H´ is large?

In both cases, burnout occurs when enough vapor is generated to cause Helmholtz instability to occur -- it does not matter whether from one side or two. Thus, when H′ is large, q_{max} is equal to 0.9 q_{max_Z} -- the <u>same</u> value in <u>both</u> cases.

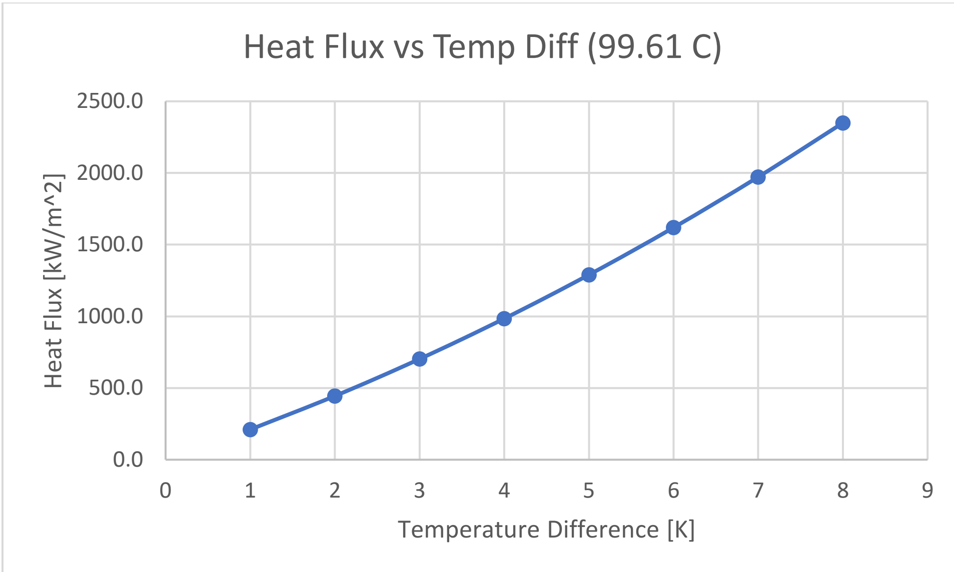
Indeed, if we have the same vapor volume at q_{max} in both cases, and if both q_{max} values are the same, then H′ must be twice as large in the insulated case. That is why the constant, 1.4, in eqn. (9.24) is exactly $2^{1/4}$ times the constant, 1.18, in eqn. (9.23). i.e.:

$$\frac{1.18}{H'^{1/4}} \equiv \frac{1.4}{(2H')^{1/4}}$$

Problem 9.37

P_vap (Pa)	T_sat (K)	T_sat (C)
1000	280.12	6.97
10000	318.96	45.81
100000	372.76	99.61

t (C)	Delta T (K)	q (kW/m^2)	h (kW/m^2K)
6.97	1	25.1	25.1
	2	52.9	26.5
	3	83.7	27.9
	4	117.2	29.3
	5	153.6	30.7
	6	192.9	32.1
	7	234.9	33.6
	8	279.8	35.0
45.81	1	113.0	113.0
	2	238.8	119.4
	3	377.3	125.8
	4	528.7	132.2
	5	692.9	138.6
	6	869.8	145.0
	7	1059.6	151.4
	8	1262.1	157.8
99.61	1	210.3	210.3
	2	444.5	222.2
	3	702.5	234.2
	4	984.2	246.1
	5	1289.8	258.0
	6	1619.2	269.9
	7	1972.4	281.8
	8	2349.4	293.7


Heat Flux vs Temp Diff (99.61 C)
2500.0
2000.0
1500.0
1000.0
500.0
0.0
Heat Flux [kW/m^2]
0
1
2
3
4
5
6
7
8
9
Temperature Difference [K]

Problem 9.38

Surface at 100 C

Delta T	P_0	Delta P	rho_0	factor	mdot (kg/m^2s)	q (MW/m^2)
0	101420.0	0	0.59817	0.000345552	0.0	0
1	105090.0	3670	0.61841	0.000345231	1.3	3
2	108870.0	7450	0.6392	0.000344907	2.6	6
3	112770.0	11350	0.66056	0.000344564	3.9	9
4	116780.0	15360	0.6825	0.000344241	5.3	12
5	120900.0	19480	0.70503	0.000343941	6.7	15
6	125150.0	23730	0.72816	0.000343615	8.2	18
7	129520.0	28100	0.7519	0.000343299	9.6	22
8	134010.0	32590	0.77627	0.000343	11.2	25
9	138630.0	37210	0.80127	0.000342697	12.8	29
10	143380.0	41960	0.82693	0.000342402	14.4	32

T_0	373.15	
p_0	101420	
R	461.404	
coef	1.6678	
sigma	0.31	
factor1	3.0329914	
hfg	2246000	treat as constant

Surface at 40 C

Delta T	P_0	Delta P	rho_0	factor	mdot (kg/m^2s)	q (MW/m^2)
0	7384.9	0	0.051242	0.000372416	0.0	0.0
1	7787.8	402.9	0.053871	0.00037186	0.1	0.3
2	8209.6	824.7	0.056614	0.000371306	0.3	0.7
3	8650.8	1265.9	0.059474	0.000370756	0.5	1.1
4	9112.4	1727.5	0.062457	0.000370213	0.6	1.5
5	9595	2210.1	0.065565	0.00036967	0.8	1.9
6	10099	2714.1	0.068803	0.000369146	1.0	2.3
7	10627	3242.1	0.072176	0.000368579	1.2	2.8
8	11177	3792.1	0.075688	0.000368067	1.4	3.2
9	11752	4367.1	0.079343	0.000367534	1.6	3.7
10	12353	4968.1	0.083147	0.000366984	1.8	4.2

40 C

T_0	313.15	
p_0	7384.9	
R	461.403996	
coef	1.6678	
sigma	0.31	
factor1	3.0329914	
hfg	2306000	treat as constant

10.1 What will be the apparent values of $\epsilon_{\lambda=0.2\mu m}$ and $\epsilon_{\lambda=0.65\mu m}$ for the sun as viewed from the earth's surface.

From Fig. 10.2 we scale

$$\epsilon_{\lambda=0.2} = \left.\frac{e}{e_b}\right|_{\lambda=0.2} = \underline{\underline{0}}$$

$$\epsilon_{\lambda=0.65} = \left.\frac{e}{e_b}\right|_{\lambda=0.65} = \underline{\underline{0.77}}$$

These are low. They show energy has been removed by the earth's atmosphere. (The sun itself is virtually black.)

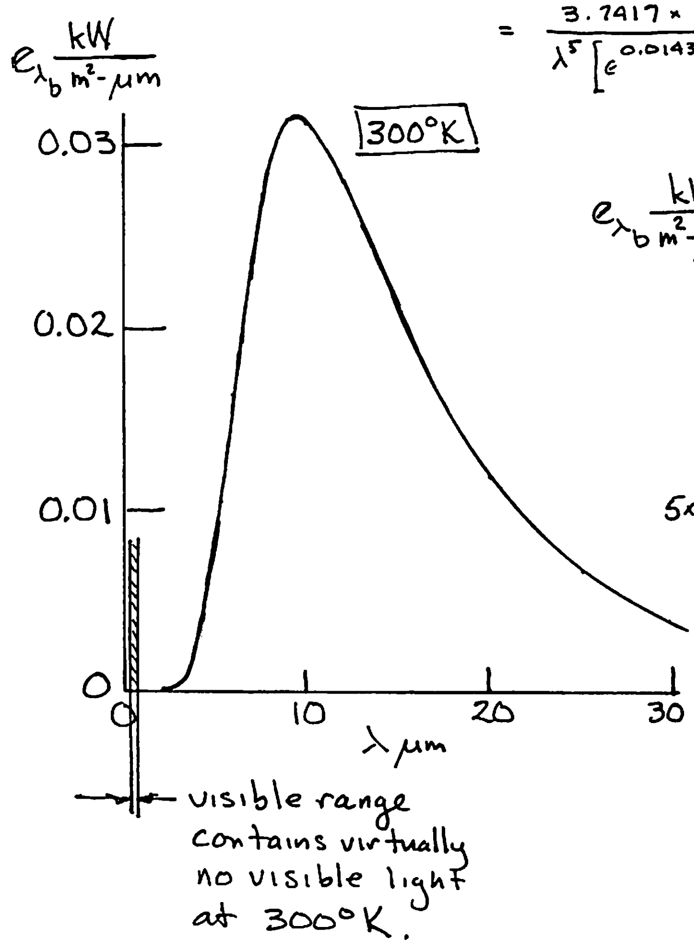
10.2 Plot $e_{\lambda,b}$ vs. T for T = 300°K and 10,000°K. What portion of the total energy is radiated in the visible range.

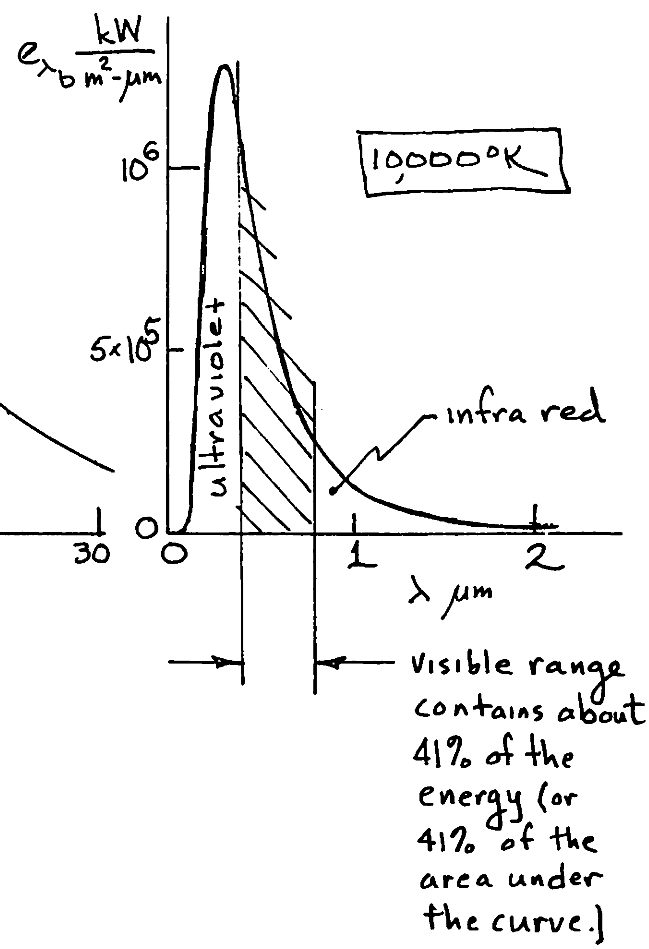
equation (1.30):

$$e_{\lambda_b} = \frac{2\pi(6.6256\times10^{-34})(2.998\times10^{8})^2 \ \frac{J\,m^2}{s}}{\lambda^5\left[e^{6.6256(2.998)10^{-26}/1.3805(10)^{-23}T\lambda}-1\right] m^5}$$

$$= \frac{3.7417\times10^{-25}}{\lambda^5\left[e^{0.01439/T\lambda}-1\right]}\ \frac{kW}{m^2\text{-}\mu m}$$

where λ is in μm and T is in °K





10.3 A 0.0006 m diam. wire ($\epsilon = 0.85$) at 950°C is on the center of a 0.07 m diam. thin metal tube ($\epsilon = 0.09$). The tube is horizontal in air at 25°C. Find T_{tube}.

First guess $\bar{h}_{conv.} = 6$ W/m^2-°C. Then from eqn. (10.14)

$$Q_{1-2} = \mathcal{F}_{1-2}\sigma A_1 (T_1^4 - T_2^4) = \frac{\sigma \pi D_1 (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{D_1}{D_2}\left(\frac{1}{\epsilon_2} + 1\right)} = \frac{5.67(10)^{-8}\pi(0.0006)(1223^4 - T_2^4)}{\frac{1}{0.85} + \frac{0.0006}{0.07}\left(\frac{1}{0.09} - 1\right)}$$

$$Q = 8.46\times10^{-11}(2.237[10]^{12} - T_2^4) = \bar{h}A\Delta T = 6\pi(0.07)(T_2 - 298)$$

Trial and error gives: $T_2 = 437°K = 167°C$

Then, evaluating all properties but β at $\frac{167+25}{2} = \underline{96°C} = 369°K$

$$Ra_D = \frac{g\beta\Delta T D^3}{\nu\alpha} = \frac{9.8\,\frac{1}{298}(167-25)\,0.07^3}{2.266\,(3.215)\,10^{-10}} = 2.20\times10^6$$

so:

$$\overline{Nu_D} = 0.36 + \frac{0.518\,(2.20\times10^6)^{1/4}}{\left[1+\left\{\frac{0.559}{0.707}\right\}^{9/16}\right]^{4/9}} = 15.1$$

and

$$h = 15.1\,\frac{0.03097}{0.07} = 6.68\ \frac{W}{m^2\,°C}$$

using 6.68 instead of 6 in the heat balance equation above, we get $T_2 = 422°K = 149°C$. That gives $h = 6.46\ \frac{W}{m^2\,°C}$. Then T_2 will drop in proportion:

$$Q = 6.68(149-25) \simeq 6.46(T_2 - 25)$$

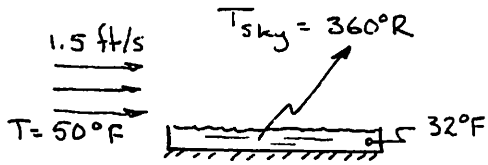
$$T_2 = T_{shield} = \underline{\underline{153\ °C}}$$

$h_{rad_{wire}} = \mathcal{F}_{w-s}\,\sigma\,\frac{T_w^4 - T_s^4}{\Delta T} = 124$, $h_{conv._{wire}}$ should be less than this.

$h_{rad_s} = \epsilon_w\,\sigma\,\frac{T_s^4 - T_\infty^4}{\Delta T} \simeq 1.0$, this is about $(1/6)^{th}$ $h_{conv.}$

Thus the present assumptions are not bad, but a refined calculation would account for convection inside and radiation outside.

10.4 A 1 ft^2 shallow pan with adiabatic sides is filled to the brim with water at 32°F. It radiates to the night sky whose temperature is 360°R while a 50°F breeze blows over it at 1.5 ft/s. Will the water freeze or warm up?



$\epsilon_{H_2O} = 0.96$

• Find q_{conv} using $\overline{Nu_L} = 0.664\, Pr^{1/3}\, Re_L^{1/2}$ (Evaluate properties at $\frac{50+32}{2} = 41°F$ $= 5°C$.)

so: $$\bar{h} = \frac{0.02493 \frac{W}{m\text{-}°C}}{0.3048\,m}\, 0.664(0.717)^{1/3}\left(\frac{1.5(0.3048)(0.3048)}{1.371\times10^{-5}}\right)^{1/2} = 4.802 \frac{W}{m^2\text{-}°C}$$

$$q_{conv} = 4.802\times[(50-32)/1.8] = \underline{48.02\ W/m^2}$$

• Find $$q_{rad} = \frac{1}{\frac{1}{\epsilon_1} + \underbrace{\frac{A_1}{A_2}}_{=0}\underbrace{\left(\frac{1}{\epsilon_2}-1\right)}_{=0}}\,\sigma\left(T_{water}^4 - T_{sky}^4\right).$$

$$= \epsilon_1\sigma\left(T_w^4 - T_s^4\right) = 0.96(5.67\times10^{-8})(273^4 - 200^4) = \underline{\underline{215\ \frac{W}{m^2}}}$$

Thus about four times as much heat radiates away as flows into the water by convection. It is, in fact, possible to freeze water in the desert in this way, on warmish nights.

10.5 Find the temperature, T_T, of a thermometer in 10°C air and 27°C walls if it and the room are black.

Let's treat the thermometer bulb as a vertical wall, 0.01m in height; and evaluate properties at 291.5°K & β at 283°K.

Then, using the simple Squire-Eckert equation (8.27)

$$\bar{h} = \frac{0.0255}{0.01}\, 0.678\left[\frac{0.713}{0.952+0.713}\right]^{1/4}\left[\frac{9.8(1/283)(T_T-10)(0.01)^3}{1.490(2.092)10^{-10}}\right]^{1/4}$$

$$\underline{q = 4.54(T_T-10)^{5/4}} = \underline{4.54[(T°R)-283]}$$

And: $$q_{rad} = -\sigma(T_T^4 - 300^4) = \underline{-5.67\times10^{-8}(T_T^4 - 8.1\times10^9)}$$

Setting these equations equal to one another and solving them simultaneously for T_T, we get

$$T_T = 292.6°K = \underline{\underline{19.6°C}}$$

Notice that we should have evaluated properties at (19.6+10)/2 = 14.8°C = 287.8°K. That's only 3°C off the mark so we let the calculation stand.

10.6 What will T_T be in Problem 11.5 if $\varepsilon_T = 0.94$ and $\varepsilon_w = 0.92$ to 0.58.

$$q_{rad} \text{ is now } = \frac{-1}{\frac{1}{\varepsilon_T} + \underbrace{\frac{AT}{Aw}\frac{1}{\varepsilon_w} - 1}_{\text{neglect}}} \sigma(T_T^4 - 300^4) = \underline{5.33\text{x}10^{-8}(T_T^4 - 300^4)}$$

The trial and error calculation now gives: $\underline{\underline{T_T = 19.4°C}}$

The black body assumption was very good in this case.

10.7 Two thin aluminum sheets, one polished (ε=0.05) and the other painted black (ε=0.9) are placed horizontally outdoors in 10°C air. $\bar{h} \simeq 5W/m^2$-°C on both the top and bottom. The top is irradiated with 750W/m^2 and it re-radiates to a sky which is at 170°K. The earth beneath each plate is black at 10°C. Find the temperature of each plate. (Assume the sky to be black).

In either case we can write:

$$\underbrace{q_{sun}}_{=750\alpha_{pl.} \atop =750\varepsilon_{pl.}} + \varepsilon_{plate}\ \sigma(\underbrace{T^4_{sky}}_{170^4} - T^4_{pl}) + \varepsilon_{pl}\ \sigma(\underbrace{T^4_{earth}}_{283^4} - T^4_{pl}$$

$$= 2\bar{h}(T_{pl} - \underbrace{T_{air}}_{283})$$

Put in the known numerical values for each of the two plates and get the following two equations.

$$\underbrace{37.5 + 20.55 + 2830}_{2888} = 5.67\text{x}10^{-9}\ T^4_{pl_1} + 10T_{pl_1}$$

and

$$\underbrace{675 + 270 + 2830}_{3875} = 1.0206\text{x}10^{-7}\ T^4_{pl_2} + 10T_{pl_2}$$

Trial and error using a pocket calculator gives

$$T_{pl_1} = 285°K = \underline{\underline{12°C}}$$

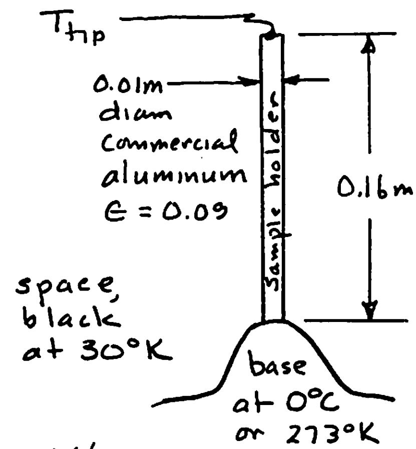
$$T_{pl_2} = 302.3°K = \underline{\underline{29.3°C}}$$

Notice that the highly reflecting surface is, as we would anticipate, almost unaffected by solar radiation. For this reason, unpainted aluminum rooting is sometimes used in sunny climates.

10.8 Find the tip temperature of the sample holder shown. Assume

$$F_{\text{sample holder-sky}} \cong 1$$

and take the holder to be a finite fin.



$$h_r = \frac{\sigma \underbrace{\mathcal{F}_{h-s}}_{=\epsilon} (T_h^4 - T_s^4)}{T_h - T_s} \quad \text{where } T_h \simeq 0°C$$

$$= \frac{5.67\cdot 10^{-8}(0.09)(273^4 - 30^4)}{(273-30)} = \underline{0.1166 \ \frac{W}{m^2 °C}}$$

Now for the fin: $mL = \sqrt{\frac{0.1166(\pi \times 0.01)}{204(\pi \times 0.005^2)}}\,(0.16) = 0.0765$

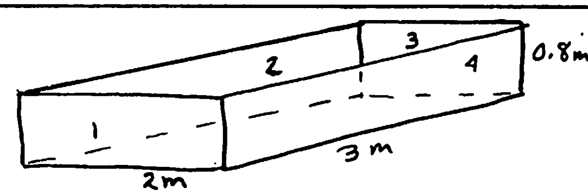
Then $\frac{T_{tip} - T_\infty}{T_{root} - T_\infty} = \frac{1}{\cosh mL} = \frac{2}{e^{0.0765} + e^{-0.0765}} = 0.9971$

so $T_{tip} = 0.9971(273-30) + 30 = 272.3\ °K$

or $\underline{\underline{T_{tip} = -0.7\,°C}}$

(Note that with so little temperature drop, it is justifiable to base h_r on a constant fin temperature.)

10.9 Find the percentages of leaving the bottom of the box that reach sides 1, 2, 3, 4, and the top.



These percentages are equal to $F_{\text{bottom}-1}$, F_{b-2}, F_{b-3}, F_{b-4}, and $F_{b-\text{top}}$.

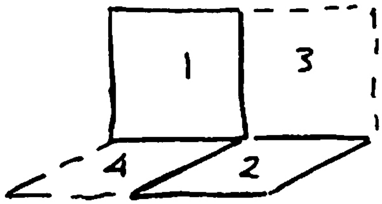
From Fig. 10.9, $a/c = \frac{2}{0.8} = 2.5$, $b/c = \frac{3}{0.8} = 3.75$, $\underline{\underline{F_{b-t} = 0.53}}$

From Fig. 10.9, $h/\ell = \frac{0.8}{3} = 0.267$, $w/\ell = \frac{2}{3} = 0.667$, $\underline{\underline{F_{b-2} = F_{b-4} = 0.14}}$

From Fig. 10.9, $h/\ell = \frac{0.8}{2} = 0.4$, $w/\ell = \frac{3}{2} = 1.5$, $\underline{\underline{F_{b-1} = F_{b-3} = 0.095}}$

Check the result $\sum_{n=0}^{5} F_{b-n} = 0.53 + 2(.14) + 2(0.095) = 1.00$

PROBLEM 10.10 Consider Fig. (10.11). Find $F_{1-(2+4)}$ and $F_{(2+4)-1}$.



SOLUTION First note that $F_{1\text{-}(24)} = F_{1\text{-}4}$. We use shape factor algebra to break this down.

$$\begin{aligned} F_{4\text{-}(123)} &= F_{4\text{-}1} + F_{4\text{-}2} + F_{4\text{-}3} \\ &= 2F_{4\text{-}1} + F_{4\text{-}2} \end{aligned}$$

$$\frac{A_{(123)}}{A_4} F_{(123)\text{-}4} = 2\frac{A_1}{A_4} F_{1\text{-}4} + \frac{A_2}{A_4} F_{2\text{-}4}$$

Divide through by $2A_1/A_4$ and rearrange:

$$F_{1\text{-}4} = \frac{A_{(123)}}{2A_1} F_{(123)\text{-}4} - \frac{A_2}{2A_1} F_{2\text{-}4}$$

$F_{2\text{-}4}$ is the subject of Problem 10.11, and the answer is $F_{2\text{-}4} = 0.255$. The other shape factor may be found using Fig. 10.9, letting surface 4 be the h surface:

$$F_{(123)\text{-}4} = 0.23 \quad \text{with} \begin{cases} h/l = 0.5/1.2 = 0.42 \\ w/l = 0.6/1.2 = 0.5 \end{cases}$$

Hence

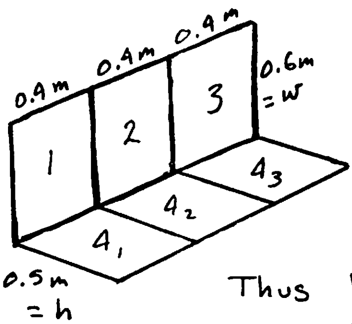
$$F_{1\text{-}4} = \frac{(1.2)(0.6)}{2(0.4)(0.6)}(0.23) - \frac{1}{2}(0.255) = 0.218$$

Then,

$$F_{(24)\text{-}1} = F_{(4)\text{-}1} = \frac{A_1}{A_4} F_{1\text{-}4} = \frac{(0.4)(0.6)}{(1.2)(0.5)}(0.218) = 0.087$$

10.11 Find F_{2-4} for the situation shown.

To solve this problem, we make use of Fig. 10.10. But notice that the upright and horizontal surfaces are inverted here.



First find F_{2-4_1}: $2F_{12-4_1 4_2} = \underbrace{F_{1-4_1} + F_{2-4_2}}_{=2F_{2-4_2}} + \underbrace{F_{2-4_1} + F_{1-4_2}}_{=2F_{2-4_1}}$

but from Fig. 10.10 (2), $\left.\begin{aligned} h/\ell &= \frac{0.5}{0.4} = 1.25 \\ w/\ell &= \frac{0.6}{0.4} = 1.5 \end{aligned}\right\} F_{2-4_2} = 0.165$

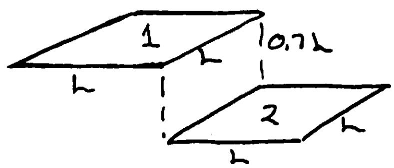
and: $\left.h/\ell = \frac{0.5}{0.8} = 0.625, \quad w/\ell = \frac{0.6}{0.8} = 0.75 \right\} F_{12-4_1 4_2} = 0.21$

Thus $F_{2-4_1} = 0.21 - 0.165 = \underline{0.045}$

Then $F_{2-4} = F_{2-4_1 4_2 4_3} = F_{2-4_1} + F_{2-4} + F_{2-4_3} = 0.045 + 0.165 + 0.045$

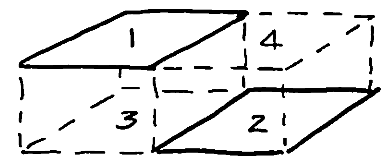
$= \underline{\underline{0.255}}$

10.12 Find F_{1-2} for the configuration shown.



Find F_{1-2}

Set up fictitious planes thus:



Therefore: $\quad A_{14} F_{14-32} = \underbrace{A_1}_{A_4} \left(\underbrace{F_{1-3}}_{F_{4-2}} + \underbrace{F_{1-2}}_{F_{4-3}} + F_{4-3} + F_{4-2} \right)$

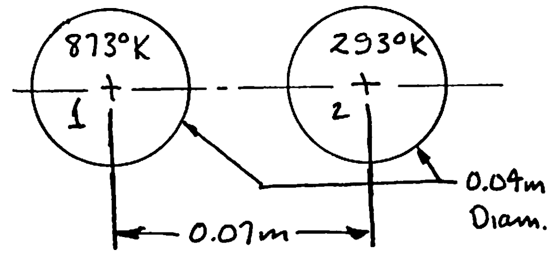
So: $\quad 2F_{1-3} + 2F_{1-2} = 2F_{14-32}$

From Fig. 10.9 we then read ①: $\left.\begin{array}{l} \frac{a}{c} = \frac{1}{0.7} = 1.43 \\ \frac{b}{c} = \frac{1}{0.7} = 1.43 \end{array}\right\}$ so $F_{1-3} = 0.3$

$\left.\begin{array}{l} \frac{a}{c} = \frac{2}{0.7} = 2.86 \\ \frac{b}{c} = \frac{1}{0.7} = 1.43 \end{array}\right\}$ so $F_{14-32} = 0.38$

It follows that: $F_{1-2} = \dfrac{2F_{14-32} - 2F_{1-3}}{2} = \underline{\underline{0.08}}$

10.13 Compute the net heat transfer between the black cylinders shown.



First find F_{1-2}. From Table 10.2 we read:

we read:

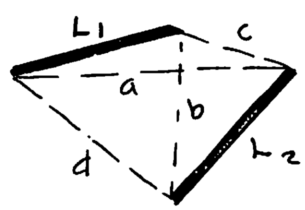
$$F_{1-2} = \frac{1}{\pi}\left[\sqrt{\left(1+\frac{0.03}{0.04}\right)^2 - 1} + \sin^{-1}\frac{1}{1+\frac{0.03}{0.04}} - 1 - \frac{0.03}{0.04}\right]$$

$$\underline{F_{1-2} = 0.0937}$$

so: $$Q = (\pi D)(0.0937)\sigma\left(T_1^4 - T_2^4\right) = \pi(0.04)(0.0937)5.67\left(8.73^4 - 2.93^4\right)$$

$$= \underline{\underline{383 \ \frac{W}{m \text{ of length}}}}$$

10.14 Develop the string method for evaluating F_{1-2} between two-dimensional surfaces.



Noting that areas are proportional to the distances shown we write view factors for the two triangles, L_1-a-c, and L^{1-b-d}, using case 4 in Table 10.2.

Thus: $$F_{1-c} = \frac{L_1 + c - a}{2L_1} \quad \text{and} \quad F_{1-d} = \frac{L_1 + d - b}{2L_1}$$

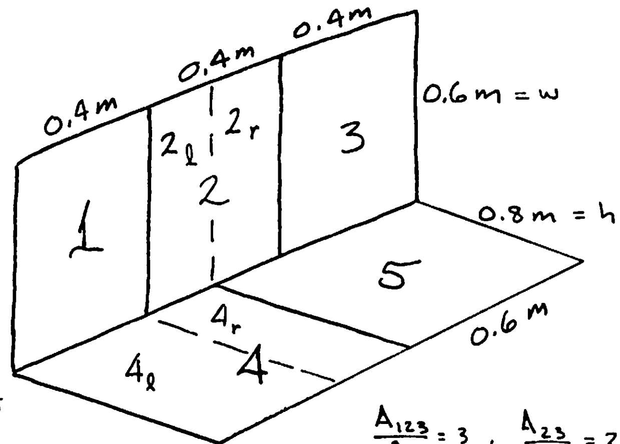
Then, since $F_{1-2} = 1 - F_{1-c} - F_{1-d}$,

$$F_{1-2} = \frac{\cancel{2L_1}}{2L_1} - \frac{\cancel{L_1} + c - a + \cancel{L_1} + d - b}{2L_1} = \underline{\underline{\frac{(a+b) - (c+d)}{2L_1}}}$$

So it would be possible to obtain F_{1-2} by comparing the difference between the lengths of the crossed strings, (a + b), and the edge strings, (c + d), with $2L_1$. Hottel and Sarofim [10.15] show that this will also work if L_1 and L_2 are curved in complicated ways.

(Note: If the student is not clever in attacking this problem, he can easily embark on some pretty complicated, albeit correct, strategies.)

10.15 Find F_{1-5} for the configuration shown:



$$\frac{A_{123}}{A_1} = 3, \quad \frac{A_{23}}{A_1} = 2$$

$$\frac{A_{12\ell}}{A_1} = \frac{3}{2}, \quad \frac{A_{2\ell}}{A_1} = \frac{1}{2}$$

We shall write:

$$F_{1-5} = F_{1-4_r5} - F_{1-4_r}$$

Then:

$$A_1 F_{1-4_r5} = A_{123} F_{123-45} - A_{23} F_{23-4_r5} - \underbrace{A_{23} F_{23-4\ell}}_{= \underbrace{A_{4\ell}}_{=A_1} \underbrace{F_{4\ell-23}}_{=F_{1-4_r5}}} - A_1 F_{1-4\ell}$$

so $\underline{F_{1-4_r5} = \frac{3}{2} F_{123-45} - F_{23-4_r5} - \frac{1}{2} F_{1-4_\ell}}$

Likewise

$$A_1 F_{1-4_r} = A_{12_\ell} F_{12_\ell-4} - A_1 F_{1-4_\ell} - \underbrace{A_{2\ell} F_{2\ell-4\ell}}_{A_{4\ell} F_{1-4_r}} - A_{2\ell} F_{2_\ell-4_r}$$

so $\underline{F_{1-4_r} = \frac{3}{4} F_{12_\ell-4} - \frac{1}{2} F_{1-4\ell} - \frac{1}{4} F_{2_\ell-4_r}}$

Finally:

$$F_{1-5} = F_{1-4_r5} - F_{1-4_r} = \underline{\frac{3}{2} F_{123-45} - F_{23-4_r5} - \frac{3}{4} F_{12_\ell-4} + \frac{1}{4} F_{2\ell-4_r}}$$

now from Fig. 10.9 ②

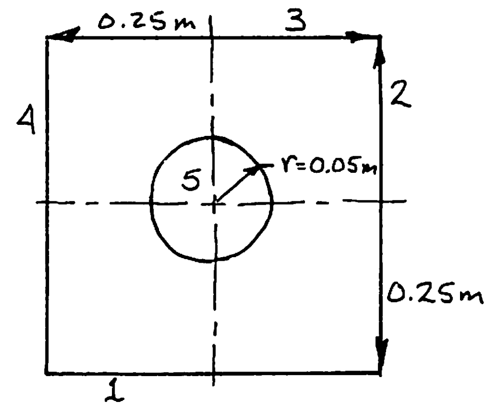
- at $h/\ell = \frac{0.8}{1.2} = 0.67$, $w/\ell = \frac{0.6}{1.2} = 0.5$, $F_{123-45} = 0.265$
- " " $= \frac{0.8}{0.8} = 1.0$, " $= \frac{0.6}{0.8} = 0.75$, $F_{23-4_r5} = 0.23$
- " " $= \frac{0.8}{0.6} = 1.333$, " $= \frac{0.6}{0.6} = 1.0$, $F_{12_\ell-4} = 0.218$
- " " $= \frac{0.8}{0.2} = 4$, " $= \frac{0.6}{0.2} = 3$, $F_{2_\ell-4_r} = 0.125$

Thus:

$$F_{1-5} = \frac{3}{2}(0.265) - 0.23 - \frac{3}{4}(0.218) + \frac{1}{4}(0.125) = \underline{\underline{0.035}}$$

This result could easily suffer 10 or 20 % error from accumulative inaccuracy of graph reading. Notice, too, that without recognizing some tricks in manipulating F's, one could have a hard time solving this one.

10.16 Find $F_{1-2,3,4}$ for the configuration shown.



$$F_{1-2,3,4} = F_{1-2,3,4}\Big|_{\text{with no blockage by the cyl., 5}} - F_{1-5}$$

However: (equation (10.12))

$$F_{1-2,3,4} = F_{1-2} + F_{1-3} + F_{1-4} = 1$$

To get F_{1-5} we use eqn. (5) in Table 10.2, with

$$r = 0.05\,\text{m},\quad b = 0.125\,\text{m},\quad c = 0.125\,\text{m}$$

and $a = -0.125$ (notice the important inclusion of a minus sign in front of a.)

SO:

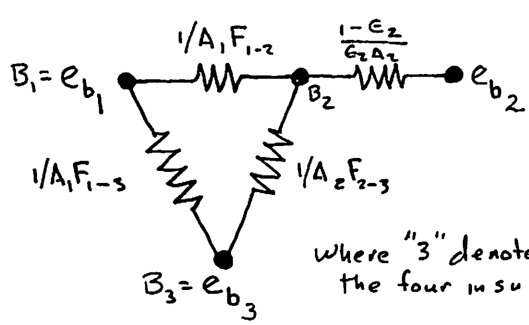
$$F_{1-5} = \frac{0.05}{0.125-(-0.125)}\left[\tan^{-1}\frac{0.125}{0.125} - \tan^{-1}\frac{(-0.125)}{0.125}\right] = 0.3142$$

Then:

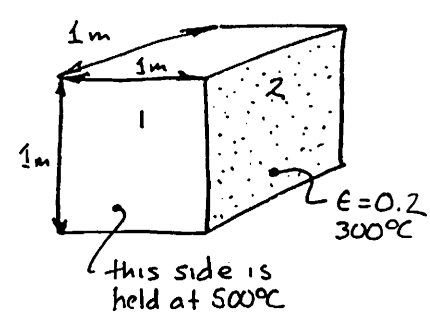
$$F_{1-2,3,4} = 1 - F_{1-5} = \underline{\underline{0.6858}}$$

(Some students will use Table 11.2 to calculate the unblocked values of $F_{1-2} = F_{1-4} = 0.293$ and $F_{1-3} = 0.4142$ and only then discover that $0.293 + 0.293 + 0.414 = 1$.)

10.17 All sides of the box, except 1 and 2 are insulated. Find Q_{1-2}.



where "3" denotes the four insulated sides.



From eqn. (11.31):

$$Q_{1-2} = \frac{e_{b_1} - e_{b_2}}{\dfrac{1-\epsilon_1}{\epsilon_1 A_1} + \dfrac{1}{\dfrac{1}{\dfrac{1}{A_1F_{1-3}} + \dfrac{1}{A_2F_{2-3}}} + \dfrac{1}{\dfrac{1}{A_1F_{1-2}}}} + \dfrac{1-\epsilon_2}{\epsilon_2 A_2}}$$

but $\epsilon_1 = 1$ so $\dfrac{1-\epsilon_1}{\epsilon_1 A_1} = 0$ and from Fig. 10.9, case (2) we read

10.17 (continued)

for $a/c = b/c = 1$, $F_{1-2} = 0.2$. It follows that $F_{1-3} = 0.8$ and, by symmetry, $F_{2-3} = 0.8$. Then, since $A_1 = A_2 = 1$ and $A_3 = 4$, the equation gives:

$$Q_{1-2} = \frac{5.67(10)^{-8}\left(773^4 - 573^4\right)}{\dfrac{1}{\dfrac{1}{\dfrac{1}{1(0.8)} + \dfrac{1}{1(0.8)}} + \dfrac{1}{\dfrac{1}{1(0.2)}}} + \dfrac{1-0.2}{0.2(1)}} = \underline{\underline{2494\text{ W}}}$$

Note: One could, alternatively, write the three nodal equations:

Node 1 : $e_{b_1} A_1 = \dfrac{5.67\times10^{-8}(773^4) - B_2}{\dfrac{1}{1(0.2)}} + \dfrac{5.67\times10^{-8}(773^4) - B_3}{\dfrac{1}{1(0.8)}}$

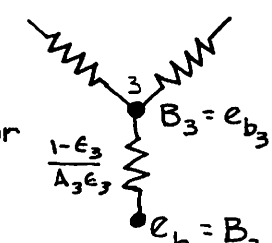
Node 2 : $\dfrac{5.67\times10^{-8}(573^4) - B_2}{\dfrac{1-0.2}{1(0.2)}} = \dfrac{B_2 - 5.67\times10^{-8}(773^4)}{\dfrac{1}{1(0.2)}} + \dfrac{B_2 - B_3}{\dfrac{1}{1(0.8)}}$

Node 3 : B_2 and B_3 are already specified. This equation is redundant.

solve for $B_2 = 16{,}088$ & $B_3 = 18{,}166$. Then $Q = \dfrac{B_1 - B_2}{\dfrac{1}{1(0.2)}} + \dfrac{B_3 - B_2}{\dfrac{1}{1(0.8)}} = \underline{\underline{2494\text{ W}}}$

10.18 Find Q_{1-2} and $T_{ins.-walls}$ for Problem 10.17 is $\epsilon_{ins.\ wall}$ is 0.6, and if it is 1.0.

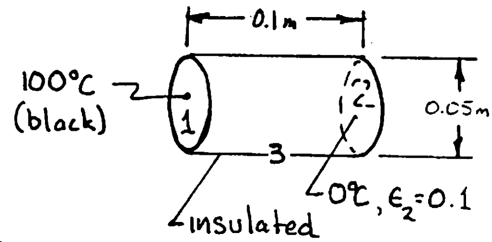
Note that, since node 3 is at an insulated wall, there is no heat flow across the thermal resistance, $(1-\epsilon_3)/A_3\epsilon_3$. Thus $e_{b,3} = B_3$ and ϵ_3 is irrelevant to the determination of either Q_{1-2} or $T_{ins.-wall}$. With reference to the solution of Problem 11.17, we can immediately write, for $\epsilon_{ins.-wall}$ equal to either 1.0 or 0.6:



$$Q_{1-2} = \underline{\underline{2494\text{ W}}}$$

$$T_{ins.\,wall} = \sqrt[4]{\frac{e_{b_3}}{\sigma}} = \sqrt[4]{\frac{B_3}{\sigma}} = \sqrt[4]{\frac{18{,}166}{5.67(10)^{-8}}} = \underline{\underline{752\text{ K} = 479^\circ\text{C}}}$$

10.19 Find F_{1-3} within the insulated cylinder shown.

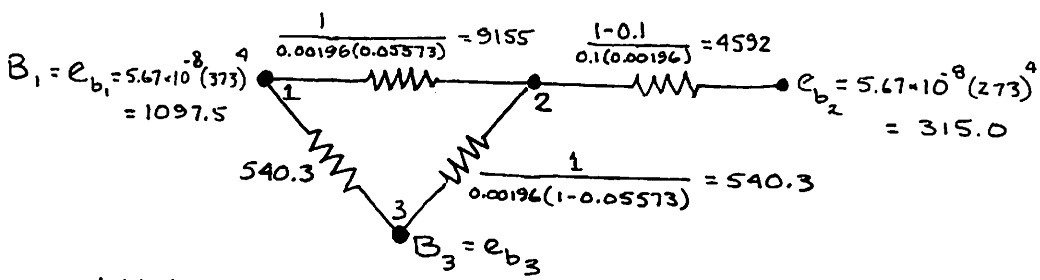


First find F_{1-2}:

$R_1 = \frac{2.5}{10} = R_2 = 0.25$, $X = 1 + \frac{1+0.25^2}{0.25^2} = 18$

so $F_{1-2} = \frac{1}{2}\left[18 - \sqrt{18^2-4}\right] = \underline{0.05573}$

Furthermore: $A_1 = A_2 = \frac{\pi}{4}(0.05)^2 = 0.00196\,m^2$, $A_3 = \pi(0.05)(0.1) = 0.0157\,m^2$



nodal balances:

$$1: \quad 0 = \frac{1097.5 - B_2}{9155} + \frac{1097.5 - B_3}{540.3}$$

$$2: \quad \frac{315 - B_2}{4592} = \frac{B_2 - 1097.5}{9155} + \frac{B_2 - B_3}{540.3}$$

$$3: \quad 0 = \frac{B_3 - 1097.5}{540.3} + \frac{B_3 - B_2}{540.3}$$

so we use the second two of these three equations:

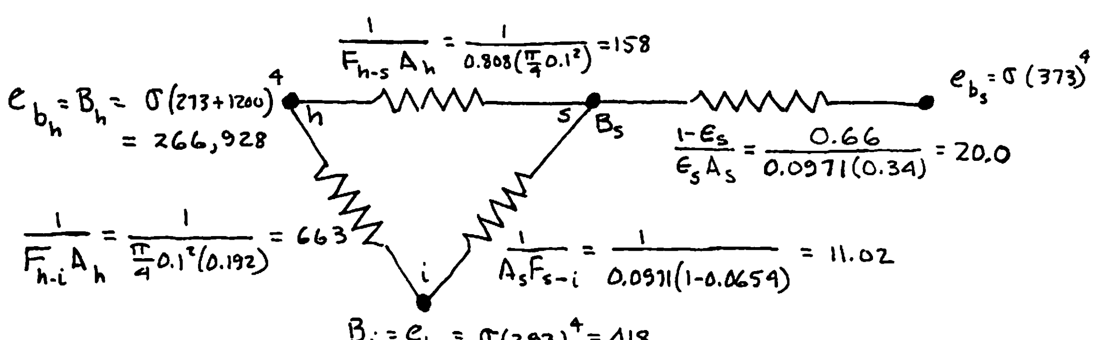
$B_3 = 548.8 + B_2/2$ & $37.06 - 0.1171B_2 - 0.05902B_2 + 64.77 - B_2 + 548.8 + B_2/2 = 0$

so. $B_2 = 961.45$, $B_3 = e_{b_3} = 1029.5$

and $T_{cylinder} = \sqrt[4]{e_{b_3}/\sigma} = \underline{\underline{367.1°K = 94°C}}$

and: $Q_{net} = \frac{B_1 - B_2}{9155} + \frac{B_3 - B_2}{540.3} = \underline{\underline{0.1408\,W}}$

10.20 Rework Example 10.3 if $\epsilon_{shield} = 0.34$. (Refer to the text for the sketch and numbers.)



Since neither h or i is adiabatic, we can only write a nodal energy balance a node s:

$$0 = \frac{B_s - 1098}{20} + \frac{B_s - 418}{11.02} + \frac{B_s - 266{,}928}{158}$$

or

$$0 = 0.14707\, B_s - 1782.25; \qquad \underline{B_s = 12{,}118}$$

Then:

$$Q_{h-s} = \frac{B_h - B_s}{\frac{1}{A_h F_{h-s}}} = \frac{266{,}928 - 12{,}118}{158} = \underline{\underline{1613\ \text{W}}}$$

$$Q_{h-i} = \frac{B_h - B_i}{\frac{1}{A_h F_{h-i}}} = \frac{266{,}928 - 418}{663} = \underline{402\ \text{W}}$$

$$Q_{s-i} = \frac{B_s - B_i}{\frac{1}{A_s F_{s-i}}} = \frac{12{,}118 - 418}{11.02} = \underline{1062\ \text{W}}$$

Thus the net cooling of the shield must be $Q_{h-s} - Q_{s-i} = \underline{\underline{551\ \text{W}}}$

Problem 10.21 A smooth gray object of emittance ε_1 and area A_1 and does not view itself and sits in a much larger isothermal environment, A_2. Suppose that the object is roughened by making many small cavities covering its entire surface, without changing the radiative properties of the material. The rough surface now has an area $A_r > A_1$. The projected area of the rough surface is a smooth surface that just touches the peaks of the cavities, and it has the same area, A_1, as the original smooth surface. Starting with eqn. (10.23), show that the roughened surface emits radiation to the surroundings as if the original smooth surface had become "blacker". Further show that the effective emittance after roughening is bounded between ε_1 and 1. *Hint:* Because the surroundings are effectively black, the value of A_2 does not affect the heat transfer: shrink A_2 until it reaches the projected surface.

Solution

The rough surface and surroundings form a two body exchange problem, with eqn. (10.23):

$$Q_{\text{net}_{1\text{-}2}} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(\dfrac{1-\varepsilon_1}{\varepsilon_1 A_r}\right) + \dfrac{1}{A_r F_{r\text{-}2}} + \left(\dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}\right)} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(\dfrac{1-\varepsilon_1}{\varepsilon_1 A_r}\right) + \dfrac{1}{A_r F_{r\text{-}2}}} \tag{*}$$

where $\varepsilon_2 = 1$ because the surrounding environment is effectively black. Because the surroundings are effectively black, the value of A_2 does not affect the heat transfer. Thus, for purposes of analysis, we can think of tightening the black surface 2 onto the roughened object so that it becomes the projected surface, with area $A_2 = A_1 < A_r$.

The projected surface, which is now surface 2, touches the top edge of each small cavity and may be considered to be stretched flat above every cavity. Therefore, surface 2 does not see itself and $F_{2\text{-}r} = 1$. We can eliminate the view factor by setting $A_r F_{r\text{-}2} = A_2 F_{2\text{-}r} = A_2$ in eqn. (*):

$$Q_{\text{net}_{1\text{-}2}} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(\dfrac{1-\varepsilon_1}{\varepsilon_1 A_r}\right) + \dfrac{1}{A_2}} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(\dfrac{1-\varepsilon_1}{\varepsilon_1 A_r}\right) + \dfrac{1}{A_1}}$$

where the second step follows since we know $A_2 = A_1$. Rearranging, we have

$$Q_{\text{net}_{1\text{-}2}} = A_1 \underbrace{\frac{1}{\dfrac{A_1}{A_r}\left(\dfrac{1}{\varepsilon_1} - 1\right) + 1}}_{=\mathcal{F}_{1\text{-}2}} \sigma\left(T_1^4 - T_2^4\right) = A_1 \mathcal{F}_{1\text{-}2} \sigma\left(T_1^4 - T_2^4\right)$$

Comparing to the expression for a small gray object in a large isothermal environment, eqn. (10.30), we see that the effective emissivity of the roughened surface is simply the transfer factor, $\mathcal{F}_{1\text{-}2}$:

$$\varepsilon_{1,\text{rough}} = \mathcal{F}_{1\text{-}2} = \frac{A_2}{A_r}\left(\frac{1}{\varepsilon_1} - 1\right) + 1$$

When $A_r \gg A_1$, $\varepsilon_{1,\text{rough}} \to 1$. When $A_r \to A_1$, $\varepsilon_{1,\text{rough}} \to \varepsilon_1$. In every case, $\varepsilon_{1,\text{rough}} > \varepsilon_1$ Thus, the rough surface is effectively "blacker" than the original surface.

Comment 1: This analysis implicitly assumes that the radiosity of the cavities is uniform, and as a result it is strictly valid only for spherical cavities (Donald K. Edwards, *Radiation Heat Transfer Notes*, [10.3]). However, the general principle applies to other cavity shapes: rougher surfaces are effectively blacker, with emissivity bounded between ε_1 and 1.

Comment 2: We have also implicitly assumed the cavities to be large compared to the wavelengths of radiation.

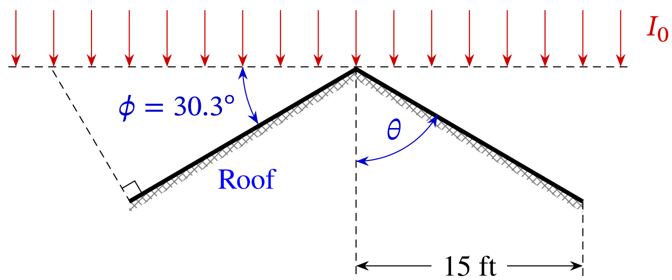
PROBLEM 10.22 A 30 ft by 40 ft house has a conventional sloping roof with a 30.3° pitch and the peak running in the 40 ft direction. Calculate the temperature of the roof in 20°C still air when the sun is overhead: (a) if the roof is made of wooden shingles; and (b) if it is commercial aluminum sheet. The incident solar energy is 670 W/m^2, the effective sky temperature is 22°C, the roofing materials are gray radiators, and the roof is very well insulated.

SOLUTION

The configuration is sketched in the figure. When the sun is directly overhead with an intensity of $I_0 = 670$ W/m^2, the incident solar radiation per unit area of roof is

$$q_{\text{sol}} = I_0 \cos\phi = 670\cos(30.3°) = 579 \text{ W/m}^2$$

where $\phi = 30.3°$ is the roof pitch.



An energy balance on the roof must account for solar energy absorption, infrared radiation exchange with the sky, natural convection to the still air, and heat transfer through the roof into the house. Since the roof is said to be very well insulated, we will neglect heat transfer into the roof. Then:

$$A_{\text{roof}}\alpha_{\text{sol}}q_{\text{sol}} = A_{\text{roof}}\left[\varepsilon_{\text{IR}}\sigma\left(T_{\text{roof}}^4 - T_{\text{sky}}^4\right) + \overline{h}_{\text{nc}}(T_{\text{roof}} - T_{\text{air}})\right] \qquad (*)$$

The natural convection heat transfer coefficient can be calculated with eqn. (8.35) by putting $g\cos\theta$ into the Rayleigh number (as discussed on pgs. 432–434), where in this case $\theta = 59.7°$:

$$\overline{\text{Nu}}_L = 0.14\ \text{Ra}_L^{1/3}\left(\frac{1 + 0.0107\ \text{Pr}}{1 + 0.01\ \text{Pr}}\right) \qquad (8.35)$$

Let's take air properties a convenient guessed value of $T_f = 310$ K (37°C) using Table A.6:

$$\text{Pr} = 0.709,\ k = 0.02684 \text{ W/m·K},\ \nu = 1.659\times10^{-5} \text{ m}^2\text{/s},\ \alpha = 2.304\times10^{-5} \text{ m}^2\text{/s}$$

The length of the roof, $L = (15 \text{ ft})(0.3048 \text{ m/ft}) / \cos(30.3°) = 5.30$ m. Then

$$\begin{aligned}
\overline{h}_{\text{nc}} &= (0.14)\frac{k}{L}\left(\frac{g\cos\theta\,\beta L^3\Delta T}{\nu\alpha}\right)^{1/3}\left(\frac{1 + 0.0107\ \text{Pr}}{1 + 0.01\ \text{Pr}}\right)\\
&= \underbrace{(0.14)\frac{0.02684}{5.30}}_{=0.0007090\ \text{W/m}^2\text{K}}\underbrace{\left(\frac{(9.806)\cos(59.7°)(1/310)(5.30)^3\Delta T}{(1.659\times10^{-5})(2.304\times10^{-5})}\right)^{1/3}}_{=1839(\Delta T)^{1/3}}\underbrace{\left(\frac{1 + 0.0107(0.709)}{1 + 0.01(0.709)}\right)}_{=1.000}\\
&= 1.304(\Delta T)^{1/3} \text{ W/m}^2\text{K}
\end{aligned}$$

where $\Delta T = (T_{\text{roof}} - T_{\text{air}})$.

We have two different roofing materials to consider, wood shingles and commercial aluminum sheet. Both materials are gray, so that solar and infrared properties are the same: $\alpha_{\text{solar}} = \varepsilon_{\text{sol}} = \varepsilon_{\text{IR}}$.

From Table 10.1, for commercial aluminum sheet we have $\alpha_{\text{sol}} = \varepsilon_{\text{IR}} = 0.09$, and for wooden shingles we take $\alpha_{\text{sol}} = \varepsilon_{\text{IR}} \cong 0.85$.

We may rearrange eqn. (*), using the information we have gotten and putting temperatures in kelvin:

$$\overline{h}_{\text{nc}}(T_{\text{roof}} - T_{\text{air}}) = \varepsilon_{\text{IR}}\big[q_{\text{sol}} - \sigma\big(T_{\text{roof}}^4 - T_{\text{sky}}^4\big)\big]$$
$$1.304(T_{\text{roof}} - 293)^{4/3} = \varepsilon_{\text{IR}}\big[579 - (5.67034 \times 10^{-8}\big(T_{\text{roof}}^4 - 295^4\big)\big]$$

This equation must be solved iteratively, guessing T_{roof} and substituting it into one side or the other. Some experimentation will show you that a stable (convergent) iteration is obtained when $\varepsilon_{\text{IR}} = 0.09$ if the substitution is on the right-hand side, but that the substitution must be on the left-hand side when $\varepsilon_{\text{IR}} = 0.85$. We stop iterating when the difference is below 0.1 K, which is well beyond the accuracy of the given information.

ε_{IR}	0.09	0.85
$T_{\text{initial guess}}$ [K]	310.0	330.0
	306.7	346.7
	308.0	324.7
	307.2	333.0
	307.7	344.6
$T_{\text{converged}}$ [K]	307.6	335.0
		⋮
		etc.
		⋮
$T_{\text{converged}}$ [K]		339.4

Summarizing:

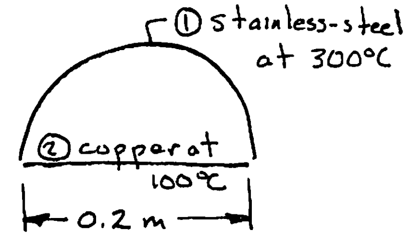
$$T_{\text{roof}} = \begin{cases} 35°\text{C} & \text{for aluminum sheet} \\ 66°\text{C} & \text{for wooden shingles} \end{cases} \quad \xleftarrow{\text{Answer}}$$

Comment 1: The uncertainty in the radiative properties of both materials is significant (and likewise for the heat transfer coefficient), and so the temperatures are clearly just approximate.

Comment 2: Our guessed film temperature is a bit high for the aluminum roof, but the properties of air don't change much over this range. We would gain little accuracy by adjusting the calculation to a different film temperature.

Comment 3: A 7/12 roof pitch — 7 inches rise per 12 inches (one foot) of run — makes a 30.3° angle.

10.23 Calculate the heat transfer between the some shown and its base, by radiation. ($\epsilon_{s.s} = 0.4$ and $\epsilon_{cu} = 0.15$)



$A_1 = 2\pi R^2$, $A_2 = \pi R^2$, $A_2/A_1 = 1/2$

$$Q = \frac{\epsilon_1 A_1}{1-\epsilon_1}\left(\sigma T_1^4 - B_1\right)$$

so we need B_1. To get it, write:

$$B_1 = \epsilon_1 \sigma T_1^4 + (1-\epsilon_1)\left[B_1 F_{1-1} + B_2 F_{1-2}\right]$$
$$B_2 = \epsilon_2 \sigma T_2^4 + (1-\epsilon_2)\left[B_2 F_{2-2} + B_1 F_{2-1}\right]$$

$$F_{1-1} + F_{1-2} = 1$$
$$F_{2-1} + \underbrace{F_{2-2}}_{=0} = 1 \Rightarrow F_{2-1} = 1$$
$$A_1 F_{1-2} = A_2 F_{2-1}, \quad F_{1-2} = \frac{A_2}{A_1} F_{2-1}$$
$$\underline{F_{1-2} = \tfrac{1}{2}}$$
$$\underline{F_{1-1} = 1 - F_{1-2} = \tfrac{1}{2}}$$

so:

$$B_1\left[1-(1-\epsilon_1)F_{1-1}\right] + B_2\left[-F_{1-2}(1-\epsilon_1)\right] = \epsilon_1 \sigma T_1^4$$
$$B_1\left[-F_{2-1}(1-\epsilon_2)\right] + B_2\left[1-(1-\epsilon_2)F_{2-2}\right] = \epsilon_2 \sigma T_2^4$$

or:

$$0.7 B_1 - 0.3 B_2 = 0.4\sigma T_1^4$$
$$-0.85 B_1 + 1 \cdot B_2 = 0.15\sigma T_2^4$$

Therefore:

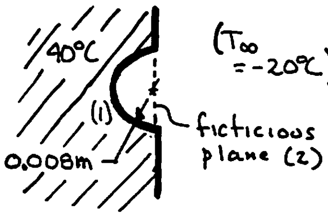
$$B_1 = \frac{\begin{vmatrix} 0.4\sigma T_1^4 & -0.3 \\ 0.15\sigma T_2^4 & 1 \end{vmatrix}}{0.7 - 0.3(0.85)} = \frac{0.4\sigma T_1^4 - 0.045\sigma T_2^4}{0.445}$$

so

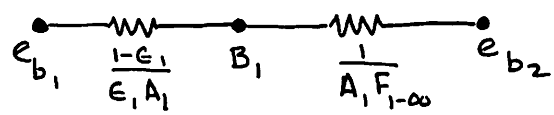
$$Q_1 = 2\pi(0.1)^2 \frac{0.4}{1-0.4} 5.67\cdot 10^{-8}\left[(573)^4\left(1-\frac{0.4}{0.445}\right) - \frac{0.045}{0.445}(373)^4\right]$$

$$\underline{\underline{Q_1 = 21.24\ \text{W}}}$$

10.24 A hemispherical indentation in a smooth wrought iron plate has a 0.008 m radius. How much heat radiates from the 40°C dent to the −20°C surroundings?



$$A_1F_{1-2} = A_2\underbrace{F_{2-1}}_{=1}\ ; \quad F_{1-2} = F_{1-\infty}\ ; \quad \underline{F_{1-\infty} = \frac{A_2}{A_1}}$$



so :
$$Q = \frac{e_{b_1} - e_{b_2}}{\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1F_{1-\infty}}} = \frac{1}{(1-\epsilon_1)\frac{A_2}{A_1} + \epsilon_1}\underbrace{\epsilon_1 A_2 \sigma (T_1^4 - T_\infty^4)}_{\text{heat flow without a dent}}$$

$$= \underbrace{\frac{1}{(1-0.35)\frac{\pi(0.008)^2}{2\pi(0.008)^2} + 0.35}}_{\text{1.48 times the heat flow that would occur with no dent.}} 0.35(\pi[0.008]^2)5.67(10)^{-8} \times [313^4 - 253^4]$$

so :

$$\underline{\underline{Q = 0.0325\,W}}$$

10.25 A conical hole in a block of metal, for which $\epsilon = 0.5$, is 5 cm in diameter at the surface and 5 cm deep. By what factor will the radiation from the area of the hole be changed by the presence of the hole?

(This following solution breaks down if the cone is very deep and slender since the the apex recieves little and we cannot use the network analogy.)

From the solution to problem 10.24 we find that

$$\frac{Q_{\text{with hole}}}{Q_{\text{no hole}}} = \frac{1}{(1-\epsilon_1)\frac{A_2}{A_1} + \epsilon_1} = \frac{1}{0.5\frac{\pi(2.5)^2}{43.9} + 0.5} = \underline{\underline{1.382}}$$

where the area of the cone is the product of its average circumference, 2.5π, and its slant height, 5/cos(tan^{-1}2.5/5) = 43.9 cm^2.

10.26 A single-pane window in a large room is 4 ft wide and 6 ft high. The room is kept at 70°F but the pane is at 67°F owing to heat loss to the colder outdoor air. Find: a) the heat transfer by radiation to the window; b) the heat transfer by natural convection to the window; and c) the fraction of heat transferred to the window by radiation.

a) $$\mathcal{F}_{\text{window to room}} = \frac{1}{\frac{1}{\varepsilon_{\text{window}}} + \underbrace{\frac{A_{\text{window}}}{A_{\text{room}}}}_{\simeq 0}\left(\frac{1}{\varepsilon_{\text{room}}} - 1\right)} = \varepsilon_{\text{window}} = \underline{0.94}$$

$$Q_{\text{rad}_{\text{window to room}}} = \varepsilon_{\text{window}}\, \sigma A_{\text{window}}\left(T_{\text{window}}^4 - T_{\text{room}}^4\right)$$
$$= 0.94(5.67)10^{-8}(4\times6)(0.3048)^2\left[292.59^4 - 294.26^4\right]$$

$$Q_{\text{rad}_{\text{room to window}}} = -Q_{\text{rad}_{\text{window to room}}} = \underline{\underline{20.06\text{ W} = 68.43\ \tfrac{\text{Btu}}{\text{hr}}}}$$

b) Use eqn. (8.13a) with $Ra_L = \dfrac{9.8(1/294.26)(1.67)(6\times0.3048)^3}{(1.508\times10^{-5})(2.117\times10^{-5})} = \underline{1.066\times10^9}$,

and $Pr = \underline{0.713}$:

$$\overline{Nu_L} = 0.68 + 0.67\,Ra_L^{1/4}\left[1+\left(\frac{0.492}{Pr}\right)^{9/16}\right]^{-4/9} = 93.64$$

so $$\bar{h} = \overline{Nu_L}\,k/L = 93.64(0.02562)/6(0.3048) = 1.31\ \frac{\text{W}}{\text{m}^2\text{-}^\circ\text{C}}$$

(this is a <u>very</u> low $\bar{h}$.)

Then: $$Q_{\text{conv.}} = \bar{h}A(\Delta T) = 1.31(6\times4)(0.3048)^2(1.67)$$
$$= \underline{\underline{4.88\text{ W} = 16.64\ \tfrac{\text{Btu}}{\text{hr}}}}$$

c) $$\%\text{ of heat transfer by radiation} = \frac{20.06}{20.06+4.88} = 0.804 = \underline{\underline{80.4\%}}$$

10.27 Suppose the window-pane temperature is unknown in Problem 10.26. The outdoor air is at 40°F and h = 62 W/m²–°C on the outside. It is night and the effective T_{sky} = 15°C. Assume $F_{window-sky}$ = 0.5 and the other surroundings are at 40°C. Evaluate T_{window} and draw the analogous electric circuit evaluating the thermal resistances. (The window is opaque to infra-red radiation but it offers little resistance to conduction so T_{window} is approximately uniform.)

$$\left(Q_{rad.} + Q_{nat'l.\,conv.}\right)_{indoors} = \left(Q_{conv.} + Q_{rad.\,sky} + Q_{rad\,\underset{surr.}{other}}\right)_{outdoors}$$

$$\epsilon_{glass} A_w \sigma\left(T_i^4 - T_w^4\right) + h_i A_w \left(T_i - T_w\right) = \bar{h}_o A_w \left(T_w - T_o\right) + \epsilon_g F_{w-s} \sigma A_w \left(T_w^4 - T_s^4\right)$$
$$+ \epsilon_g F_{w-o.s.} \sigma A_w \left(T_w^4 + T_s^4\right)$$

Let's cut through a potentially terrible lot of trial and error computation by noting that natural convection is not going to be very important on the inside. (We take our cue in this from the solution of Problem 10.26. Therefore we'll guess that h_i is 3 W/m²–°C and correct this assumption later if we must. Then divide that equation above by $\epsilon_g \sigma A_w$ and get (in S.I. units):

$$294.24^4 - T_w^4 + \frac{3(10)^8}{0.94(5.67)}(294.4 - T_w) = \frac{62(10)^8}{0.94(5.67)}(T_w - 277.6) + \frac{1}{2}\left(2T_w^4 - 277.6^4 - 263.1^4\right)$$

or

$$7.496(10)^9 - T_w^4 + 339.5(10)^9 - 1.22(10)^9 T_w = T_w^4 - 5.387(10)^9$$

so:

$$T_w^4 + 0.61(10)^9 T_w = 176.2(10)^9$$

solving by trial and error we get: $T_w = 279.0°K$
$= 42.53°F$

Now check $\bar{h}_{nat'l\,conv.}$ using eqn. (8.13a). (Details are given in solution to Problem 10.26.)

$$Ra_L = \underbrace{1.066 \times 10^9}_{\text{when } \Delta T = 1.67°C} \frac{294.24 - 279.0}{1.67} = 9.728(10)^9$$

so:

$$\overline{Nu}_L = 0.68 + 0.67\, Ra_L^{1/4}\left[1 + \left(\frac{0.492}{.713}\right)^{0.5625}\right]^{-0.4444} = 162.3$$

and $\bar{h} = 162.3(0.0256)/6(0.3048) = \underline{2.27\ W/m^2\text{–}°C}$

Going back through the trial and error solution based on this value of $\bar{h}$ we get 42.35°F -- almost no change.

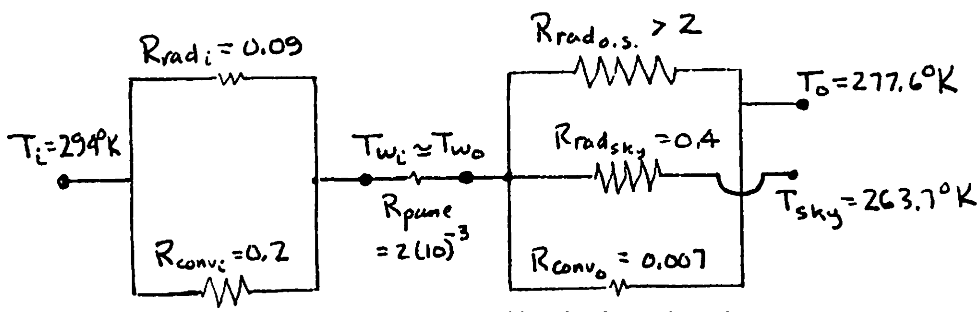
so $\underline{\underline{T_w = 42.35°F}}$

10.27 (continued)

Next calculate resistances; $R_{rad_i} = \frac{1}{h_{rad_i} A_w} = \frac{\Delta T}{Q_{rad_i}} = \frac{15.2}{170.7} = \underline{\underline{0.0893 \frac{^\circ C}{W}}}$

$R_{rad_{sky}} = \frac{30.54}{72.71} = \underline{\underline{0.420}}$; $R_{rad_{o.s.}} = \frac{16.64}{7.17} = \underline{\underline{2.32}}$; $R_{conv_o} = \frac{1}{h_o A_w} = \underline{\underline{0.00723}}$,

the hitherto neglected $R_{pane} = \frac{\text{thickness}}{k A_w} = \underline{\underline{0.001725}}$; & $R_{conv_i} = \underline{\underline{0.1975}}$.



Heat is basically short-circuited to the window by radiation.

Heat is short-circuited to the outdoors by convection.

So convection is irrelevant inside. Radiation is irrelevant outside. And conduction through the glass causes no ΔT.

10.28 An effective low-temperature insulation is made by evacuating the space between metal sheets. Calculate q between 150°K and 100°K for: (a) two sheets of <u>highly</u> polished al., (b) three sheets of highly polished al., and (b) three sheets of rolled sheet steel.

In all cases, $\mathcal{F}_{1-2} = \dfrac{1}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$ or $\dfrac{1}{\frac{1}{\varepsilon_1} + 1\left(\frac{1}{\varepsilon_2} - 1\right)} = \dfrac{1}{\frac{2}{\varepsilon} - 1}$

Case a.) $\mathcal{F}_{1-2} = \dfrac{1}{\frac{2}{0.04} - 1} = 0.02041$ (where we use ε_{al} for a higher temp.)

$$q = \mathcal{F}_{1-2}\sigma(T_1^4 - T_2^4) = \underbrace{0.02041(5.67)10^{-8}}_{1.157(10)^{-9}}(150^4 - 100^4) = \underline{\underline{0.470 \frac{W}{m^2}}}$$

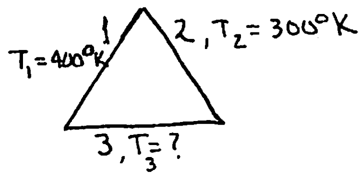
b.) $q = 1.157(10)^{-9}[150^4 - T_{middle}^4] = 1.157(10)^{-9}[T_{middle}^4 - 100^4]$

$T_{middle} = [(150^4 + 100^4)/2]^{1/4} = \underline{131.95^\circ K}$

$$q = 1.157(10)^{-9}[150^4 - 131.95^4] = \underline{\underline{0.235 \frac{W}{m^2}}}$$

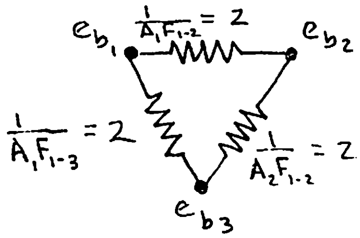
c.) $\mathcal{F}_{1-2} = \dfrac{1}{\frac{2}{0.66} - 1} = 0.4925$, $q = 0.4925\,\sigma[131.95^4 - 100^4] = \underline{\underline{5.67 \frac{W}{m^2}}}$

10.29 Three parallel black walls, 1m wide, form an equilateral triangle. One wall is held at 400°K, one at 300°K, and the third is insulated. Find Q W/m and the temperature of the third wall.



By symmetry, we see: $F_{1-2} = F_{1-3} = F_{2-1} = F_{2-3} = \frac{1}{2}$

$$\& \quad e_{b_1} = \sigma 400^4 = 1452 \text{ W/m}^2$$
$$e_{b_2} = \sigma 300^3 = 459.3 \text{ W/m}^2$$



Using equation (10.35) we get:

$$Q = \frac{1452 - 459.3}{\left(\frac{1}{2+2} + \frac{1}{2}\right)^{-1}} = \underline{\underline{744 \text{ W/m}}}$$

And to get e_{b_3} we write for the node #3:

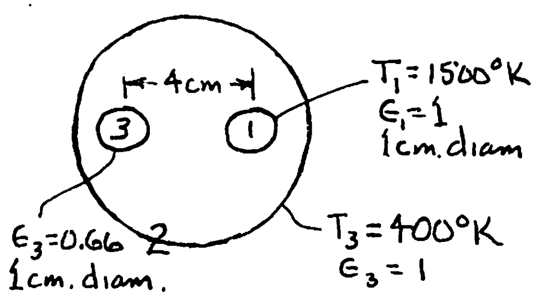
$$0 = \frac{e_{b_1} - e_{b_3}}{2} + \frac{e_{b_2} - e_{b_3}}{2}$$

or

$$e_{b_3} = (e_{b_1} + e_{b_2})/2 = 955.7 = \sigma T_3^4$$

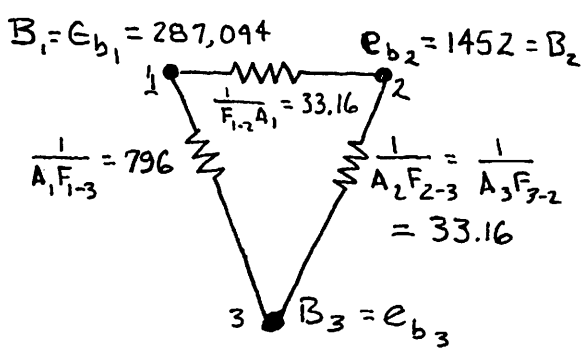
so $\underline{\underline{T_3 = 360.3°K}}$

10.30 Two 1cm diameter rods run parallel with centers 4 cm apart. One is at 1500°K and black. The other is unheated and ε = 0.66. They are both encircled by a cylindrical black radiation shield at 400°K. Evaluate Q W/m and the temperature of the unheated rod.



From Table 10.2 #6, $X = 1 + \frac{3}{1} = 4$

$$F_{1-3} = \frac{1}{\pi}\left[\sqrt{15} + \sin^{-1}\frac{1}{4} - 4\right] = \underline{0.0400}$$
$$= F_{3-1}$$
$$F_{1-2} = 1 - F_{1-3} = \underline{0.960} = F_{3-2}$$
$$A_1 = A_3 = 0.031416 \text{ m}^2/\text{m}$$



Using equation (10.35) we get

$$q_{1-2} = \frac{287{,}044 - 1452}{\cfrac{1}{\cfrac{1}{796 + 33.16} + \cfrac{1}{33.16}}} = \underline{8957 \frac{\text{W}}{\text{m}}}$$

$$Q = 0.031416(8957) = \underline{\underline{281.4 \frac{\text{W}}{\text{m}}}}$$

Now at Node #1, $q = -8957 = \frac{B_2 - B_1}{33.16} + \frac{B_3 - B_1}{796}$

so $B_3 = 12{,}858 = \sigma T_3^4$

$\underline{\underline{T_3 = 690°K}}$

(We note that ϵ_3 does not enter the problem.)

10.31 A small diameter heater is centered in a large cylindrical shield. Discuss the relative improtance of the emittance of the shield during specular and diffuse radiation.

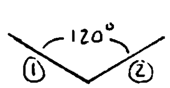
In this case $A_{inside}/A_{outside}$ is very small so (see eqn. (10.30):

$$\mathcal{F}_{1-2}\Big|_{diffuse} \Rightarrow \frac{1}{\frac{1}{\epsilon_1}+0} = \epsilon_1 \quad ; \quad \mathcal{F}_{1-2}\Big|_{specular} = \frac{1}{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_2}-1}$$

In the pure diffuse limit, ϵ_2 is irrelevant, while in the pure specular limit, ϵ_1 and ϵ_2 are equally important.

10.32 Two, 1m wide, commercial aluminum sheets are joined at a 120° angle along one edge. The back (or 240°-angle)side is insulated. The plates are both held at 120°C. The 20°C surroundings are distant. What is the net radiant heat transfer from the left hand plate: to the right hand side, and to the surroundings.

The business about heat transfer from left to right is a red-herring. We see at once that symmetry requires this to be zero. We may therefore treat the plates a common heater, calculate Q from both, and then divide it by two.



$\epsilon_1 = \epsilon_2 = 0.09$

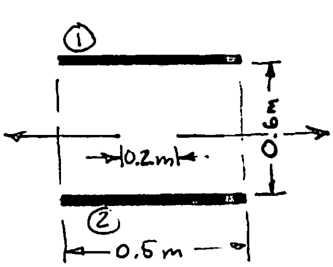
$$A_{1\&2}F_{1\&2-s} = 2A_1F_{1-s} = 2(1-F_{1-2}) = 1.732$$

$F_{1-2} = 1-\sin\frac{120}{2} = 0.134$, from Tab. 10.2, #2

Then eqn. (11.24) gives:

$$Q = \frac{1}{2}\left[\frac{(5.67)10^{-8}\left([120+273]^4-[20+273]^4\right)}{\frac{1-0.09}{0.09(2)}+\frac{1}{1.732}+\frac{1}{\infty}}\right] = \underline{\underline{82.96\,W/m}}$$

10.33 Two parallel discs of diameter equal to 0.5m are separated by an infinite parallel plate, midway between them, with a 0.2m diameter hole in it. What is the view factor between the two discs, if they are 0.6 m apart.



$$F_{1-2} = F_{1-hole} = \frac{1}{2}\left(X - \sqrt{X^2 - 4(R_{hole}/R_1)^2}\right)$$ Table 10.3 No. 3

$$X = 1 + \frac{\left(1+\left[\frac{R_{hole}}{h}\right]^2\right)}{(R_1/h)^2} = 1 + \frac{1+(0.2/0.6)^2}{(0.5/0.6)^2} = 2.6$$

$$F_{1-hole} = \frac{1}{2}\left(2.6 - \sqrt{2.6^2 - 4(0.2/0.6)^2}\right)$$

$$= \underline{\underline{0.0435}}$$

10.34 An evacuated spherical cavity, 0.3 m in diameter in a zero-gravity environment, is kept at 300°C. Saturated steam at 1 atmosphere is then placed in the cavity. a) What is the initial flux of radiant heat transfer to the steam? b) Determine how long it will take for $q_{conduction}$ to become less than $q_{radiation}$. (Correct for the rising steam temperature if it is necessary to do so.)

In this case: $pL = 0.3$ so at 373°K, $\varepsilon_g = f_1 f_2 = 0.275(1.21) = \underline{0.333}$

$\dot{\varepsilon} \cdot pL \frac{573}{373} = 0.46$ at 573°K, $\alpha_g = \underbrace{\varepsilon_g}_{0.325(1.21)} \left(\frac{373}{573}\right)^{0.45} = \underline{0.324}$

Then $q_{net} = q_{w-g} - q_{g-w} = 0.333\sigma(573)^4 - 0.324\sigma(373)^4 = \underline{\underline{1680 \text{ W/m}^2}}$

and from eqn. (5.54) $\quad q_{cond} = \dfrac{k_{stm}\Delta T}{\sqrt{\pi \alpha_{stm} t}} = \dfrac{0.0237(200)}{\sqrt{\pi\, 2.032 \times 10^{-5}\, t}}$

Thus q_{cond} will equal 1680 after $\underline{\underline{t = 0.353 \text{ sec}}}$

Should we have accounted for temperature rise? It would appear not, but let's check:

Heat capacity of steam $= \rho c_p \text{Vol} = 0.597(2030)\frac{4\pi}{3}(0.15)^3$
$= 17.13$ W/°C

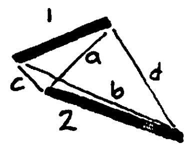
$Q_{rad}\, t = 1680(4\pi(0.15)^2)\, 0.353 = 167.7$ W

$\int Q_{cond}\, dt = A\underbrace{q(t)}_{1680}\, 2t = 2(167.7)$ W

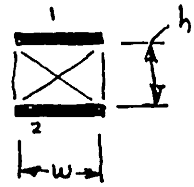
so $\quad \Delta T = \dfrac{Q_{total}}{\text{Heat cap}} = \dfrac{3(167.7)}{17.13} = \underline{29.4°C}$

That's a lot more ΔT than one might first expect, but it's still a small number. We can probably ignore it.

10.35 Verify cases 1, 2, and 3 in Table 10.2 using the "string method" described in Problem 10.14.



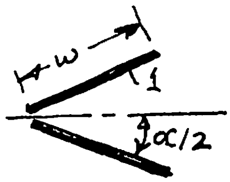
$$F_{1-2} = \frac{(a+b)-(c+d)}{2L_1}$$

1) 

$c = d = h$

$a = b = \sqrt{h^2+w^2}$

$$F_{1-2} = \frac{(a+b)-(c+d)}{2w} = \underline{\underline{\sqrt{1+\left(\frac{h}{w}\right)^2} - \frac{h}{w}}}$$

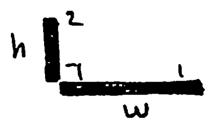
2) 

$a = b = w$

$c = 0$

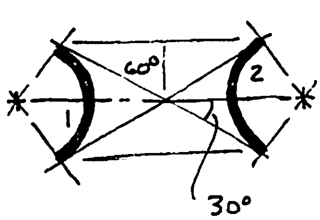
$d = 2w\sin\frac{\alpha}{2}$

$$F_{1-2} = \underline{\underline{1 - \sin\alpha/2}}$$

3) 

$$F = \frac{(h+w)-\left(0+\sqrt{h^2+w^2}\right)}{2w} = \underline{\underline{\frac{1}{2}\left[1+\frac{h}{w}-\sqrt{\left(\frac{h}{w}\right)^2+1}\right]}}$$

10.36 Two long parallel heaters consist of 120° segments of 10 cm diameter parallel cylinders, whose centers are 20 cm apart. The segments are those nearest each other, symetrically placed on the line connecting their centers. Find F_{1-2} using the "string method" described in Problem 10.14



$$F_{1-2} = \frac{(a+b)-(c+d)}{2L_1}$$ (see general sketch in solution above.)

$a = 20\sin 60° = b$

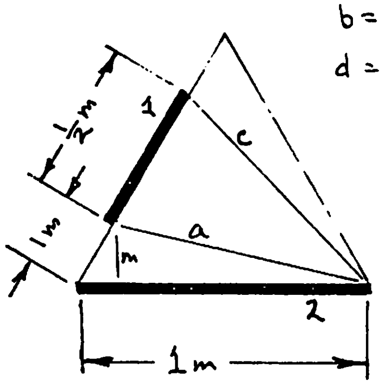
$c = (20\sin 60°)\sin 60° = d$

$L_1 = \frac{1}{3}(\pi \times 10)$

so: $$F_{1-2} = \frac{40\sin 60° - 40\sin 60° \times \sin 60°}{20\pi/3} = \frac{6}{\pi}\sin 60°(1-\sin 60°)$$

$$\underline{\underline{F_{1-2} = 0.2216}}$$

10.37 Two long parallel strips of rolled steel sheet lie along sides of an imaginary 1 m equilateral triangular cylinder. One piece is 1 m wide and kept at 20°C. The other is (1/2) m wide, centered in an adjacent leg, and kept at 400°C. The surroundings are distant and they are insulated. Find Q. (You will need a shape factor. It can be found using the method described in Problem 10.14.)



$b = \frac{3}{4}$ m

$d = \frac{1}{4}$ m

$$m = \underbrace{d\sin 60°}_{\frac{1}{4}\,0.866} = \sqrt{a^2 - \underbrace{(1 - \underbrace{d\cos 60°}_{1/8})^2}_{49/64}} = 0.2165$$

so $a = 0.9014$

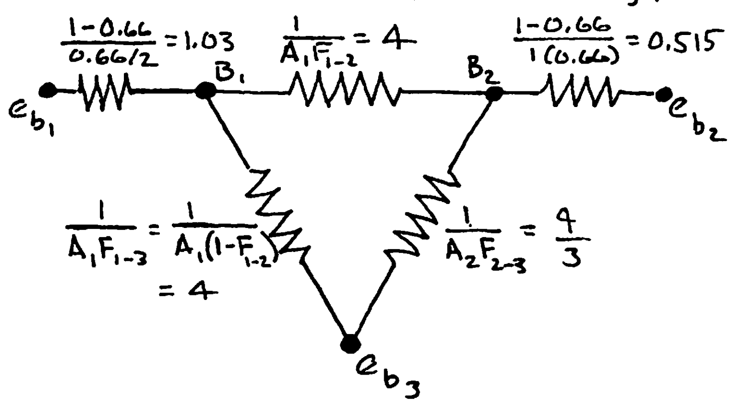
& $c = 0.9014$ by symmetry

Then:

$$F_{1-2} = \frac{(a+b)-(c+d)}{2h_1} = \frac{0.9014 + 0.75 - 0.9014 - 0.25}{2(0.5)}$$

$$= \frac{0.5}{2(0.5)} = \underline{0.5}$$

using $\varepsilon_1 = 0.66 = \varepsilon_2$, & calling the surroundings, 3 :



$$A_1 F_{1-2} = A_2 F_{2-1}$$

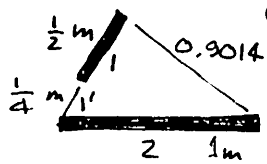
but $F_{2-1} = \frac{1}{2}\frac{1}{2} = \frac{1}{4}$

so $\underline{F_{2-3} = 1 - F_{2-1} = \frac{3}{4}}$

Then eqn. (10.35) gives:

$$Q = \frac{\sigma(673^4 - 293^4)}{1.03 + \dfrac{1}{\dfrac{1}{4 + \frac{4}{3}} + \dfrac{1}{4}} + 0.515} = \underline{\underline{2927\ W}}$$

10.38 Find the shape factor from the hot to the cold strip in Problem 11.37 using, not the string method, but Table 10.2. If your instructor asks you to do so, complete Problem 10.37 when you have F_{1-2}.



(see details of dimensions in Problem 10.37 solution.)

Now: $$A_{1,1'}F_{1,1'-2} = A_1 F_{1-2} + A_{1'} F_{1'-2}$$

But from Table 10.2, #4: $$F_{1,1'-2} = \frac{3/4 + 1 - 0.9014}{2\left(\frac{3}{4}\right)} = 0.5657$$
$$F_{1'-2} = \frac{1/4 + 1 - 0.9014}{2(1/4)} = 0.6972$$

so: $$\frac{3}{4}(0.5657) = \frac{1}{2}F_{1-2} + \frac{1}{4}(0.6972) \quad \text{so} \quad \underline{\underline{F_{1-2} = 0.5}}$$

(From this point forward, details of solution are given in Prob 10.37.)

11.39 Prove that, as the figure becomes very long, the view factor for the 2nd case in Table 10.3 reduces to that given for the 3rd case in Table 10.2.

In this case $H \to 0$, $W \to 0$, $H/W \to h/w$, *so the equation reduces to*

$$F_{1-2} \to \frac{1}{\pi}\left(\tan^{-1}\infty + \frac{h}{w}\tan^{-1}\infty - \sqrt{\left(\frac{h}{w}\right)^2+1}\ \tan^{-1}\infty + \frac{1}{4}\left(\ln 1 + 0 \ln 1 + 0 \ln 1\right)\right)$$
$$= \frac{1}{\pi}\left(\frac{\pi}{2} + \frac{h}{w}\frac{\pi}{2} - \sqrt{\left(\frac{h}{w}\right)^2+1}\ \frac{\pi}{2} + 0\right) = \underline{\underline{\frac{1}{2}\left(1 + \frac{h}{w} - \sqrt{1+\left(\frac{h}{w}\right)^2}\right)}}$$

This is given in Table 10.2.

10.40 Show that F_{1-2} for the first case in Table 10.3 reduces to the expected result when plates 1 and 2 are extended to infinity.

X and $Y \to \infty$ so

$$F_{1-2} \to \frac{2}{\pi}\left[\underbrace{\frac{\ln \frac{XY}{\sqrt{X^2+Y^2}}}{XY}}_{\to 0} + \underbrace{1\tan^{-1}\frac{X}{Y} + 1\tan^{-1}\frac{Y}{X}}_{\text{but } \tan^{-1}a + \tan^{-1}\frac{1}{a} = \frac{\pi}{2}} - \underbrace{\frac{\tan^{-1}X}{Y}}_{\to 0} - \underbrace{\frac{\tan^{-1}Y}{X}}_{\to 0}\right]$$

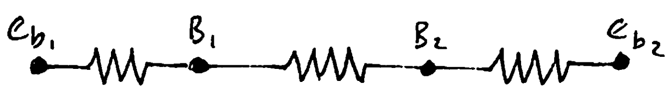
so $\underline{\underline{F_{1-2} = 1}}$ which is correct for two infinite facing plates

10.41 10.42 In problem 2.26 you were asked to neglect radiation in showing that q was equal to 8227 W/m^2 as the result of conduction alone. Discuss the validity of the assumption, quantitatively.

In this case we have: $q_{rad} = \dfrac{\sigma}{\frac{2}{\epsilon} - 1}\left([1000+273]^4 - [200+273]^4\right)$

where ϵ = the emittance of <u>both</u> plates. When $q_{cond} = q_{rad}$ $= 8227$, $\epsilon = 0.1066$. This is a reasonable value for polished metal, but it doesn't give a <u>negligible</u> value of q_{rad}. If q_{rad} = only 10% $q_{cond} = 822.7$ then $\underline{\epsilon = 0.0112}$. This would be hard to achieve. It would require, for example, a very highly polished silver.

10.42 A 100°C sphere with $\varepsilon = 0.86$ is centered within a second sphere at 300°C with $\varepsilon = 0.47$. The outer diameter is 0.3 m and the inner diameter is 0.1 m. What is the radiant heat flux?



$\dfrac{1-0.47}{\pi(0.3)^2\, 0.47} = 3.988$ $\quad \dfrac{1}{A_1 F_{12}} = \dfrac{1}{\pi(0.3)^2\left(\frac{0.05}{0.15}\right)^2} = 31.83$ $\quad \dfrac{1-0.86}{\pi(0.1)^2 0.86} = 5.182$

(We get F_{1-2} from Table 10.3, case 4.)

Then, using eqn. (10.35), we get:

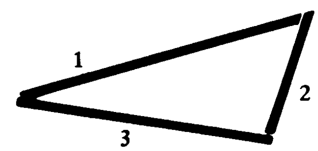
$$Q = \frac{\sigma(573)^4 - \sigma(373)^4}{3.988 + 31.83 + 5.182} = \underline{122.3 \text{ W}}$$

$$q = \frac{Q}{A} = \frac{122.3}{\pi(0.3)^2} = \underline{\underline{432.6 \ \frac{\text{W}}{\text{m}^2}}}$$

PROBLEM 10.43 Verify $F_{1\text{-}2}$ for case 4 in Table 10.2. *Hint:* This can be done without integration.

SOLUTION

The configuration of three flat surfaces is shown in the figure. We can apply the summation and reciprocity rules for view factors, eqns. (10.12) and (10.15), for the three surfaces; and then we can do algebra to find $F_{1\text{-}2}$. Note that $F_{i\text{-}i} = 0$ for flat surfaces.



The rules lead to this set of relationships, which provide 6 equations for 6 unknowns:

$$F_{1\text{-}2} + F_{1\text{-}3} = 1, \quad F_{2\text{-}1} + F_{2\text{-}3} = 1,$$
$$F_{3\text{-}1} + F_{3\text{-}2} = 1$$
$$A_1 F_{1\text{-}2} = A_2 F_{2\text{-}1}, \quad A_1 F_{1\text{-}3} = A_3 F_{3\text{-}1},$$
$$A_2 F_{2\text{-}3} = A_3 F_{3\text{-}2}$$

We can eliminate $F_{3\text{-}1}$ and $F_{3\text{-}2}$ with the last two reciprocity relationships

$$F_{3\text{-}2} = (A_2/A_3)F_{2\text{-}3}, \quad F_{3\text{-}1} = (A_1/A_3)F_{1\text{-}3}$$

and the fact that they sum to 1:

$$F_{3\text{-}1} + F_{3\text{-}2} = (A_1/A_3)F_{1\text{-}3} + (A_2/A_3)F_{2\text{-}3} = 1$$

We may rearrange the second sum rule and combine with the first reciprocity rule

$$F_{2\text{-}3} = 1 - F_{2\text{-}1} = 1 - (A_1/A_2)F_{1\text{-}2}$$

We can solve for $F_{1\text{-}3}$ by substituting this result into the previous equation

$$(A_1/A_3)F_{1\text{-}3} + (A_2/A_3)[1 - (A_1/A_2)F_{1\text{-}2}] = 1$$
$$\begin{aligned} F_{1\text{-}3} &= (A_3/A_1) \\ &\quad - (A_3/A_1)(A_2/A_3)[1 - (A_1/A_2)F_{1\text{-}2}] \\ &= (A_3/A_1) - (A_2/A_1) + F_{1\text{-}2} \end{aligned}$$

Finally, we may substitute the last relationship for $F_{1\text{-}3}$ into the first sum rule

$$F_{1\text{-}2} + F_{1\text{-}3} = 1 = F_{1\text{-}2} + (A_3/A_1) - (A_2/A_1) + F_{1\text{-}2}$$

Solving

$$F_{1\text{-}2} + F_{1\text{-}2} = 1 - (A_3/A_1) + (A_2/A_1)$$

$$F_{1\text{-}2} = \frac{1 - (A_3/A_1) + (A_2/A_1)}{2} = \frac{A_1 + A_2 - A_3}{2A_1} \quad \xleftarrow{\text{Answer}}$$

Problem 10.44 Consider the approximation made in eqn. (10.30) for a small gray object in a large isothermal enclosure. How small must A_1/A_2 be in order to introduce less than 10% error in $\mathcal{F}_{1\text{-}2}$ if the small object has an emittance of $\varepsilon_1 = 0.5$ and the enclosure is: a) commercial aluminum sheet; b) rolled sheet steel; c) rough red brick; d) oxidized cast iron; or e) polished electrolytic copper. Assume that both the object and its environment have temperatures in the range of 40 to 90°C.

Solution

For an object 1 enclosed by a surface 2, eqn. (10.30) suggests an approximation to the transfer factor defined in eqn. (10.27) when $A_1/A_2 \ll 1$:

$$\mathcal{F}_{1-2} = \frac{1}{\dfrac{1}{\varepsilon_1} + \underbrace{\dfrac{A_1}{A_2}}_{\ll 1}\left(\dfrac{1}{\varepsilon_2} - 1\right)} \cong \varepsilon_1$$

Equation (10.27) requires that object 1 does not see itself, so A_1/A_2 must always be < 1. Further, as $\varepsilon_2 \to 1$, the approximation of an effectively black surrounding is more accurate for any value of A_1/A_2.

The problem asks us to find the value of A_1/A_2 for which

$$\left|\varepsilon_1 - \left[\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)\right]^{-1}\right| < 0.10$$

for $\varepsilon_1 = 0.5$ and various values of ε_2. We see that the first term inside the absolute value is always larger than the second term, so we can drop the absolute value signs, substitute $\varepsilon_1 = 0.5$ and rearrange:

$$0.5 - \left[2 + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)\right]^{-1} < 0.10$$

$$0.4 < \left[2 + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)\right]^{-1}$$

and, with $0.4 = 2/5$,

$$\frac{5}{2} > 2 + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)$$

This gives us a bound on A_1/A_2 as a function of ε_2:

$$\frac{1}{2\left(\dfrac{1}{\varepsilon_2} - 1\right)} > \frac{A_1}{A_2} \qquad (*)$$

Before charging ahead, let's remember that $A_1/A_2 < 1$. If we substitute $A_1/A_2 = 1$ in eqn. (*) and solve for ε_2, we find that

$$\varepsilon_2 < \frac{2}{3}$$

In other words, $\varepsilon_2 = 0.667$ is the largest value for which we get a 10% error in the approximation. *For any larger value of ε_2, the error will be less than 10% even if $A_1/A_2 \to 1$!*

From here, we can make a table using the emissivities listed in Table 10.1. The area ratio must be smaller than the value in the last column, which is calculated from eqn. (*) for $\varepsilon_2 < 0.667$ and is simply 1 for larger values of ε_2.

Case	Material	ε_2	$A_1/A_2 <$
a)	commercial aluminum sheet	0.09	0.05
b)	rolled sheet steel	0.66	0.97
c)	rough red brick	0.93	1
d)	oxidized cast iron	0.57–0.66	0.66–0.97
e)	polished electrolytic copper	0.02	0.01

Problem 10.45 Derive eqn. (10.45), starting with eqns. (10.39–10.41).
In older versions of AHTT, the first term in the right-hand side of eqn. (10.45) is incorrect.

Solution

This problem is all algebra. The approach is to substitute eqn. (10.40) for B_i into eqn. (10.41) for H_i. That expression can be rearranged as a matrix equation in terms of H_j and σT_j^4. Likewise, eqn. (10.40) can be substituted into eqn (10.39) to obtain an expression for $Q_{\text{net},i}$ in terms of H_i and σT_i^4. The two derived expressions can be combined to eliminate H_j, leaving a matrix equation for $Q_{\text{net},j}$ in terms of σT_j^4.

Here we go. Equations (10.40) and (10.41) are:

$$B_i = (1-\varepsilon_i)H_i + \varepsilon_i\,\sigma T_i^4 \tag{10.40}$$

$$H_i = \sum_{j=1}^{n} B_j F_{i\text{-}j} \tag{10.41}$$

Elimination of B_j from eqn. (10.41) gives

$$H_i = \sum_{j=1}^{n}\Big[(1-\varepsilon_j)H_j + \varepsilon_j\,\sigma T_j^4\Big]F_{i\text{-}j} \tag{10.41}$$

or, noting that $\sum_j H_i\delta_{ij} = \sum_j H_j\delta_{ij} = H_i$,

$$\sum_{j=1}^{n}\big[\delta_{ij} - F_{i\text{-}j}(1-\varepsilon_j)\big]H_j = \sum_{j=1}^{n}\varepsilon_j F_{i\text{-}j}\,\sigma T_j^4 \tag{*}$$

Next we can write $Q_{\text{net}_i} = q_{\text{net}_i}A_i$ with eqn. (10.39) and substitute eqn. (10.40) for B_i:

$$Q_{\text{net}_i} = A_i(B_i - H_i) = A_i(\varepsilon_i\,\sigma T_i^4 - \varepsilon_i H_i) \tag{10.39}$$

so that

$$H_i = \sigma T_i^4 - \frac{Q_{\text{net}_i}}{\varepsilon_i A_i}$$

Now eliminate H_j in eqn. (*) using the above equation:

$$\sum_{j=1}^{n}\big[\delta_{ij} - F_{i\text{-}j}(1-\varepsilon_j)\big]\left(\sigma T_j^4 - \frac{Q_{\text{net}_j}}{\varepsilon_j A_j}\right) = \sum_{j=1}^{n}\varepsilon_j F_{i\text{-}j}\,\sigma T_j^4$$

$$\sum_{j=1}^{n}\big[\delta_{ij} - F_{i\text{-}j}(1-\varepsilon_j)\big]\sigma T_j^4 - \sum_{j=1}^{n}\varepsilon_j F_{i\text{-}j}\,\sigma T_j^4 = \sum_{j=1}^{n}\big[\delta_{ij} - F_{i\text{-}j}(1-\varepsilon_j)\big]\frac{Q_{\text{net}_j}}{\varepsilon_j A_j}$$

$$\sum_{j=1}^{n}\big[\delta_{ij} - F_{i\text{-}j}(1-\varepsilon_j)\big]\frac{Q_{\text{net}_j}}{\varepsilon_j A_j} = \sum_{j=1}^{n}\big(\delta_{ij} - F_{i\text{-}j}\big)\sigma T_j^4$$

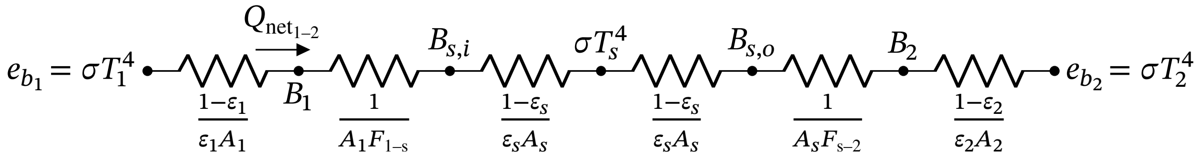
Multiplying through by A_i gives the solution, eqn. (10.45):

$$\boxed{\sum_{j=1}^{n}\left[\frac{\delta_{ij}}{\varepsilon_i} - \frac{(1-\varepsilon_j)}{\varepsilon_j A_j}A_i F_{i\text{-}j}\right]Q_{\text{net}_j} = \sum_{j=1}^{n}\big(A_i\delta_{ij} - A_i F_{i\text{-}j}\big)\sigma T_j^4} \tag{10.45}$$

Problem 10.46 (a) Derive eqn. (10.31), which is for a single radiation shield between two bodies. Include a sketch of the radiation network. (b) Repeat the calculation in the case when two radiation shields lie between body 1 and body 2, the second just outside the first.

Solution

a) The radiation network connects the black body emissive power of body 1, $e_{b_1} = \sigma T_1^4$, to that of the shield, e_{b_s}, to that of body 2, $e_{b_2} = \sigma T_2^4$. The emissive powers are separated from the radiosities by a surface resistance, and the radiosities are separated by a geometrical resistance.



We assume that body 1 views only the inside of the radiation shield and that the outside of the radiation shield views only body 2. Thus, $F_{1-s} = 1$ and $F_{s-2} = 1$.

All net heat transfer leaving body 1 goes to body 2, so a single current, $Q_{\text{net}_{1-2}}$, net travels through all resistors. To calculate this current, we simply divide the sum of the resistances into the difference between the two given emissive powers. The result is eqn. (10.31):

$$Q_{\text{net}_{1-2}} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}\right) + \underbrace{2\left(\dfrac{1-\varepsilon_s}{\varepsilon_s A_s}\right) + \dfrac{1}{A_s}}_{\text{added by shield}}} \tag{10.31}$$

b) The second radiation shield adds three more resistances: i) a geometrical resistance between the first and second shield; ii) a surface resistance on the inside of the second shield; and iii) a surface resistance on the outside of the second shield. The three resistances are simply added in series between the original shield and body 2.

We may reasonably assume the inner shield sees only the outer shield (so that the view factor is one), and that the outer shield sees only the body 2 (so that the view factor is also one). Further, we assume that the emittances of shield 1 and shield 2 are the same. The heat flow is reduced to:

$$\boxed{Q_{\text{net}_{1-2}} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}\right) + \underbrace{2\left(\dfrac{1-\varepsilon_s}{\varepsilon_s A_s}\right) + \dfrac{1}{A_s}}_{\text{added by shield 1}} + \underbrace{2\left(\dfrac{1-\varepsilon_s}{\varepsilon_s A_{s_2}}\right) + \dfrac{1}{A_{s_2}}}_{\text{added by shield 2}}}}$$

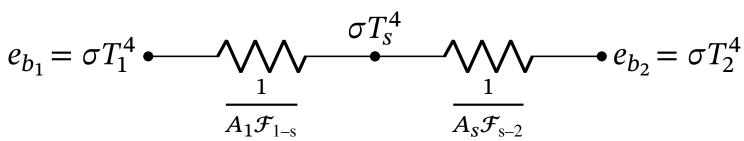
Comment: Radiation shields are often made of reflective metals with low ε, but even a shield that is black adds additional geometrical resistance that can significantly lower the radiation heat transfer.

Problem 10.47 Use eqn. (10.32) to find the net heat transfer from between two specularly reflecting bodies that are separated by a specularly reflecting radiation shield. Compare the result to eqn. (10.31). Does specular reflection reduce the heat transfer?

Solution

Equation (10.32) provides the radiation heat transfer between two specularly reflecting bodies, one of which encloses (body 2) the other (body 1). We may be consider the added radiation shield as a body which encloses body 1 and is enclosed by body 2. Then we can apply eqn. (10.32) separately to body 1 and the shield and to the shield and body 2.

The network, using the transfer factors inside and outside the shield, is



To calculate the heat transfer, we simply divide the sum of the two resistances into the difference between the two given emissive powers and substitute eqn. (10.32) for $\mathcal{F}_{1\text{–s}}$ and $\mathcal{F}_{\text{s}–2}$:

$$
\begin{aligned}
Q_{\text{net}_{1-2}} &= \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1}{A_1\mathcal{F}_{1\text{–s}}} + \dfrac{1}{A_s\mathcal{F}_{\text{s}–2}}} \\
&= \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1}{A_1}\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_s} - 1\right) + \dfrac{1}{A_s}\left(\dfrac{1}{\varepsilon_s} + \dfrac{1}{\varepsilon_2} - 1\right)}
\end{aligned}
$$

For the purpose of comparing to eqn. (10.31), we can rearrange the denominator by adding and subtracting one:

$$
\begin{aligned}
Q_{\text{net}_{1-2}} &= \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1}{A_1}\left(\dfrac{1}{\varepsilon_1} - 1 + 1 + \dfrac{1}{\varepsilon_s} - 1\right) + \dfrac{1}{A_s}\left(\dfrac{1}{\varepsilon_s} - 1 + 1 + \dfrac{1}{\varepsilon_2} - 1\right)} \\
&= \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1} + \dfrac{1-\varepsilon_s}{\varepsilon_s A_1}\right) + \left(\dfrac{1-\varepsilon_s}{\varepsilon_s A_s} + \dfrac{1}{A_s} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_s}\right)} \\
&= \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_s}\right) + \left(\dfrac{1-\varepsilon_s}{\varepsilon_s A_1} + \dfrac{1-\varepsilon_s}{\varepsilon_s A_s} + \dfrac{1}{A_s}\right)}
\end{aligned}
$$

The two areas in red, A_s and A_1, are smaller than the corresponding areas in eqn. (10.31), which were A_2 and A_s, respectively. That difference will tend to *increase* the two resistances and *decrease* the heat fluxes.

Specular reflection decreases the heat transfer. $\xleftarrow{\text{Answer}}$

PROBLEM 10.48 Some values of the monochromatic absorption coefficient for liquid water, as $\rho\kappa_\lambda$ (cm^{-1}), are listed in Table 10.6 [10.5]. For each wavelength, find the thickness of a layer of water for which the monochromatic transmittance is 10%. On this basis, discuss the colors one might see underwater and water's infrared emittance.

λ (μm)	$\rho\kappa_\lambda$ (cm^{-1})	*Color*
0.3	0.0067	
0.4	0.00058	violet
0.5	0.00025	green
0.6	0.0023	orange
0.8	0.0196	
1.0	0.363	
2.0	69.1	
2.6–10.0	> 100.	

SOLUTION

We may use Beer's Law in the form of eqn. (10.49)

$$\tau_\lambda = \exp(-\rho\kappa_\lambda L) \tag{10.49}$$

Setting $\tau_\lambda = 0.10$ and solving for L:

$$L = -\frac{\ln \tau_\lambda}{\rho\kappa_\lambda} = -\frac{\ln(0.10)}{\rho\kappa_\lambda} = \frac{2.303}{\rho\kappa_\lambda}$$

We may add a column to the table for the value of L. We also convert L from cm to m for convenience (dividing L by 100):

λ (μm)	$\rho\kappa_\lambda$ (cm^{-1})	L (m)	*Color*
0.3	0.0067	3.44	
0.4	0.00058	39.71	violet
0.5	0.00025	92.1	green
0.6	0.0023	10.01	orange
0.8	0.0196	1.18	
1.0	0.363	0.063	
2.0	69.1	0.0003	
2.6–10.0	> 100.	< 0.0002	

Colors in the green to violet range are transmitted the farthest. Reds and yellows are quickly absorbed. So, underwater one would see mainly bluish-green light.

Infrared wavelengths (see Table 1.2) are absorbed in a few mm or less of water. As such, water's infrared absorptance will be close to one if the water has any significant depth. From Kirchhoff's law (Section 10.2), the infrared emittance on those wavelengths will also be close to one.

Problem 10.49 The sun has a diameter of 1.3914×10^6 km. The earth has a mean diameter of 12,742 km and lies at a mean distance of 1.496×10^8 km from the center of the sun. (a) If the earth is treated as a flat disk normal to the radius from sun to earth, determine the view factor $F_{\text{sun–earth}}$. (b) Use this view factor and the measured solar irradiation of 1361 ± 0.5 W/m^2 to show that the effective black body temperature of the sun is 5772 K.

The physical data in this problem were updated in v5.20. Sources in Comment 4 below.

Solution

a) The sun has a spherical field of view of which earth is a tiny part. We can treat earth as a circular disk of diameter $D_e = 12,742$ km sitting on the spherical surface, S, at a radius of $R = 1.496 \times 10^8$ km. The entire area of S is $4\pi R^2$, and the fraction of that area occupied by the earth is the view factor:

$$F_{\text{sun–earth}} = \frac{\pi D_e^2/4}{4\pi R^2} = \frac{(12,742)^2}{16(1.496 \times 10^8)^2} = 4.534 \times 10^{-10} \quad \xleftarrow{\text{Answer}}$$

b) Satellite data show that the solar irradiation normal to the sun-earth axis is $q_{\text{irrad}} = 1361$ W/m^2 above the atmosphere. If we treat the sun as a spherical black body of diameter D_s, the heat transfer *from* the sun *to* the disk of earth's diameter, is

$$Q_{\text{sun to earth}} = (\pi D_s^2) F_{\text{sun–earth}} \sigma T_{\text{sun}}^4 = q_{\text{irrad}}(\pi D_e^2/4)$$

(This heat transfer does not consider the [tiny] radiation from the earth to the sun.) Solving for T_{sun},

$$\begin{aligned} T_{\text{sun}} &= \left[\frac{q_{\text{irrad}}(\pi D_e^2/4)}{\pi D_s^2 \sigma F_{\text{sun–earth}}} \right]^{1/4} \\ &= \left[\frac{(1367)(12,742)^2(1000)^2}{4(1.3914 \times 10^6)^2(1000)^2(5.670376 \times 10^{-8})(4.534 \times 10^{-10})} \right]^{1/4} \\ &= 5772 \text{ K} \quad \xleftarrow{\text{Answer}} \end{aligned}$$

Comment 1: Avoid the temptation to treat the sun and earth as two disks facing each other (Table 10.3, item 3)! That approach understates the area viewed by the sun by a factor of two. That calculation must also be done to very high precision, or by a binomial expansion, to obtain an accurate, nonzero result from the equation in Table 10.3.

Comment 2: Part b) can be done without the view factor and earth's radius by a simple energy balance between the heat leaving the sun's surface and the heat reaching the sphere of radius R:

$$(\pi D_s^2)\sigma T_{\text{sun}}^4 = q_{\text{irrad}}(4\pi R^2)$$

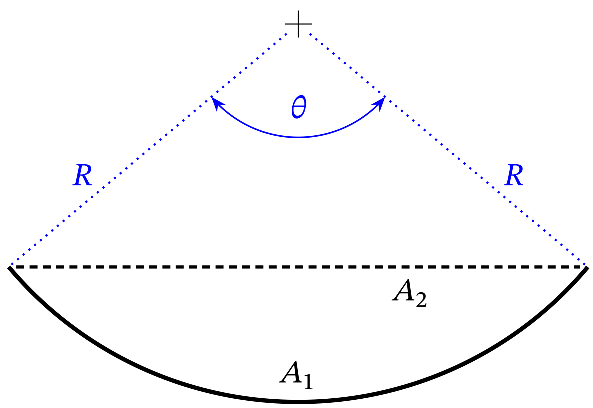
$$T_{\text{sun}} = \left[\frac{q_{\text{irrad}}(4\pi R^2)}{\pi D_s^2 \sigma} \right]^{1/4} = \left[\frac{(1361)(4)(1.496 \times 10^8)^2}{(1.3914 \times 10^6)^2(5.670376 \times 10^{-8})} \right]^{1/4} = 5772 \text{ K}$$

Comment 3: The total solar irradiance varies with seasons by $\pm 3.5\%$, as the distance between earth and sun changes, and rises during the 11 year sun spot cycle (amounting to about 0.1%). A mean annual value of 1367 W/m^{2}K was in use ca. 1982. As satellite instrumentation has improved, the value has been adjusted. Recent data are lower, 1361 ± 0.5 W/m^{2}K, and are referenced to a "quiet sun" condition, with minimal sun spot activity. At the time of this writing, NASA had an ongoing mission to measure solar irradiance: https://lasp.colorado.edu/home/tsis/data/tsi-data/.

Comment 4: The International Astronomical Union provides the solar data used here: Prša et al., "Nominal Values for Selected Solar and Planetary Quantities: IAU 2015 Resolution B3," *Astronomical Journal* **152**:41, 2016, doi:10.3847/0004-6256/152/2/41. Mean earth radius is due to the International Union of Geodesy and Geophysics (Geodetic Reference System, 1980).

Problem 10.50 A long, section of cylindrical shell has a radius R, but it does not form a complete circle. Instead, the cylindrical shell forms an arc spanning an angle θ less than 180°. Because the shell is curved, the inside surface of the shell (call this surface 1) views itself. Derive an expression for the view factor $F_{1\text{-}1}$, and evaluate $F_{1\text{-}1}$ for $\theta = 30°$.

Solution



Let surface 2 be a flat surface that lies across the open portion of the arc (see figure). Then $F_{2\text{-}2} = 0$, so that $F_{2\text{-}1} = 1$. Reciprocity gives

$$A_1 F_{1\text{-}2} = A_2 F_{2\text{-}1} = A_2 \quad \text{so that} \quad F_{1\text{-}2} = \frac{A_2}{A_1}$$

Then

$$F_{1\text{-}1} = 1 - F_{1\text{-}2} = 1 - \frac{A_2}{A_1}$$

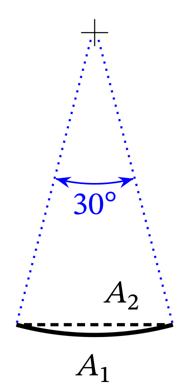
From geometry, the area of the curved surface 1 is $A_1 = \theta R$ per unit length. The area per unit length of the *flat* surface 2 is found by trigonometry, as $A_2 = 2R\sin(\theta/2)$. Hence:

$$\begin{aligned} F_{1\text{-}1} &= 1 - \frac{2R\sin(\theta/2)}{\theta R} \\ &= 1 - \frac{\sin(\theta/2)}{(\theta/2)} \qquad \xleftarrow{\text{Answer}} \end{aligned}$$

For $\theta = 30° = 30(2\pi/360)$ rad $= 0.5236$ rad,

$$F_{1\text{-}1} = 1 - \frac{\sin(0.5236/2)}{(0.5236/2)} = 1 - 0.9886 = 0.0114 \qquad \xleftarrow{\text{Answer}}$$

Comment: Note that $A_2 \to A_1$ as $\theta \to 0$ so that $F_{1\text{-}1} \to 0$ as $\theta \to 0$. The case drawn has $\theta = 100°$ and $F_{1\text{-}1} = 0.1222$. Each point on A_1 sees other points on A_1, but only at large angles relative to the normal direction (see Fig. 10.4). The situation for $\theta = 30°$ is sketched at right.



Problem 10.51 Solve Problem 1.46, finding the Stefan-Boltzmann constant in terms of other fundamental physical constants.

Problem 1.46 Integration of Planck's law, eqn. (1.30) over all wavelengths leads to the Stefan-Boltzmann law, eqn. (1.28). Perform this integration and determine the Stefan-Boltzmann constant in terms of other fundamental physical constants. *Hint:* The integral can be written in terms of Riemann's zeta function, $\zeta(s)$, by using this beautiful relationship between the zeta and gamma functions

$$\zeta(s)\,\Gamma(s) = \int_0^\infty \frac{t^{s-1}}{e^t - 1}\,dt$$

for $s > 1$. When s a positive integer, $\Gamma(s) = (s-1)!$ is just a factorial. Further, several values of $\zeta(s)$ are known in terms of powers of π and can be looked up.

Solution

$$\begin{aligned}
e_b(T) &= \int_0^\infty e_{\lambda,b}\,d\lambda \\
&= \int_0^\infty \frac{2\pi h c_o^2}{\lambda^5\left[\exp(hc_o/k_B T\lambda) - 1\right]}\,d\lambda \\
&= \int_0^\infty \frac{2\pi h \nu^3}{c_o^2\left[\exp(h\nu/k_B T) - 1\right]}\,d\nu \\
&= \frac{2\pi k_B^4 T^4}{h^3 c_o^2}\int_0^\infty \frac{x^3}{e^x - 1}\,dx
\end{aligned}$$

We are given

$$\zeta(s)\,\Gamma(s) = \int_0^\infty \frac{t^{s-1}}{e^t - 1}\,dt$$

For our case, $s = 4$ and $\Gamma(4) = 3! = 6$. Hence:

$$\begin{aligned}
e_b(T) &= \frac{2\pi k_B^4 T^4}{h^3 c_o^2}\,\zeta(4)\,3! \\
&= \frac{12\pi k_B^4}{h^3 c_o^2}\zeta(4)\,T^4
\end{aligned}$$

Zeta is a famous function, and the value at 4 has been established to be:

$$\zeta(4) = \frac{\pi^4}{90}$$

Hence:

$$\begin{aligned}
e_b(T) &= \left(\frac{2\pi^5 k_B^4}{15 h^3 c_o^2}\right) T^4 \\
&= \sigma\,T^4
\end{aligned}$$

which gives us the Stefan-Boltzmann constant in terms of fundamental physical constants:

$$\sigma = \frac{2\pi^5 k_B^4}{15 h^3 c_o^2} \quad \xleftarrow{\text{Answer}}$$

PROBLEM 10.52: The fraction of blackbody radiation between wavelengths of 0 and λ is

$$f = \frac{1}{\sigma T^4} \int_0^\lambda e_{\lambda,b}\, d\lambda \tag{11}$$

a) Work Problem 10.51.
b) Show that

$$f(\lambda T) = \frac{15}{\pi^4} \int_{c_2/\lambda T}^{\infty} \frac{t^3}{e^t - 1}\, dt \tag{12}$$

where c_2 is the second radiation constant, hc/k_B, equal to 1438.8 µm·K.
c) Use the software of your choice to plot $f(\lambda T)$ and check that your results match Table 10.7.

SOLUTION. Following the solution to Problem 10.51:

$$\begin{aligned}
f &= \frac{1}{\sigma T^4} \int_0^\lambda e_{\lambda,b}\, d\lambda && (13)\\
&= \frac{1}{\sigma T^4} \int_0^\lambda \frac{2\pi h c_o^2}{\lambda^5 \left[\exp(hc_o/k_B T\lambda) - 1\right]}\, d\lambda && (14)\\
&= \frac{1}{\sigma T^4} \int_{c_o/\lambda}^{\infty} \frac{2\pi h \nu^3}{c_o^2 \left[\exp(h\nu/k_B T) - 1\right]}\, d\nu && (15)\\
&= \frac{1}{\sigma T^4} \frac{2\pi k_B^4 T^4}{h^3 c_o^2} \int_{c_2/\lambda T}^{\infty} \frac{t^3}{e^t - 1}\, dt && (16)\\
&= \frac{15}{\pi^4} \int_{c_2/\lambda T}^{\infty} \frac{x^3}{e^x - 1}\, dx && (17)\\
&= \frac{15}{\pi^4} \int_0^{\infty} \frac{x^3}{e^x - 1}\, dx - \frac{15}{\pi^4} \int_0^{c_2/\lambda T} \frac{x^3}{e^x - 1}\, dx && (18)\\
&= 1 - \frac{15}{\pi^4} \int_0^{c_2/\lambda T} \frac{x^3}{e^x - 1}\, dx && (19)
\end{aligned}$$

The numerical integration can be done in various ways, depending on the software available. (On a sophisticated level, the last integral can be written in terms of the Debye function which is available in the Gnu Scientific Library.) This equation is plotted in Fig. 1.

PROBLEM 10.53: Read Problem 10.52. Then find the central range of wavelengths that includes 80% of the energy emitted by blackbodies at room temperature (300 K) and at the solar temperature (5772 K).

SOLUTION. From Table 10.7, $f = 0.10$ at $\lambda T = 2195$ µm·K and $f = 0.90$ at $\lambda T = 9376$ µm·K. Dividing by the absolute temperatures gives:

T [K]	$\lambda_{0.1}$ [µm]	$\lambda_{0.9}$ [µm]
300	7.317	31.25
5772	0.380	1.62

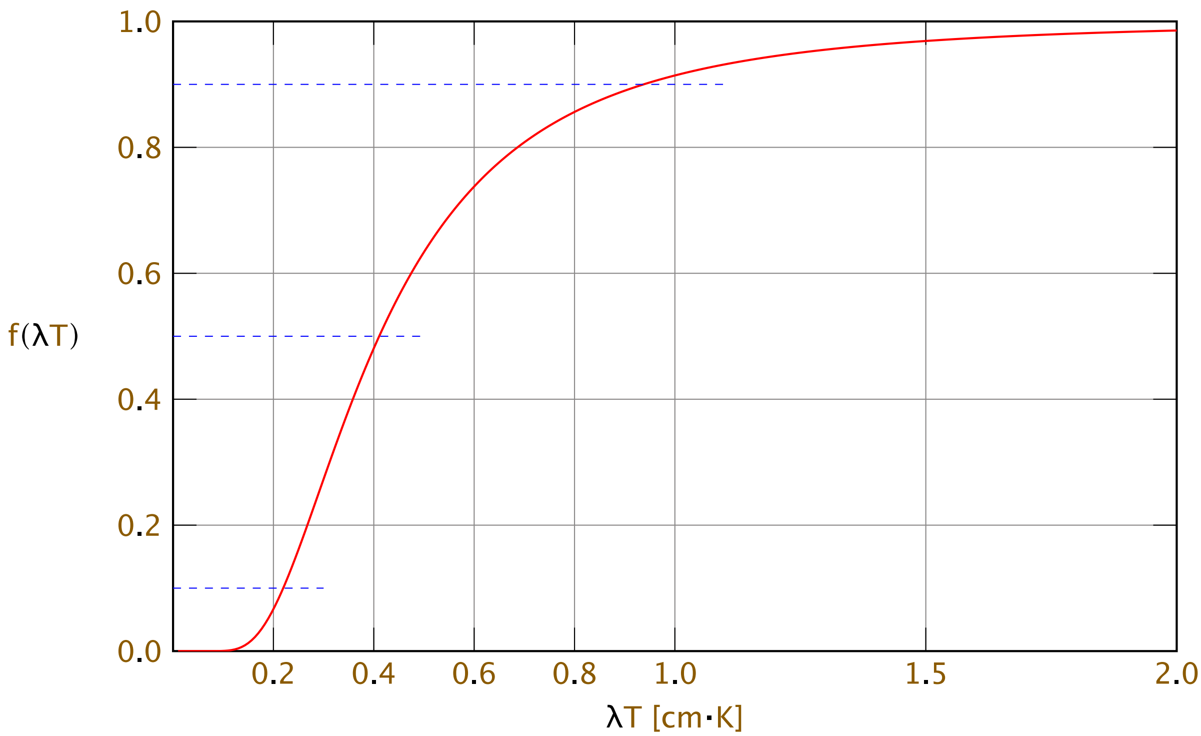


Figure 1. The radiation fractional function

Problem 10.54: Read Problem 10.52. A crystalline silicon solar cell can convert photons to conducting electrons if the photons have a wavelength less than $\lambda_{\text{band}} = 1.11$ µm, the *bandgap* wavelength. Longer wavelengths do not produce an electric current, but simply get absorbed and heat the silicon. For a solar cell at 320 K, make a rough estimate of the fraction of solar radiation on wavelengths below the bandgap? Why is this important?

Solution. The relevant temperature is that of the sun, 5772 K, not that of the solar cell. We approximate the sun as a blackbody at 5777 K, ignoring atmospheric absorption bands.

$$\lambda_{\text{band}} T = (1.11)(5772)\ \mu\text{m} \cdot \text{K} = 6407\ \mu\text{m} \cdot \text{K}$$

Referring to Table 10.7, a bit less than 80% of solar energy is on these shorter wavelengths (with a more exact table, 77%). This is significant because the solar cell can convert less than 80% of the solar energy to electricity; additional considerations lower the theoretical efficiency still further, to less than 50%.

PROBLEM 10.55 Two stainless steel blocks have surface roughness of about 10 μm and $\varepsilon \approx 0.5$. They are brought into contact, and their interface is near 300 K. Ignore the points of direct contact and make a rough estimate of the conductance across the air-filled gaps, approximating them as two flat plates. How important is thermal radiation? Compare your result with Table 2.1 and comment on the relative importance of the direct contact that we ignored.

SOLUTION The gaps are very thin, so little circulation will occur in the air. Heat transfer through the air will be by conduction. Radiation and conduction act in parallel across the gap. The temperature difference across the gap will likely be small, so we may use a radiation thermal resistance. The conductance is the reciprocal of the thermal resistance, per unit area, so $h_{\text{gap}} = h_{\text{cond}} + h_{\text{rad}}$.

Letting the gap width be $\delta = 10$ μm and taking $k_{\text{air}} = 0.0264$ W/m·K, we can estimate

$$h_{\text{cond}} \approx \frac{k}{\delta} = \frac{0.0264}{10 \times 10^{-6}} = 2,640 \text{ W/m}^2\text{K}$$

With eqns. (2.29) and (10.25):

$$\mathcal{F}_{1\text{–}2} = \left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)^{-1} = \left(\frac{2}{0.5} - 1\right)^{-1} = \frac{1}{3}$$

$$h_{\text{rad}} = 4\sigma T_m^3 \mathcal{F}_{1\text{–}2} = 4(5.67 \times 10^{-8})(300)^3(0.3333) = 2.041 \text{ W/m}^2\text{K}$$

Then

$$h_{\text{gap}} = h_{\text{cond}} + h_{\text{rad}} = 2640 + 2.041 = 2,642 \text{ W/m}^2\text{K}$$

This conductance is on the lower end of the range of given in Table 2.1. Conduction through contacting points will add significantly to the heat transfer, although it will be highly multidimensional and not easily calculated. Thermal radiation, however, is negligible.

PROBLEM 10.56 A 0.8 m long cylindrical combustion chamber is 0.2 m in diameter. The hot gases within it are at a temperature of 1200°C and a pressure of 1 atm, and the absorbing components consist of 12% by volume of CO_2 and 18% H_2O. Determine how much cooling is needed to hold the walls at 730°C if they are black. *Hints*: For this small optical depth, the emissivities of CO_2 and H_2O may be added without correction. The gas mixture is approximately ideal, with vol% of a = mole fraction, $x_a = p_a/p$.

SOLUTION The geometrical mean beam length, L_0, for this cylindrical enclosure may be calculated from eqn. (10.58):

$$L_0 = \frac{4\,(\text{volume of gas})}{\text{boundary area that is irradiated}} = \frac{(4)(0.8)\pi(0.2/2)^2}{\pi(0.2)(0.8) + 2\pi(0.2/2)^2} = 0.178 \text{ m} = 17.8 \text{ cm} \tag{10.58}$$

(If we had neglected end effects, from Table 10.4, $L_0 = D = 20$ cm.)

According to the hint, with a total pressure of 1 atm, the partial pressure of CO_2 is 0.12 atm and the partial pressure of H_2O is 0.18 atm.

We can start by finding the gas emissivities. We need the temperatures in kelvin: $T_g = 1200 + 273 = 1473$ K and $T_w = 730 + 273 = 1003$ K. For CO_2,

$$p_a L = (0.12 \text{ atm})(1.013 \text{ bar/atm})(17.8 \text{ cm}) = 2.16 \text{ bar-cm}$$

At 1473 K, Fig. 10.22 gives $\varepsilon^0 = 0.052$. Figure 10.24 gives the pressure correction factor as $C = 0.995$ with $P_E = 1.03$ bar, so that

$$\varepsilon_{CO_2} = C\varepsilon^0 \cong 0.052$$

For H_2O,

$$p_a L = (0.18)(1.013)(17.8) = 3.25 \text{ bar-cm}$$

At 1003 K, Fig. 10.23 gives $\varepsilon^0 = 0.070$. Figure 10.25 gives $C \cong 1.15$ with $P_E = 1.90$ bar, so that

$$\varepsilon_{H_2O} = C\varepsilon^0 \cong 0.081$$

As suggested in the hints, we add the two emissivities to obtain $\varepsilon_g = 0.133$.

The absorptivity is computed using eqn. (10.57). This calculation requires us to find ε_g using different temperatures and partial pressures. We add the emissivities for the two gases, then compute the absorptivity of the mixture.

$$\alpha_g = \varepsilon_g\left(p_a L \frac{T_w}{T_g}, p, T_w\right) \times \left(\frac{T_g}{T_w}\right)^{1/2} \tag{10.57}$$

The adjusted pressure-paths are:

$$p_a L \frac{T_w}{T_g} = \begin{cases} 2.16(1003)/(1473) = 1.47 \text{ bar-cm} & \text{for } CO_2 \\ 3.25(1003)/(1473) = 2.21 \text{ bar-cm} & \text{for } H_2O \end{cases}$$

The emissivities from Figs. 10.22–10.24 are:

$$\varepsilon(1.47 \text{ bar-cm}, 1 \text{ atm}, 1003 \text{ K}) = C\varepsilon^0 = (0.99)(0.062) = 0.061 \quad \text{for } CO_2$$
$$\varepsilon(2.21 \text{ bar-cm}, 1 \text{ atm}, 1003 \text{ K}) = C\varepsilon^0 = (1.15)(0.056) = 0.064 \quad \text{for } H_2O$$

wherein P_E takes the same values as before. Adding these and using eqn. (10.57) we have

$$\alpha_g = \varepsilon_g \left(\frac{T_g}{T_w} \right)^{1/2} = \underbrace{(0.061 + 0.064)}_{=0.125} \sqrt{\frac{1473}{1003}} = 0.151$$

Finally, we are ready to compute the heat transfer from the gas to the wall, which is the cooling load:

$$\begin{aligned} Q_{\text{net}_{\text{g-w}}} &= A_w \left(\varepsilon_g \sigma T_g^4 - \alpha_g \sigma T_w^4 \right) \\ &= [\pi(0.2)(0.8) + 2\pi(0.2/2)^2](5.67034 \times 10^{-8}) \left[(0.133)(1473)^4 - (0.151)(1003)^4 \right] \\ &= 15 \text{ kW} \quad \xleftarrow{\text{Answer}} \end{aligned}$$